QUASI-MONTE CARLO SIMULATION OF NONLINEAR EULER-BERNOULLI BEAMS WITH DYNAMIC ABSORBERSUNDER UNCERTAIN MOVING LOADS

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Abstract - In this paper, a Quasi-Monte Carlo Simulation (QMCS) method is used to perform a stochastic analysis of a nonlinear Euler-Bernoulli beam with a dynamic Vibration Absorber loaded with uncertain moving loads. Dynamic absorbers and system of nonlinear beams are modeled by simplified governing equations coupled by each other and solved numerically by Finite Difference Method (FDM). A low-discrepancy sampling sequence is used to incorporate the uncertainty in the moving load excitation via QMCS. The mean deflection, standard deviation, probability density function (PDF) and the response range are used to assess the stochastic response, for each coefficient of variation (COV). It is observed that the higher levels of uncertainty cause a large increase in the beam response variance, whereas the beam response means are only slightly different from the deterministic solutions. A COV of 15% resulted in the maximum beam deflection of about 147.51% of the deterministic deflection, highlighting the significance of stochastic analysis in the design of structures. The outcomes demonstrate the efficiency and accuracy of the proposed QMCS method for nonlinear beam structures with moving uncertain loads.

Keywords: Quasi-Monte Carlo Simulation, Stochastic Dynamic Analysis, Uncertainty quantification, Nonlinear Euler–Bernoulli beams, Dynamic vibration absorbers, Moving loads.

I. INTRODUCTION

The dynamic response of beams carrying moving loads has been a topic of great interest in structural and mechanical engineering and is prevalent in many engineering designs such as bridges, railway tracks, crane systems, and transportation infrastructures. Moving loads generate complex vibration responses which could result in resonance, fatigue damage and structural failure if vibration control measures are not employed [1–3]. Dynamic absorbers are therefore widely adopted and proven to be a useful tool to reduce excessive beam vibrations in such systems.

There are several sources of uncertainty that can influence beams equipped with dynamic absorber systems, such as the uncertainty in the material properties, the uncertainty in the geometric dimensions, the uncertainty in the properties of the dynamic absorber system and the uncertainty in the moving load parameters such as magnitude and velocity [4]. Additional uncertainties resulting from assumptions of the model, numerical approximations, boundary conditions, manufacturing tolerances, and installation imperfections. These factors can affect the dynamic response of the structure and cannot be captured by deterministic methods to model all aspects of structural response. For the realistic structural analysis and reliability assessment, therefore, the stochastic modeling techniques are gaining importance.

In recent years, considerable research efforts have focused on the stochastic modeling of beam structures with uncertain parameters. Previous studies investigated the effects of random material properties, stochastic moving loads, and dynamic excitations on beam vibrations using stochastic finite element methods and Monte Carlo simulation approaches [5–16]. However, most existing works considered beam systems without vibration control devices or neglected the fully coupled interaction between nonlinear beam behavior, moving loads, and dynamic absorbers under uncertain conditions. Moreover, conventional Monte Carlo-based methods often require high computational effort and may suffer from slow convergence when applied to nonlinear stochastic systems [17].

To address these limitations, the present study employs Quasi-Monte Carlo Simulation (QMCS) to investigate the stochastic dynamic response of nonlinear Euler–Bernoulli beams equipped with dynamic absorbers and subjected to uncertain moving loads. The QMCS approach utilizes low-discrepancy sampling sequences to improve computational efficiency and convergence accuracy compared with traditional Monte Carlo techniques. To the authors' knowledge, limited studies have examined the stochastic nonlinear interaction between moving loads and beam with dynamic absorber systems using QMCS-based formulations.

The rest of this paper is organized as follows. In Section 2, the QMCS method is introduced. The mathematical formulation of the nonlinear beam with dynamic absorber model and thestochastic formulation and the Stochastic Finite Difference Methodis presentedin Section 3.In Section 4 the numerical implementation procedure is described,

while Section 5 discusses the obtained numerical results. Finally, the main conclusions are summarized in Section 6.

II. QUASI-MONTE CARLO SIMULATION (QMCS)

Quasi-Monte Carlo Simulation (QMCS) is an advanced stochastic numerical simulation methodthat can provide a more efficient numerical evaluation of systems with uncertain parameters than traditional Monte Carlo Simulation (MCS). QMCS uses low-discrepancy sequences, such as Sobol and Halton sequences instead of the traditional random sampling to provide more uniform sampling and faster convergence rate [18,19]. Recently, QMCS has been widely used to study the effect of uncertainties on the dynamic response of beams and vibration-control systems in the realm of structural dynamics [17,20].

