A TWO-PHASE SOLUTION METHOD FOR THE INTEGRATED HUB LOCATION AND VEHICLE ROUTING PROBLEM

JI UNG SUN

Department of Industrial and Management Engineering, Hankuk University of Foreign Studies, South Korea
E-mail: jusun@hufs.ac.kr

Abstract- This paper deals with a hub location-routing problem which integrates the hub location problem and the multi-depot vehicle routing problem. To solve these problems simultaneously we present asolution framework which divides the problem into two sub-problems: the hub location problem in the first stage and the multi-depot vehicle routing problem in the second one. The two-phase approach hybridizes two meta-heuristics, an ant colony optimization algorithm for the first stage with a genetic algorithm for the second stage, respectively. We determine the locations of prescribed *p* hubs within a set of candidate potential hubs, the allocation of each customer node to each selected hub as well as the vehicle routes for each hub in order to minimize the sum of the fixed hub costs, traveling costs and fixed vehicle costs. The performance of the proposed solution method is investigated through a computational experiment.

Keywords- Hub Location-Routing, Meta-Heuristic, Ant Colony Optimization, Genetic Algorithm.

I. INTRODUCTION

The location-routing problem (LRP) is one of the most important planning issues in designing and operating supply chain networks, communication networks and air cargo networks in order to facilitate the traffic among nodes.The LRP is commonly tackled in two phase: facility location for a long term planning and vehicle routingfor the operational issues. These two components can be treated separately, but may lead to suboptimal solutions [1].Classical facility location models assume that each customer is served on a straight-and-back basis, in other words, the flow of commodities from each customer is directly connected with the facilities. However, in many applications arising in real practice, multiple customers are served in a single route and distribution cost depends on the sequence of customers on the route [2].
In this case, to compute accurately the distribution cost, the LRP should be solved simultaneously. The LRP integrates facility location problem which determines the depot locations and allocation of customers to each selected depot, and vehicle routing decision which constructs the vehicle routes foreach selected depot.

This study is motivated by the observation made in the parcel service.Hub-and-spoke structure is a well-known configuration implemented in parcel delivery systems. It provides economies of scale by consolidating the traffic flows at the hubs then transferring to another hub and distributing to the destinations rather than carrying from origins to destinations directly. In general, the flow of each origin-destination pair consists of three components, in other words, collection, transfer and distribution respectively. In the standard LRP, we assume a structure of facilities serving a number of customers, who are connected to their depot by means of vehicle tours. No routes connect facilities to each other. On the other hand, this research considers more complex network structure than the standard LRP.

We consider a variant of the LRP known as thehub location-routing problem (HLRP). Kuby and Gray [3] consider the hub network design problem where they determine the least cost set of direct or stopover routes for the traffic from a given set of points to a predetermined hub. Aykin [4] addresses the HLRP in continuous solution space where the hubs can be located anywhere in the service region encountered in air cargo transportation. He presents an algorithm solving a series of shortest path problems and a multi-facility location problem in an iterative manner. Even though these two studies consider routing possibilities, their approaches are restricted in parallel with their assumptions. Nagi and Salhi [5] introduce the many-to-many location-routing problem similar to the HLRP where several customers send goods to others and the locations of each hub are to be determined. In their study, a hierarchical heuristic solution framework based on the concept of nested methods is presented. Bruns et al. [6] consider a problem arising in the parcel delivery operations of a postal service.

They determine the locations of delivery hubs and the allocation of customer area to delivery hubs. In the case of routing costs, related with pickup and delivery route, they present a route length estimation formula. The problem studied by Wasner and Zapfel [7] is closely related to the HLRP. However, all inter-hub flow must go through a central hub rather than allowing all hubs to be directly connected to each other. Cetiner et al. [8] consider the combined hubbing and routing problem in postal delivery systems and develop an iterative two-state solution procedure for the problem in order to produce a route-compatible hub configuration. Catanzaro et

al.[9] investigate the partitioning-HLRP, a peculiar version of the hub location problem (HLP) involving graph partitioning and routing features and mainly arises from the deployment of an internet routing protocol. The partitioning-HLRP consists of partitioning a given network into sub-networks, locating at least one hub in each sub-network and routing the traffic within the network at minimum cost.

