PROGRESS ON THE PARALLELING-IN-ORDER BASED ORTHOGONAL EXPANSION IN TIME-DOMAIN METHODS: A REVIEW

[1]BAO-YUE YAN, [2]ZHENG-YU HUANG, [3]XUE-QI ZHENG, [4]FENG JIANG, [5]XIU-ZHEN GONG, [6]QINGSHENG ZENG

Key Laboratory of RadarImaging and Microwave Photonics, Nanjing University of Aeronautics and Astronautics, Nanjing, China
E-mail: huangzynj@163.com

Abstract - In recent years, the orthogonal expansions in time-based finite-difference time-domain (FDTD) methods have emerged as an important branch of unconditionally stable algorithms. By expanding electromagnetic fields and their time derivatives using various orthogonal basis functions—such as Associated Hermite (AH), Chebyshev (CS), and Legendre (LD)—a number of enhanced FDTD schemes have been developed. These methods overcome the stability constraints of traditional FDTD by enabling larger time steps, and they demonstrate excellent accuracy and efficiency in solving multiscale and complex electromagnetic problems. This paper reviews the theoretical foundations and advances of these methods, with a particular focus on the development of CS-FDTD and AH-FDTD. Their advantages and limitations are discussed, and potential future research directions and applications are outlined.

I. INTRODUCTION

Among the existing methods, the extended FDTD method based on Weighted Laguerre Polynomials (WLP) orthogonal functions shows a unique advantage in multiscale structural simulation by circumventing the Courant-Friedrichs- Lewy(CFL) condition [1], and its marching-on-in-order approach echoes with the Hermite Rodriguez (HR) method, which exhibits robust stability in two-dimensional unconditionally stable solutions [2].

On this basis, researchers have developed paralleling-in-order (PIO) methods, among which LD FDTD, as a typical PIO method, outperforms other methods in long time simulation [3].AH FDTD and CS FDTD, as the latest advances in PIO methods, further extend the application of unconditionally stabilized FDTD and provide more choices in the solution of complex electromagnetic problems.In this paper, we will review the unconditionally stabilized FDTD methods based on PIO orthogonal basis expansion in recent years, as shown in Fig. 1.

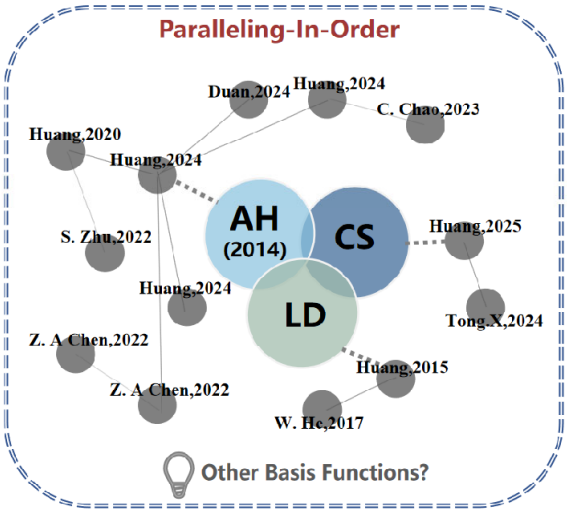

Fig. 1. Literature on PIO Techniques

II. DIFFERENT TYPES OF PIO METHODS AND RECENT DEVELOPMENTS

A. AH FDTD Methodology

In 2018, Huang et al. proposed for the first time the AH differential transform operator as a fundamental element of the time-frequency bridge ($\frac{\partial}{\partial t} \to \alpha$, $\frac{\partial^2}{\partial t^2} \to \alpha^2$, etc.), unified the construction formulae and generalized them to a wider range of physical modeling [4]. In 2020, the 3-D AH-FDTD method was proposed. By combining the CFS-PML absorbing boundary with the Kronecker product technique to generate the coefficient matrix, it shortens the computational time and has a significant advantage in aggregate parameter modeling analysis [5]. In 2022,Huang et al. combined the 3-D AH-FDTD and split-field techniques to obtain a 13-point coefficient matrix equation that eliminates the phase shift, and utilized Floquet theory to deal with the periodic boundary overflow problem [6]. In the same year, in thin wire modeling, Huang et al. applied and extended Riku M. M¨akinen's thin wire model to the AH FDTD method [7], and Z. -A. Chen et al. combined the contour path formula of Faraday's law with the AH domain operator to obtain the final solution of thin wire modeling along the Cartesian axis [8]. In addition, when studying the static field model of infinite-length thin wires, the coefficient matrix equation of