III. STOCHASTIC FORMULATION AND NUMERICAL SOLUTION FOR NONLINEAR BEAM WITH DYNAMIC ABSORBER

3.1. Stochastic Formulation

The system analyzed in this paper is a simply supported nonlinear beam attached to a small mass via a linear spring and a linear viscous damper, which is excited by a moving point load as shown in Fig.1 [21, 22].

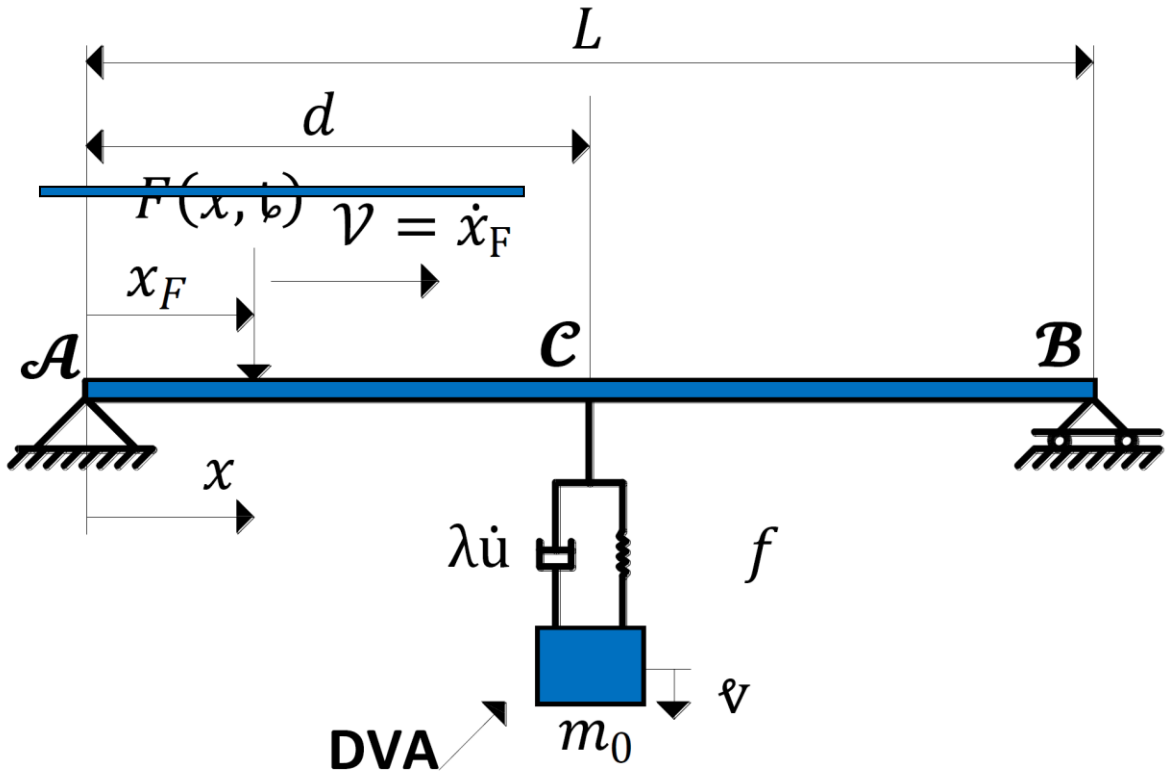


Fig.1. The model of beam with dynamic absorber.

The dynamic behavior of the beam is governed by the partial differential equation (PDE) presented in Eq. (1), together with the boundary conditions defined in Eq. (2) and the initial conditions given in Eq. (3). The governing equations of the deterministic system can be expressed as follows [23].

$$EIw_{xxxx} + mw_{tt} - \frac{EA}{L}\left(\frac{1}{2}\int_0^L (w_x)^2 dx\right) w_{xx} + [f(u) + \lambda u_t(t)]\delta(x-d) = F(x,t), x \in (0,L), t > 0, \quad (1)$$

$$w(0,t) = 0, \quad w(L,t) = 0, \quad w_{xx}(0,t) = 0, \quad w_{xx}(L,t) = 0, \quad (2)$$

$$w(x,0) = 0, \quad w_t(x,0) = 0. \quad (3)$$

The vibration absorber's governing equation is given by:

$$m_0 v_{tt}(t) - f(u) - \lambda u_t(t) = 0$$
$$v(0) = 0, \quad v_t(0) = 0, \quad t > 0, \quad (4)$$