The HLRP is very difficult to be solved since it is composed of two NP-hard problems, the HLP and the multi-depot vehicle routing problem (MD-VRP).Therefore, the relatively large number of papers devoted to develop the approximation methods and the heuristic approaches can be classified into three categories such as clustering-based, iterative and hierarchical heuristics, respectively. Clustering-based methods begin by partitioning the customer set into clusters, one cluster per potential depot or one per vehicle route. Then, they locate a depot in each cluster and then solve a VRP for each cluster [10, 11]. Iterativeheuristics decompose the problem into its two sub-problems.

Then, the methods iteratively solve the sub-problems, feeding information from one phase to the other [12, 13]. Hierarchical heuristics considers the location problem as the main problem and routing as a subordinate problem where main algorithm is devoted to solving the location problem and refers in each step to a subroutine that solves the routing problem [14, 15]. Nagi and Salhi [16] indicated that hierarchical methods may provide better solutions. Based on their observation, this paper introduces a novel two-phase framework that integrates the hub location problem and routing decisions of the HLRP. The two-phase approach hybridizes an ant colony optimization (ACO) algorithm with a genetic algorithm (GA) in a computationally efficient manner.The objective is to determine thelocations of prescribed p hubs, the allocation of each customer node to hubs as well as the vehicle routes for each hub so as to minimize the sum of the fixed hub costs, traveling costs and fixed vehicle costs.

II. ACO ALGORITHM FOR HLP

The problem under study is hard to solve exactly when the size of problems increases. Therefore, we propose a hierarchical solution frameworkbased on two efficient meta-heuristics to get the near optimal solutions of the HLRP within a reasonable computational time. The hierarchical approach divides the problem into two sub-problems: the HLP in the first stage and the MD-VRP in the second one. The proposed two-phase meta-heuristic approachis applied to these two sub-problems hierarchically.
Dorigo and Stutzle [17] introduced the concepts and fundamental backgrounds of the ACO algorithm.ACO based algorithms have been widely applied on combinatorial optimization problems and showed the distinguished results. Our proposed ACO algorithm solves the HLP, one sub-problem in the HLRP.

2.1. Selection of Hub Locations

The solution construction in the HLP might be divided into two phases, which involve the selection of hub locations and the allocation of customer nodes to hubs. Then the solution of the hub selection can be constructed by the following probabilistic equations.

$$k = \begin{cases} \arg\max_{k \in L_h^s} \left(\tau_k^s (\eta_k)^\alpha \right), & if\ q \le q_0 \\ K, & \text{otherwise} \end{cases} \quad (1)$$

$$K : P_k^h = \frac{\tau_k^s (\eta_k)^\alpha}{\sum_{k \in L_h^s} \tau_k^s (\eta_k)^\alpha} \quad (2)$$

where L_h^s is the set of candidate hub sites which are not selected yet by ant h in the sth iteration, τ_k^s is the amount of pheromone on each candidate hub k in the sth iteration. The heuristic information η_k needs to be defined considering various problem specific characteristics. In this paper we consider the capacities of candidate hubs, traveling costs and fixed hub costs as the following equation (3).

$$\eta_k = \frac{\pi_k}{\sum_i \sum_j w_{ij} \left(\chi c_{ik} + \delta c_{kj} \right) + \sum_{l \in W_h^s} \gamma c_{lk} + f_k} \quad (3)$$

where χ , δ and γ are the collection coefficient from origin to its hub, the distribution coefficient from the hub to its destination and the transfer coefficient between two hub nodes, respectively. Further, W_h^s is the set of hub sites which are selected by ant h in the $s$thiteration. Intuitively, each candidate hub node with large capacity, low fixed cost and incurring low traveling cost is likely to be selected as the hub.

The parameter α establishes the importance of heuristic information in comparison to pheromone quantity in the selection process. The value q is a random uniform variable [0, 1] and the value q_0 is a parameter. When each selection decision is made, the ant selects the node with the highest value from equation (1) if $q \le q_0$. Otherwise, the ant selects a random variable K to be the next hub using the probability distribution defined in equation (2).