This work was supported in part by National Key Laboratory Foundation of China under Grant JCKYS2023LD5, Aeronautical Science Foundation of China (Grant No. 20240018052002),National Natural Science Foundation of China under Grant 61801217 and Natural Science Foundation of Jiangsu Province under Grant BK20180422.

dielectric-coated thin wires in the AH domain is derived, and a dielectric-coated thin wire model based on the AH-FDTD method is proposed [9]. In 2024, Huang's research team proposed several major innovations. The 3D AH-FDTD algorithm was improved to realize the plane wave incidence technique [10]; in terms of mesh generation, a mesh generation algorithm based on the FDTD framework was proposed to generate Yee cells on the surface of the model using the random filling method and to generate an internal mesh using electromagnetic energy distribution [11]; in non-uniform mesh computation, the fivepoint coefficient matrix equation of the electromagnetic field components was obtained based on the coordinate transformation, which improves the inadequacy of the traditional FDTD method in non-uniform mesh computation [12]; Huang et al. innovatively proposed the ML-AH-FDTD solving method, which dramatically improves the speed of generating datasets by combining it with machine learning [13]. Compared with traditional FDTD, its main features are as follows :

- Handling of dispersive media: AH transform naturally provides a time-frequency conversion bridge, and it is more convenient to handle the frequency dependence of media in the AH domain.
- Complex cases such as thin lines and periodic structures: thin-line antennas and periodic electromagnetic structures can be efficiently modeled by combining the contour path method or Floquet theory.
- Parallel solving and efficiency improvement: the use of eigenvalue transformation and banded matrix LU decomposition reduces memory and CPU consumption, making large-scale simulation possible.

B. CS FDTD Methodology

Compared to AH orthogonal functions, Chebyshev polynomials also have good orthogonality and recursive properties, which can be used to construct another unconditionally stable FDTD method, CS FDTD [14]. The basic procedure is to expand the Maxwell equations under the Chebyshev basis functions to form an implicit update on a closed interval, followed by numerical simulation of long-time or resonant scenarios by means of segmental reconstruction and Clenshaw recursive algorithm. The method needs to balance the basis function order with the computational cost while maintaining high accuracy. In recent years, CS-FDTD methods have made significant progress in both the temporal and spatial domains. Huang et al. proposed the Piecewise Calculation Scheme (PCS) to solve the memory occupation and accuracy problems caused by the large order of the basis function in long time simulation, which is better than the AH FDTD method in some complex scenarios [15]. In addition, Tong, X. et al. introduced the Chebyshev pseudo-spectrum (CPS) technique in the spatial domain and innovatively used Gauss-Chebyshev-Lobatto (GCL) points to construct an adaptive non-uniform mesh, which is able to more accurately approximate complex physical fields compared to the traditional uniform mesh [16].

III. CONCLUSION

A review of unconditionally stabilized FDTD methods based on Associated Hermite, Chebyshev, and other orthogonal basis functions shows that they have achieved remarkable results in overcoming the limitations of traditional FDTD by CFL conditions and realizing accurate simulation of dispersive media and complex electromagnetic structures. However, these methods still need to be further explored in terms of algorithm complexity, adaptive mesh and parallelization implementation.

REFERENCES

[1] M. Yang, Y. H. Tan, E. B. Li, C. Ma, and Y. C. You, "A New Marchingon- in-Order Based 2-D Unconditionally Stable FDTD Method," IEICE Trans. Electron., vol. E99.C, no. 10, pp. 1080–1087, 2016.

[2] Z. Y. Huang, L. H. Shi, and Y. H. Zhou, "A new marching-on-in-order unconditionally stable FDTD method based on orthogonal basis," in Proc. 2015 7th Asia-Pacific Conf. Environ. Electromagn. (CEEM), 2015, pp. 54–57.