$$u(t) = w(d,t) - v(t), \quad f(u) = ku \quad (5)$$

where,

$[f(u) + \lambda u_t]\,\delta(x-d)$is the force from the vibration absorber. In this case, f(u)represents the

restoringstiffness force, as given in Eq. (5), while λu_t corresponds to the viscous damping force. The Dirac delta function $\delta(x-d)$is used to mark the position of the absorber along the beam, andthe function$F(x,t)$is the external moving load on the beam. The motion of the vibration absorber is governed by Eq. (4). In addition, $w(x,t)$is the transverse displacement of the beam in the downward direction, and$v(t)$is the absolute displacement of the attached massm_0, located at$x = d$.The viscous damping coefficient is denoted by λ, and the mass of the absorber ism_0, which is relatively small compared with the total beam mass. Increasing the absorber mass gives better vibration reduction, but also to higher static beam deflection,so an optimum mass ratio has to be used. In the current research, the mass of the absorber mass is taken as 5% of the beam mass [21].

In order to analyze the coupled system of dynamics (1) and (4), the above partial differential equation is projected onto a full orthonormal basis. The eigenfunctions for linear operator of a simply supported beam with no attachments are used for this purpose.

$$\varphi_r(x) = \frac{\sqrt{2}}{\sqrt{mL}}\sin\left(\frac{r\pi x}{L}\right), \omega_r = r^2\pi^2\frac{\sqrt{EI}}{\sqrt{mL^4}} \quad , r = 1, 2, \ldots \quad (6)$$

where $\omega_r$denotes the natural frequency ofr^{th}mode, and its eigenfunctions $\varphi_r(x)$ satisfies orthonormality conditions.

$$\int_0^L m\varphi_i(x)\varphi_j(x)dx = \delta_{ij}, \int_0^L \varphi_i(x)\left(EI\varphi_j''(x)\right)'' dx = \omega_j^2\delta_{ij} \quad , i, j = 1, 2, \ldots \quad (7)$$

Here,δ_{ij} denotes the Kronecker deltafunction. Accordingly, the transverse displacement of the beam can be expressed as follows:

$$w(x,t) = \sum_{r=1}^{\infty} a_r(t)\varphi_r(x) \quad (8)$$

In practical engineering applications, the beam with dynamic absorber system is influenced by several sources of uncertainty. These uncertainties may originate from fluctuations in load amplitude, moving speed, and external dynamic excitations. Consequently, a stochastic formulation is adopted to provide a more realistic representation of the system behavior. The deterministic governing equations are therefore modified to incorporate these uncertainties as follows:

$$\begin{aligned} &a_{tt_p}(t;\theta) + 2\xi\omega_p a_{t_p}(t;\theta) + \omega_p^2 a_p(t;\theta) \\ &\quad + \frac{A}{2mIL}\omega_p^2 a_p^3(t;\theta) \\ &\quad + \left\{k\left[\sum_{r=1}^{\infty} a_r(t;\theta)\varphi_r(d) - v(t;\theta)\right]\right. \\ &\quad + \lambda\left[\sum_{r=1}^{\infty} a_{t_r}(t;\theta)\varphi_r(d)\right. \\ &\quad \left.\left. - v_t(t;\theta)\right]\right\}\varphi_p(d) = \tilde{F}(t;\theta) \quad , p = 1,2,\ldots \end{aligned} \quad (9)$$

$$\begin{aligned} &m_0 v_{tt}(t;\theta) - k\left[\sum_{r=1}^{\infty} a_r(t;\theta)\varphi_r(d) - v(t;\theta)\right] \\ &\quad + \lambda\left[v_t(t;\theta) - \sum_{r=1}^{\infty} a_{t_r}(t;\theta)\varphi_r(d)\right] \\ &\quad = 0 \end{aligned} \quad (10)$$

The transient response of the system is obtained by integrating the dynamical equations given in Eqs. (9) and (10) after truncating the expansion series in Eq. (8). The required number of retained terms is selected based on a convergence study, and three modes are found to be sufficient for the present analysis.