2.2. Allocation of Customers to Hubs

After the hub selection phase is completed each customer node is successively allocated to the hub by the following probabilistic formula,

$$r = \begin{cases} \arg\max_{r \in W_h^s} \left(\xi_{ir}^s \left(\psi_{ir} \right)^\beta \right), & if\ q \le q_0 \\ R, & \text{otherwise} \end{cases} \quad (4)$$

$$R : P_r^h = \frac{\xi_{ir}^s \left(\psi_{ir} \right)^\beta}{\sum_{r \in W_h^s} \xi_{ir}^s \left(\psi_{ir} \right)^\beta} \quad (5)$$

Where *i* is the index of the current customer node that is going to be assigned to hub and *r* is the index of any potential hub. A flow potential vector ***W*** is computed in which the *i*-th component W_i represents the sum of the flows from node *i* to all other nodes. We sort the nodes in decreasing order of flow potentials and at each construction step an ant *h* selects the next, still unassigned, customer node *i* as a candidate node for allocation. In addition, ψ_{ir} is defined as equation (6). The heuristic information, intuitively, should prefer the hub which incurs the low vehicle routing costs with the already allocated customer nodes.

$$\psi_{ir} = \begin{cases} \dfrac{1}{\min_{l \in A_r^{sh}} (c_{il})}, & \text{if the capacity is not violated} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where A_r^{sh} is the set of customer nodes which are allocated to hub *r* by ant *h* in the *s*th iteration. Note that A_r^{sh} also includes the hub *r*.

If the solution violates the hub capacity constraint, it is disregarded for the remainder of the iteration and the ant is considered as a dead ant. This construction process continues until all customer nodes are allocated to the hub.And then the next phase of solution method, that is GA for MH-VRP, is invoked.

III. GA FOR MH-VRP

The MD-VRP considered in this paper assumes that all the vehicles have an identical capacity. All customers can be visited only once by a single vehicle for collection purpose and be visited only once by a single vehicle for delivery purpose, respectively. When a new route of vehicle is created a fixed cost per vehicle is considered in order to minimize the total number of vehicles used.

GA is a stochastic search method based on the mechanism of natural selection and natural genetics. The bio-inspired computing based on GA has already been proved efficient for many combinatorial optimization problems.Our proposed GAsolves the MH-VRP, the other sub-problem of the HLRP in the second stage of solution framework

3.1. Genetic Representations

In order to solve the MD-VRP, each route of the vehicle should be determined based on the clusters obtained by the application of ACO algorithm in the previous phase. The solution representation method is very critical to the successful implementation of GA to the application area. In this paper, one-dimensional array is used to genetically represent individuals for MH-VRP. The length of an individual is equal to the number of customers. Let n_i be the number of customers who are assigned to hub *i* = 1, 2, …,*p*.**Fig.1.** illustrates the genetic representation of an individual in the population for the multi-hub VRP.

$$\left[g_{11}\ g_{12} \cdots g_{1n_1} \mid g_{21}\ g_{22} \cdots g_{2n_2} \mid \cdots \mid g_{p1}\ g_{p2} \cdots g_{pn_p} \right]$$

Fig.1.Genetic representation of an individual

The genetic representation of each cluster is encoded as a permutation of customer nodes already assigned to the hub.Its route is constructed by incrementally selecting customers in consideration of the demand of each customer and the loading capacity of vehicles until all customers have been visited. Initially, each vehicle starts at each hub and the set of customers included in its tour is empty. The unassigned customers are considered one by one in the order they appear in the permutation and the storage capacity of the vehicle is updated before another customer is selected. The vehicle returns to the depot when the capacity constraint of the vehicle is met, or when all customers are visited. This decoding method is applied to each cluster successively. It ensures that the feasibility conditions are not violated by the crossover operations such that every permutation results in a feasible solution.

3.2. Genetic Operators

Each individual in the population is considered for genetic operations such as selection, crossover and mutation to generate new offspring for the next generation. The higher an individual has fitness value, the higher it has the chances to participate in mating pool. In this paper, a stochastic remainder sampling without replacement method [18] is used for selection operator. After selection process is completed, members of the newly generated chromosomes are mated at random. Then, the crossover operator is applied to produce two offspring. In this study, we presenta modified order crossover (MOX) operator and its procedure can be explained as the following.

