MATHEMATICAL MODEL TO CALCULATE THE PERFORMANCE OF LOW POWER VERTICAL AXIS WIND TURBINE

[1]D.SH. AKHMEDOV, [2]D.I. ERYOMIN, [3]N.I. YAGFAROVA, [4]M.K. TURARBEKOV

Doctor of technical science, magister, magister
E-mail: lacp@mail.ru, denis.e@bk.ru, nadiya1006@gmail.com

Abstract- Vertical axis wind turbines do not require airflow direction position control system, which is an advantage over horizontal axis wind turbines. However, wind power absorption efficiency of vertical axis wind turbines is much less, so it is important to assess the capacity of the wind turbine.
This article describes mathematical model to calculate the performance of low power vertical axis wind turbine.

Index Terms- Mathematical Model, Power Factor, Swept Area.

I. INTRODUCTION

Mathematical model

The mathematical model is based on the analysis of the wind turbine characteristics and the area. Theoretical studies are designed to assess the capacity of the perfect wind turbine.

Calculation of the input data

Input data determine the wind turbine and area characteristics and are divided into meteorological station data and wind turbine parameters.
The parameters of the wind turbines are the following:

- Turbine height (*h* m);
- Rotor diameter (*d* m);
- Blade length (*l* m);
- Blades number (*n*);
- Nominal power ($P_{ном}$ kW);
- Rated wind speed ($\upsilon_{ном}$ m/s).

Meteorological station data include annual average wind speed on the territory of Kazakhstan, provided by the web-site http://atlas.windenergy.kz/. Web-version was developed by the UNDP project "Kazakhstan – Wind Power Market Development Initiative." [1]

Power factor calculation

The energy absorbed by the wind-driven power plant $E_{ген}$ is the difference between air flow energy before installation $E_1 = \frac{m\upsilon_1^2}{2}$ and after it $E_2 = \frac{m\upsilon_2^2}{2}$ [2]:

$$E_{ген} = E_1 - E_2 \quad (1)$$

Owing to the **equation of continuity of air flow stream** air flow mass is constant. Consequently:

$$m = \rho \cdot \upsilon_1 \cdot S = const \quad (2)$$

S - swept area of the wind wheel, geometric projection of wind turbines surface area on a plane perpendicular to the wind velocity vector [3].

ρ - air density (1,225 $кг / м^3$).

After transformations, wind turbines capacity formula becomes:

$$P_{ген} = \frac{\rho S \cdot \upsilon_1^3}{2} \cdot \frac{\left(1 + \frac{\upsilon_2}{\upsilon_1} - \frac{\upsilon_2^2}{\upsilon_1^2} - \frac{\upsilon_2^3}{\upsilon_1^3}\right)}{2} \quad (3)$$

The first part of the equation (3) represents wind flow power formula $P_{вет} = \frac{\rho S \cdot \upsilon_1^3}{2}$, and the second part of the equation is wind power efficiency C_p.

$$C_p = \frac{\left(1 + \frac{\upsilon_2}{\upsilon_1}\right) \cdot \left(1 - \frac{\upsilon_2^2}{\upsilon_1^2}\right)}{2} \quad (4)$$

Replace the ratio of speeds υ_2 and υ_1 with the parameter β and the equation (4) becomes:

$$C_p = \frac{(1+\beta) \cdot (1-\beta^2)}{2} \quad (5)$$

Wind power efficiency C_p is the main factor characterizing wind-powered generator efficiency. Consider power efficiency as a function [4]:

$$C_p(\beta) = \frac{(1+\beta) \cdot (1-\beta^2)}{2} \quad (6)$$

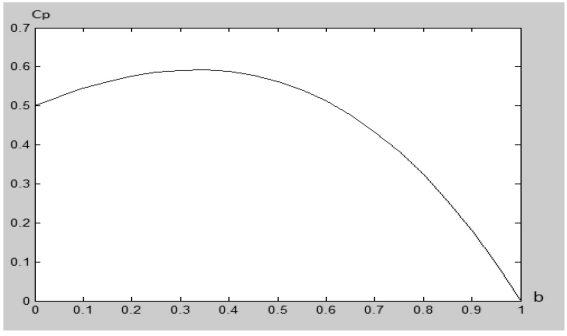


Figure 1 shows a function graph (6):

Figure 1 – Power efficiency function graph Solving the problem by finding extrema of the function determines the value $\beta = \frac{1}{3}$. Substituting this value into the main function (6) we obtain the peak value which power factor can take C_p.

$$C_p = \frac{16}{27} \approx 0,593 \quad (7)$$

Resulting value is called Betz criterion. Practical results of the factor take smaller value and are withdrawn by the nominal power ratio of the wind turbine to the wind flow power:

$$C_p = \frac{2 \cdot P_{ном}}{\rho S \cdot \upsilon_{ном}^3} \quad (8)$$

Calculation of the wind turbine swept area

Calculation of the vertical axis wind turbine swept area differs from calculation methods for the horizontal axis type and needs special attention.

Figure 2 shows horizontal plan of the wind turbine, which is considered as an equilateral polygon with the sides $A_1A_2, ..., A_{n-1}A_n$.