3.2. Solution of the problem Using FDM

Applying the Finite Difference Method (FDM) to Eqs. (9) and (10) yields the following discretized equations:

$$\begin{aligned} a^{n+1} = (\Delta t)^2 * &\left[\bar{F}(t) - \frac{A}{2mIL}\omega_p{}^2(a^n)^3\right. \\ &- \left\{k * \left[\sum_{r=1}^{\infty} a_r^n\varphi_r(d) - v^n\right]\right. \\ &+ \lambda \\ &* \left[\sum_{r=1}^{\infty}\left(\frac{a_r^n - a_r^{n-1}}{\Delta t}\right)\varphi_r(d)\right. \\ &\left.\left.\left. - \left(\frac{v^n - v^{n-1}}{\Delta t}\right)\right]\right\}\varphi_p(d) \\ &\left. - \left[\omega_p{}^2 + \left(\frac{2\xi\omega_p}{\Delta t}\right) - \frac{2}{(\Delta t)^2}\right] * a^n\right. \\ &\left. - \left[\frac{1}{(\Delta t)^2} - \left(\frac{2\xi\omega_p}{\Delta t}\right)\right] * a^{n-1}\right], \quad p = 1,2,\ldots \end{aligned} \quad (11)$$

$$\begin{aligned} v^{n+1} &= \left(\frac{(\Delta t)^2}{m_0}\right) * \left[k * \left[\sum_{r=1}^{\infty} a_r^n\varphi_r(d) - v^n\right] - \lambda\right. \\ &* \left[\left(\frac{v^n - v^{n-1}}{\Delta t}\right) - \sum_{r=1}^{\infty}\left(\frac{a_r^n - a_r^{n-1}}{\Delta t}\right)\varphi_r(d)\right] + m_0 \\ &\left. * \left(\frac{2v^n - v^{n-1}}{(\Delta t)^2}\right)\right] \end{aligned} \quad (12)$$

IV. NUMERICAL IMPLEMENTATION

Equations (11) and (12) numerically solved using MATLAB [24]. The beam domain was discretized using spatial and temporal step sizes of$\Delta x = 0.1\text{m}$ and $\Delta t = 0.001\text{s}$, respectively. The beam parameters were selected as follows: beam length $L = 17.4$ m, Young's modulus $= 209 \times 10^9\text{Pa}$, second moment of area $I = 0.5358851675\text{m}^4$, cross-sectional area $A = 6.8\text{m}^2$, and mass per unit length $m = 16300\text{kg/m}$. The beam was subjected to a moving point load of magnitude $F_0 = 215732\text{N}$, traveling at constant velocity $\mathcal{V} = 73.89\text{m/s}$. A linear dynamic absorber with mass$m_0 = 14181\text{kg}$, stiffness $k = 100 \times 10^6\text{N/m}$, and viscous damping coefficient $\lambda = 0.05\text{Ns/m}$was attached at $x_d = 8.352\text{m}$. The beam's response was evaluated at $x = 8.7\text{m}$, with a structural damping ratio $\xi = 0.04$. The simulation was carried out over a time duration of$t = 3$ s, and the maximum deflection was observed at $t = 0.118$ s. These parameter values were adopted from [25] to validate the present numerical solution against previously reported results, as presented in Table 1.

Nonlinear beam	Maximum deflection without any attachment	Maximum deflection with dynamic vibration absorber
Nadia Parandeh Afshar, et al. [25]	0.2357	0.2083
Present study	0.2356928	0.20837
|Difference|(%)	0.0030	0.0336

Table 1:Comparison between the maximum beam deflections obtained in this paper and those reported by Nadia Parandeh Afshar et al. [25]

V. RESULTS AND DISCUSSION

The stochastic dynamic response of the nonlinear Euler-Bernoulli beam with a dynamic vibration absorber and loaded with uncertain moving loads was investigated using the Quasi-Monte Carlo Simulation (QMCS) approach.The probabilistic analysis was performed at each discretized grid point along the beam domain. The statistical response quantities considered in the present study include the mean deflection μ_w,standard deviation σ_w,estimated minimum value (Min), estimated maximum value (Max), response range R,and deterministic deflection w_D.The main statistical characteristics of the maximum beam deflection (w_{max}) for different coefficients of variation (COV), denoted by $\mathcal{K}$, are summarized in Table 2.

To illustrate the stochastic behavior of the system, several graphical results were obtained. Figures 2(a) and 2(b) present the variation of the mean deflection and the standard deviation along the beam length for COV values of 0.05, 0.10, and 0.15, respectively. In addition, Figure 3(a) illustrates the probability density function (PDF) of the maximum beam deflection obtained using QMCS, while Figure 3(b) shows the variation of the mean deflection and standard deviation at the observation point $x = 8.7$ m for different uncertainty levels.