- ***Step 1:*** Two individuals (P1 and P2) in the current neighborhood are selected at random.
- ***Step 2:*** A vehicle number used in one parent (P1) is randomly chosen. The corresponding genes, which are the sequence of customer indices for the chosen vehicle, are copied to the beginning of the new offspring. The indices of already inherited genes from P1 are deleted from the other parent (P2).
- ***Step 3:*** The remaining genes in P2 are copied into the offspring in the order in which they

appear in P2. This process is applied to each cluster successively.

The proposed MOX operator is illustrated with an example as follows. Assume that there are 12 customer nodes of the first cluster only for simplicity, nodes 1, 5, 7 and 9 are assigned to vehicle 1, nodes 2, 4, 8, 11 and 10 are assigned to vehicle 2, nodes 3, 6 and 12 are assigned to vehicle 3, respectively. Assume that the randomly selected vehicle in the parent 1 is vehicle number 1. Then possible generation of offspring can be described as shown in **Fig.2.**

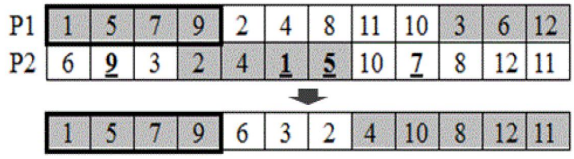


Fig.2. An example of the proposed MOX operator

In addition, a swap mutation operator is used which selects randomly two nodes in different clusters and swaps their assigned hubs. If the feasibility is violated by the mutation operator a penalty is imposed to the fitness function when the individual is evaluated. To improve the quality of solutions for the various combinatorial optimization problems, various local search methods have been applied to the GA [19, 20]. In this paper we adopt the 2-opt exchange method to improve the quality of solution for the MH-VRP. For each pair of customers in the route of each vehicle, the 2-opt exchange heuristic swaps their locations in the route and finds the best combination resulting the most great improvements. It has been applied to only the elite individual of the population in every generation after crossover and mutation operators are finished. invoked.

IV. EXPERIMENTAL RESULTS

This section evaluates the effectiveness of the proposed two-phase bio-inspired approach through a comparative study. For this purpose, the proposed algorithm is programmed in C++ and run on an IBM PC Pentium with 4GB RAM and 3.0 GHz microprocessor running Windows 7. Computational experiments are conducted to evaluate the relativeperformance of the proposed algorithm over multiple ACO (Maco) algorithm which is one of the most recent LRP heuristic proposed by Ting and Chen [21]. We construct a test data set that consists of a variety of problem instances.For the test data, the number of customers was set to 100 and 200, whereas the number of candidate hubs was set to 10 and 20. The two factors related to the spatial distribution of customers such as the number of clusters and clustering ratio is considered.

The number of clusters is set to 0, 3, and 5. The clustering ratio is defined as the ratio of number of customers that belong to the cluster to the total number of customers. This factor is set to 0.50 and 0.75.The demands of each customer were created from a uniform distribution in the range of [0, 20]. And vehicle capacity is set to 150 which is corresponds to an average 10 customers on a route. In all of these instances, fixed hub cost was identically set to 10.0 and fixed vehicle cost was set to 1.0, respectively.

The comparison of computational results with $\chi = 3, \gamma = 0.75, \delta = 2$ is given in **Table 1.**The parameter valuesused in ACO algorithm have been determined through the preliminary experiments: $q_0 = 0.6$, $\alpha = \beta = 1$, $\rho = \rho' =$ 0.1, and w (the number of ants) = 50. The genetic parameter values obtained in the experiments: population size = 100, crossover rate = 0.8, mutation rate = 0.1. In the Table 1, columns 1, 2, 3, 4, and 5 indicate the problem number of test problems, the number of customers, the number of candidate hub locations, the number of clusters, and clustering ratio, respectively. The sixth and seventh columns show the solution values and CPU times (in seconds) obtained from the mACO algorithm. The results from the proposed algorithm are reported in the eighth to tenth columns. The improvement ratio in column ten indicates the relative improvement ratio of solutions obtained by the proposed algorithm over ACO. Each entry of the solution value obtained by the both algorithms is the average of the 10 runs. Though we tried to solve the proposed HLRP using Xpress-MP 7.6, commercial optimization software, we found that the mathematical formulation for the HLRP could not get the optimal solutions for the test problems with more than 10 nodes in a reasonable computational time.