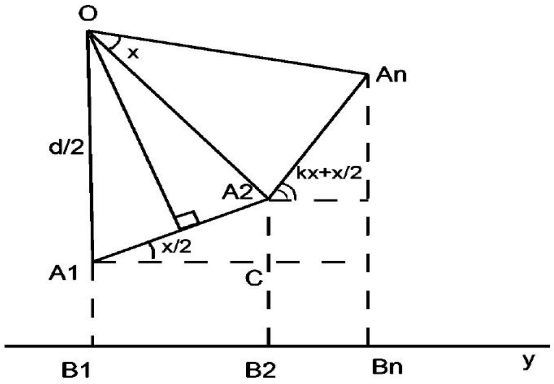


Figure 2 – Horizontal plan

This variant of the swept area calculation does not take account of the blade width, and its projection as a consequence, since most manufacturers do not provide the information about the width of the blade. Due to man-made assumption, the formula for the area calculation will give a smaller result, which compensates the wind turbine efficiency error on the other hand.

When the number of blades is even the formula to calculate the swept area is enumerated by multiplying the blade length (l) by the rotor diameter (d):

$$S = d \cdot l \quad (9)$$

When the number of blades is odd the formula becomes:

$$S = 2l \cdot \sum_{k=0}^{m} d \cdot \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2} + kx\right) \quad (10)$$

where $m = \frac{n}{4}$ is the number of sides in a quarter of the polygon.

Calculation of wind velocity at the height of the wind turbine

Taking into account the fact that the largest part of Kazakhstan territory is the steppe, velocity defect law of steppe is used to assess the wind velocity change at the height of the wind turbine [5].

Where

υ - wind velocity at the height of the wind turbine;

υ_0 - wind velocity obtained from the meteorological station data;

H - the height of meteorological station assessment, 80 m;

h - the height of the wind turbine.

The formula to calculate the wind turbine power in the area

After defining all the parameters for power calculation the main formula can be developed:

$$P = P_{ном} \cdot \frac{\left(\upsilon_0 \cdot \left(\frac{h}{H}\right)^{0.2}\right)^3}{\upsilon_{ном}^3} \quad (12)$$

This formula calculates power output of the wind turbine per calendar day.

Consider the calculation of the wind turbine performance shown in Figure 3 and having the following characteristics [6]:

Turbine height (h m) = 15 m
Rotor diameter (d m) = 4 m
Blade length (l m) = 4 m
Blades number (n) = 3
Nominal power ($P_{ном}$ kW) = 5 kW
Nominal wind velocity ($\upsilon_{ном}$ m/s) = 12 m/s
Latitude = 61,648°
Longitude = 48,808°



Figure 3 – Vertical axis wind-powered generator F16-05

Wind velocity at the height of 80 m (υ_0) = 6.4 m/s.

Calculation of the wind velocity at the height of the wind turbine:

$$\upsilon = 6,4 \cdot \left(\frac{15}{80}\right)^{0,2} = 4,58 \text{ m/s} \quad (13)$$

Swept area calculation:

$$S = 2 \cdot 4 \cdot 4 \cdot \sin\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{3}\right) = 13,85 м^2 \quad (14)$$

Power factor calculation:

$$C_p = \frac{2 \cdot 5000}{1,225 \cdot 13,85 \cdot 12^3} = 0,34 \quad (15)$$

Power calculation:

$$P = 5000 \cdot \frac{4,58^3}{12^3} = 278 \text{ W} \quad (16)$$

CONCLUSION

This paper is a study on the creation of mathematical model for the calculation of performance and characteristics of the vertical axis wind turbines.

Peak value which power factor can take was defined. Practical results of the power factor did not exceed Bets criterion and on the average took on a value less than 0.4.

A distinction between the vertical axis wind turbines and horizontal axis ones is the calculation of the swept area of the rotor. The solution of this task was based on the basic geometrical calculation methods. Two formulas for calculating the surface area with odd and even number of blades were developed.

The cube correspondence between the wind velocity and power was determined. Accordingly, the change of wind velocity has a strong influence on the power generated.

REFERENCES

[1] Kazakh Electrical Association, Committee on the renewable power sources [Digital resource] http://www.windenergy.kz.

[2] International Journal of Mathematics and Soft Computing Vol.4, No.1 (2014), 61 – 66, P. O. Ochieng, A. W. Manyonge, A. O. Oduor " Mathematical analysis of tip speed ratio of a wind turbine and its effects on power coefficient".

[3] National State Standard 27.002-89 The reliability of the technique. Basic concepts. Terms and definitions.

[4] University of Illinois at Urbana-Champaign, 216 Talbot Laboratory, USA, Magdi Ragheb and Adam M. Ragheb "Wind Turbines Theory - The Betz Equation and Optimal Rotor Tip Speed Ratio".

[5] Infinite energy. Book 2. Wind-power engineering / V.S. Krivtsov, A.M. Oleinikov, A.I. Yakovlev. — Kharkov, Ukraine "KAI", 2004.

[6] Vertical wind-powered generators Fairwind, [Digital resource], http://winder.ua/ru/

★★★