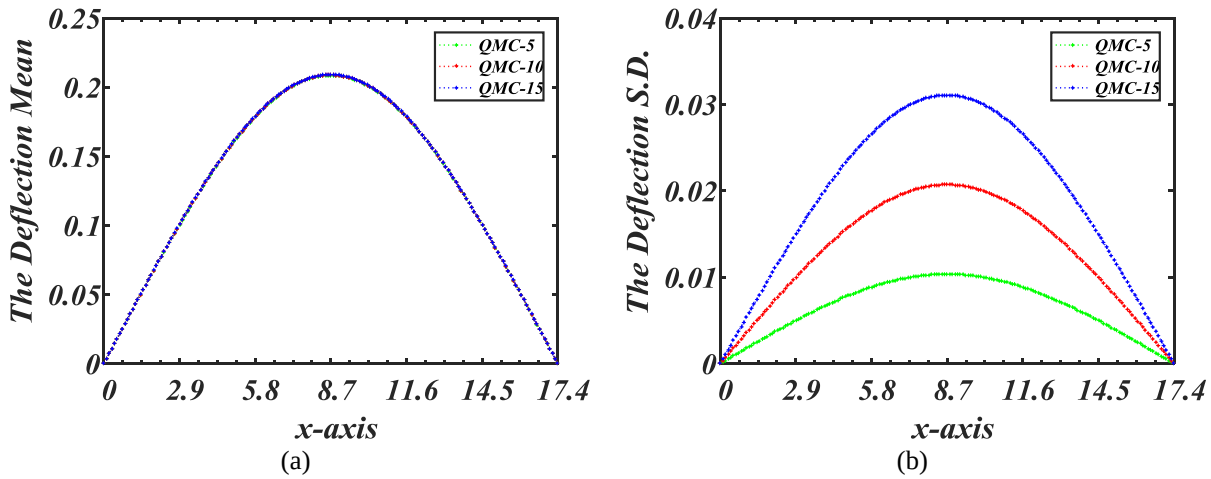
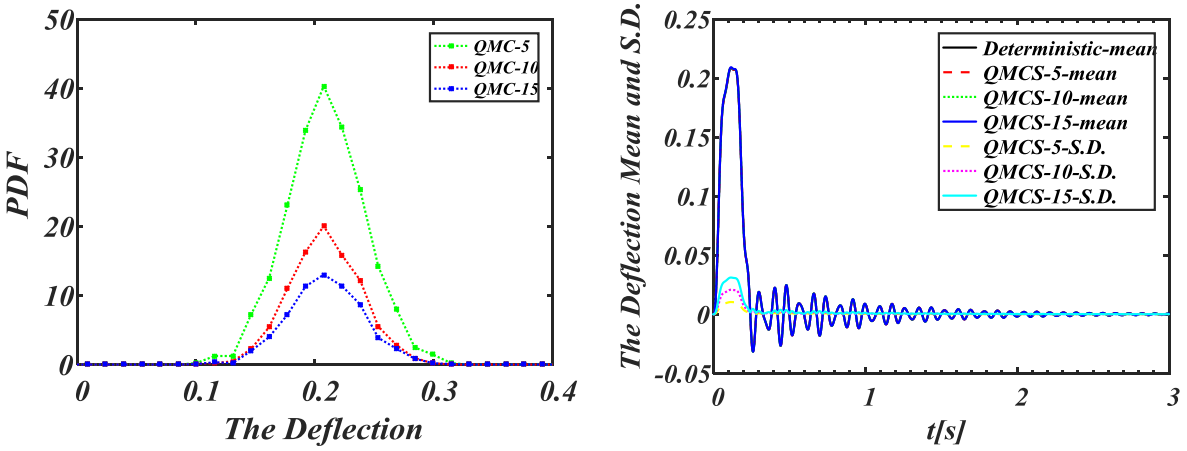


Fig. 2. (a) The mean value of deflection using QMCS method, (b) The S.D. of deflection using QMCS method.



(a) (b)

Fig. 3. (a) The PDF of the maximum deflection using QMCS, (b) The deflection mean and S.D. at x = 8.7m using QMCS with OV = 5%, 10%, 15% .

COV	Solution approach	$\mu_{w_{max}}$	$\mu_{w_{max}}/w_D$	$\sigma_{w_{max}}$	$\sigma_{w_{max}}/\mu_{w_{max}}$	Min	Max	R
$\mathcal{K}$ = 5%	QMC-5	0.2086	1.0014	0.0104	0.0497	0.1716	0.2417	0.0701
$\mathcal{K}$ = 10%	QMC-10	0.2089	1.0028	0.0207	0.0992	0.1349	0.2751	0.1402
$\mathcal{K}$ = 15%	QMC-15	0.2092	1.0041	0.0311	0.1486	0.0983	0.3086	0.2103

Table 2:The beam's key statistics for stochastic w_{max}.