The results of computational experiments indicate that the proposed two-phase bio-inspired technique outperforms the existing multiple ACO algorithm and generates 0.78% ~ 8.44% improved solutions. As shown in Table 1, the proposed hierarchical bio-inspired computing technique provided better solutions in all test problems than the existing ACO algorithm. These encouraging results are due to the successful hybridization of ACO with GA for each problem. It seems that the proposed approach can take advantage of a proper balance between the parallel search with GA and the sequential search with ACO algorithm. This enhanced capability of exploitation together with the sequential and parallel search capability results in finding better quality solutions than existing hierarchical approach based on pure ACO algorithm. Because the costs related to the design and operation of hub and spoke networkaccount for a large portion of the company costs in most applications, it is justified to invest more computational time in order to improve the

solution quality even by a small amount. It can be observed that the effect of the number of clusters and clustering ratio seems to be insignificant to the solution quality. The computational time required by both algorithms increases by approximately the same factor as the problem size increases.

Table1:Computational results for the HLRP

P	N	H	C	R	mACO		Proposed		
					Cost	CPU	Cost	CPU	IR (%)
1	100	10	0	-	1634.2	2.78	1543.7	3.76	5.54
2	100	10	3	0.50	1566.7	2.45	1523.2	2.38	2.78
3	100	10	3	0.75	1583.9	3.50	1544.5	4.22	2.49
4	100	10	5	0.50	1493.2	2.84	1379.8	3.94	7.59
5	100	10	5	0.75	1523.0	3.12	1477.2	3.69	3.01
6	100	20	0	-	1824.5	3.67	1745.9	5.12	4.31
7	100	20	3	0.50	1677.3	3.42	1546.8	3.86	7.78
8	100	20	3	0.75	1734.9	4.77	1721.4	4.61	0.78
9	100	20	5	0.50	1356.8	4.63	1255.6	5.38	7.46
10	100	20	5	0.75	1475.3	5.02	1391.2	5.44	5.70
11	200	10	0	-	3254.7	12.48	3113.7	11.63	4.33
12	200	10	3	0.50	2944.6	15.33	2744.8	18.75	6.79
13	200	10	3	0.75	3120.3	13.72	3012.8	17.42	3.45
14	200	10	5	0.50	2839.4	16.80	2672.1	15.77	5.89
15	200	10	5	0.75	3023.5	15.23	2768.3	22.32	8.44
16	200	20	0	-	3827.9	14.21	3682.5	18.45	3.80
17	200	20	3	0.50	3653.8	18.25	3566.9	24.93	2.38
18	200	20	3	0.75	3569.5	22.37	3340.1	20.61	6.43
19	200	20	5	0.50	3766.2	16.77	3472.0	17.02	7.81
20	200	20	5	0.75	3012.7	24.30	2859.5	28.08	5.09

* P: problem number, N: number of customers, H: number of candidate hubs, C: number of clusters, R: clustering ratio

CONCLUSIONS

In this research, we addressed a capacitated HLRP which is composed of two sub-problems, HLP and MD-VRP, encountered in parcel delivery service. We proposed a new hierarchical solution framework based on a bio-inspired meta-heuristic which hybrids ACO algorithm with GAto solve the problem simultaneously.In the HLRP, the hub location and customer allocation decision will affect the total cost of vehicle routes and the structure of vehicle routes will also affect the decision of hub selections and allocation of customers to hubs.The experimental results show that the proposed algorithm provides substantial savings over the pure ACO algorithm. The encouraging results suggest that the hybrid meta-heuristics can be a valuable approach to design the parcel delivery network.

In future research, an integrated approach such as endosymbiotic evolutionary algorithm can be considered to conduct the comparative study with the proposed hybrid meta-heuristic approach.

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