[3] W. He, Z. Y. Huang, Y. B. Li, Z. Sun, and Y. H. Zhou, "A new parallelingin- order based unconditionally stable FDTD method using Legendre polynomials," Int. J. Appl. Electromagn. Mech., vol. 55, no. 1, pp. 35–44, 2017.

[4] Z. Huang, L. Shi, Z. Huang and Z. Sun, "A Theoretical Extension of AH FDTD Method and Applications in Various Physical Fields," in IEEE Journal on Multiscale and Multiphysics Computational Techniques, vol. 3, pp. 312-318, 2018.

[5] Z. -Y. Huang, L. -H. Shi and B. Chen, "The 3-D Unconditionally Stable Associated Hermite Finite-Difference Time-Domain Method," in IEEE Transactions on Antennas and Propagation, vol. 68, no. 7, pp. 5534- 5543, July 2020.

[6] S. Zhu, Z. -Y. Huang, C. Li, F. Jiang and L. -H. Shi, "The Unconditionally Stable Associated Hermite FDTD Method for 3-D Periodic Structure Analysis," in IEEE Antennas and Wireless Propagation Letters, vol. 21, no. 10, pp. 1950-1954, Oct. 2022.

[7] Z. Chen, Z. Huang, S. Liu, Q. Zeng and R. E. V. Lorena, "The Associated Hermite FDTD Method With the Thin-Wire Modeling in Low-Frequency Cases," in IEEE Journal on Multiscale and Multiphysics Computational Techniques, vol. 7, pp. 56-60, 2022.

[8] Z. -A. Chen, S. -B. Liu, Z. -Y. Huang, L. -H. Shi and Y. -T. Hou, "The Unconditionally Stable Associated Hermite FDTD Method With Thin Wire Modeling," in IEEE Microwave and Wireless Components Letters, vol. 32, no. 4, pp. 265-268, April 2022.

[9] C. Chao, Z. -Y. Huang, Z. -A. Chen, J. Feng and H. -Y. Duan, "The Unconditionally Stable Associated Hermite FDTD Method With Dielectric Coating Thin Wires Model," in IEEE Microwave and Wireless Technology Letters, vol. 33, no. 12, pp. 1595-1598, Dec. 2023.

[10] Z. -Y. Huang, F. Jiang, Z. -A. Chen, S. -B. Liu, C. Li and L. -H. Shi, "A 3-D Total-Field/Scattered-Field Plane-Wave Source for the Unconditionally Stable Associated Hermite FDTD Method," in IEEE Antennas and Wireless Propagation Letters, vol. 23, no. 2, pp. 628-632, Feb. 2024 .

[11] Z. -Y. Huang, H. -Y. Duan, Y. -Q. Liang, J. -C. Zhao and S. -B. Liu, "A Novel FDTD Mesh Generation Method Based on

Itself Framework," in IEEE Antennas and Wireless Propagation Letters, vol. 23, no. 12, pp. 4832-4836, Dec. 2024.

[12] H. -Y. Duan et al., "A Novel Nonuniform Grid AH FDTD Method for Fine Structure by Using the Coordinate Transformation," in IEEE Antennas and Wireless Propagation Letters, vol. 23, no. 10, pp. 3118- 3122, Oct. 2024.

[13] Z. -Y. Huang, X. -Z. Gong, Y. -R. Zheng, H. -Y. Duan and Q. -S. Zeng, "AH FDTD Method and Its Application in Solving Electromagnetic Forward and Inverse Problems Based on Machine Learning," in IEEE Antennas and Wireless Propagation Letters.

[14] C. Li, Z. Y. Huang, Z. A. Chen, S. Zhu, A. W. Yang, and Z. J. Wang, "Unconditionally stable FDTD method based on Chebyshev polynomials—Chebyshev (CS) FDTD," in Proc. 2021 Int. Appl. Comput. Electromagn. Soc. (ACES-China) Symp., 2021.

[15] Z. -Y. Huang et al., "Piecewise Calculation Scheme for the Unconditionally Stable Chebyshev Finite-Difference Time-Domain Method," in IEEE Transactions on Microwave Theory and Techniques.

[16] Tong, X.; Sun, Y. A Hybrid Chebyshev Pseudo-Spectral Finite- Difference Time-Domain Method for Numerical Simulation of 2D Acoustic Wave Propagation. Mathematics 2024, 12, 117.

★★★