The obtained results clearly demonstrate the considerable influence of load uncertainty on the dynamic response of the nonlinear beam with dynamic absorber system. Based on the QMCS results shown in Table 2 and Figs. 2–3, the mean maximum deflection $\mu_{w_{max}}$ increased slightly compared with the deterministic deflection w_D. The observed increases were approximately 0.14%, 0.28%, and

0.41% for COV values of 5%, 10%, and 15%, respectively. Although the increase in the mean response is relatively small, the variability and dispersion of the response become significantly larger as the uncertainty intensity increases.

The ratio $\sigma_{w_{max}}/\mu_{w_{max}}$ was found to be approximately 99% of the prescribed COV values, indicating strong agreement between the input uncertainty and the resulting stochastic response. This observation confirms the capability and robustness of the QMCS method in accurately capturing the probabilistic characteristics of nonlinear structural systems subjected to uncertain moving excitations. Furthermore, the resulting probability density functions exhibit an approximately Gaussian distribution, which simplifies the prediction of response statistics for other uncertainty levels and reduces the computational effort required for probabilistic assessment.

A substantial spread in the beam deflection response was also observed with increasing uncertainty levels. The estimated minimum and maximum deflections varied within the ranges of (82.26%, 115.87%), (64.58%, 131.69%), and (46.99%, 147.51%) of the mean deflection for COV values of 5%, 10%, and 15%, respectively. Correspondingly, the response range R increased from 0.0701 to 0.2103 as the coefficient of variation increased. These findings clearly indicate that the uncertainty level has a pronounced effect on both the variability and dispersion of the beam response.

Additionally, the presented results highlight the importance of incorporating stochastic load modeling into the analysis and design of beam systems subjected to moving loads. In particular, for a COV value of 15%, the maximum beam deflection reached approximately 147.51% of the deterministic response. This significant increase demonstrates that deterministic analysis may considerably underestimate the actual structural response and may therefore lead to unconservative and potentially unsafe structural designs. Consequently, neglecting uncertainty effects in moving load problems may compromise the structural safety and serviceability of engineering systems.

In general, the present QMCS-based framework provides an efficient and reliable stochastic analysis tool for predicting the probabilistic dynamic behavior of nonlinear beam systems with vibration absorbers under uncertain moving load excitations. The adopted QMCS approach offers improved convergence characteristics and computational efficiency compared with conventional Monte Carlo methods, making it suitable for large-scale stochastic structural dynamics applications.

CONCLUSIONS

In this study, the stochastic dynamic response of a nonlinear Euler-Bernoulli beam with a dynamic Vibration Absorber loaded with uncertain moving loads was investigated using the Quasi-Monte Carlo Simulation (QMCS) approach. The governing nonlinear equations were formulated using modal analysis and solved numerically through the Finite Difference Method (FDM). The probabilistic behavior of the system was evaluated for different coefficients of variation representing uncertainty in the moving load excitation.

The obtained results demonstrated that uncertainty in the moving load parameter has a significant influence on the beam dynamic response. Although the mean deflection showed only slight increases compared with the deterministic response, the variability and dispersion of the response increased considerably with increasing uncertainty levels. The ratio between the standard deviation and the mean response closely matched the prescribed coefficients of variation, confirming the robustness and accuracy of the adopted QMCS framework. In addition, the resulting probability density functions exhibited approximately Gaussian distributions for all investigated cases.

The numerical results further revealed that the maximum beam deflection may substantially exceed the deterministic prediction under high uncertainty levels. For a coefficient of variation of 15%, the maximum deflection reached approximately 147.51% of the deterministic response, indicating that deterministic analysis may underestimate the actual structural response and potentially lead to unsafe structural designs. Moreover, the stochastic response range increased significantly as the uncertainty intensity increased, highlighting the importance of uncertainty quantification in vibration and reliability analysis of beam systems subjected to moving loads.
In general, the present study demonstrates that the QMCS method provides an efficient and reliable stochastic computational framework with improved convergence and reduced computational effort compared with conventional Monte Carlo techniques. The proposed methodology can therefore serve as an effective tool for the probabilistic analysis, design, and safety assessment of nonlinear beam structures with dynamic absorbers under uncertain operating conditions.

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