MINIMIZING THE COST OF AN INTEGRATED MODEL WITH NONPARAMETRIC MULTIVARIATE APPROACH TO PROCESS CONTROL AND MAINTENANCE USING GENETIC ALGORITHM

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Abstract- This paper is aimed to find the optimal values of four variables (n_1,n_2,h_1,h_2) that minimize and the economic design cost model based on Chen [2]. The mathematical model is used to analyze the cost of the integrated model before the genetic algorithms (GA) approach is operated out in order to calculate the optimal values of four variables (n_1,n_2,h_1,h_2) that minimize the hourly cost. Finally, they are subjected to the nonparametric linear regression test in order to verify the optimal values of four variables. A comparison between four policies of integrated model NMKST and VP MEWMA models indicates that the integrated NMKST model has a better economic behavior when it is distribution-free or unknown distribution.

Index Terms- Integrated Model, Nonparametric Multivariate, Genetic Algorithms.

I. INTRODUCTION

In the past work, the quality control was widely used in diverse industry areas. The extensive application on control chart was used in producing an economic design of control charts. Montgomery [1] stated that the economic design of control charts referred the control charts that have been designed for statistical criteria only.
We apply the control chart used nonparametric multivariate case distribution-free or unknown distribution, the economic design cost model based on Chen [2]. In recent years, a related research, such as Nakto and Pongpullponsak [3]: Chen, Liao, and Chen [4] also applied on Chen [2].

The objective of the paper is studied to find the optimal values of four variables (n_1,n_2,h_1,h_2) that are minimizing the cost of an integrated systems approach to process control and maintenance model and the paper is organized as follows; Section II Details Experimental, Section III. Results and Discussion, Section IV conclusion.

II. DETAILS EXPERIMENTAL

A. Nonparametric Multivariate Kolmogorov-Smirnov Two Sample (NMKST)

In this section, the study used and applied For Higgins [5], assume we have a m experimental measure of treatment 1 and n for treatment 2. The measure is assumed to be selected at random from two populations. The vector of observations for the two treatments as below,

$\text{treatment 1}: \mathbf{F}' = (F_{i1},F_{i2},F_{i3},\ldots,F_{ik}), i=1,2,3,\ldots,m$

$\text{treatment 2}: \mathbf{G}' = (G_{i1},G_{i2},G_{i3},\ldots,G_{ik}), i=1,2,3,\ldots,n$

the vectors of sample mean by

$\text{treatment 1}: \bar{\mathbf{F}}' = (\bar{F}_1,\bar{F}_2,\bar{F}_3,\ldots,\bar{F}_k)$

$\text{treatment 2}: \bar{\mathbf{G}}' = (\bar{G}_1,\bar{G}_2,\bar{G}_3,\ldots,\bar{G}_k)$

The covariance between the uth and vth response variable on treatment 1 and 2

$$\mathbf{C}_{Fuv} = \frac{\sum_{i=1}^{m}(F_{iu}-\bar{F}_u)(F_{iv}-\bar{F}_v)}{m-1} \quad (1)$$

and $$\mathbf{C}_{Guv} = \frac{\sum_{i=1}^{n}(G_{iu}-\bar{G}_u)(G_{iv}-\bar{G}_v)}{n-1} \quad (2)$$

Let $\mathbf{C}=[C_{uv}]$ denote the $k\times k$ matrix of pooled covariance is

$$\mathbf{C}_{uv} = \frac{(m-1)C_{Fuv}+(n-1)C_{Guv}}{m+n-2}. \quad (3)$$

A multivariate statistic for two treatments is approximation T^2 defined by

$$T^2 = \frac{mn}{m+n}(\bar{\mathbf{F}}-\bar{\mathbf{G}})'\mathbf{C}^{-1}(\bar{\mathbf{F}}-\bar{\mathbf{G}}) \quad (4)$$

Then the statistic

$$F = \frac{m+n-k-1}{(m+n-2)k}T^2 \quad (5)$$

has an F-distribution with degrees of freedom k and $m+n-k-1$.

B. Approximation T^2 of NMKST Control Chart

This section proposes a method to search for the appropriate products for the NMKST control chart following a distribution-free or unknown distribution, and used method of nonparametric multivariate approximation by T^2. Therefore the lower control limit is zero (LCL = 0), we used the upper control (UCL) limit as below,

$$UCL = F = \frac{m+n-k-1}{(m+n-2)k}T^2$$
$$\rightarrow T^2 = \frac{(m+n-2)k}{m+n-k-1}F_{k,m+n-k-1}(\alpha). \quad (6)$$

The approximation of T^2 the NMKST control chart, and approximation T^2 statistic defined by Refer to (4). denoted,

$$\bar{\mathbf{F}} = |F_u - F_v| - D_{mn}(F), \bar{\mathbf{G}} = |G_u - G_v| - D_{mn}(G) \quad (7)$$

from application of the kolmogorov-smirnov two sample, was proposed by Conover [6]and Gibbons [7] which stated the kolmogorov-smirnov two samples to show possible different populations, and the experimental wisher to determine whether the two distribution functions associated with the two populations are identical or not, D_{mn} is a maximum absolute difference and called the empirical distribution function (edf),

$$D_{mn}(F) = \max|F_u - F_v|, D_{mn}(G) = \max|G_u - G_v| \quad (8)$$

For two random samples of size m and n are defined continuous populations, and order statistics.

C. Development of cost model

The model that describes the operation of the NMKST chart is an extension of the model presented by Chen [2] for the NMKST chart. It assumes that the production process can be represented as a series of stochastic identical cycles, as follows:

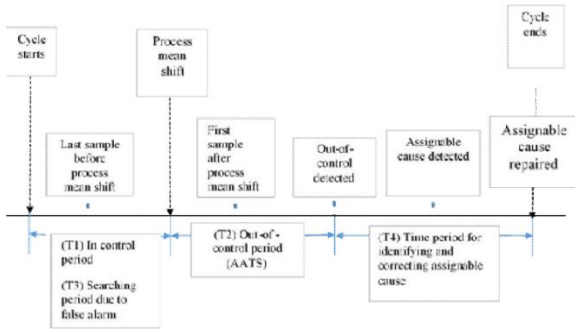


Fig. 1. Process cycle considered in the cost model by Chen [2]

Fig. 1. presents the process cycle, which is divided into four time intervals of in-control period, out-of-control period, searching period due to false alarm, and the time period for identifying and correcting the assignable cause. Individual are now illustrated before they are grouped in four periods.

(T1) The expected length of in-control period is $1/\lambda$.

(T2) The expected length of out-of-control shows the average time is necessary for the control chart to produce a signal after the process mean shift. This average time be called the adjusted average time to signal (AATS), which is the utter widely used statistical measure for comparing the efficiencies of different control schemes when the sampling interval are variables. The memoryless property of the exponential distribution allows the calculation of AATS used the markov chain approach. At each sampling point during the period M, one of the five transient states is reached according to the status of the process (in-control or out-of-control) and the position of approximation T^2 (relaxing, warning and action region, see Table 1 and Fig. 2.):

Table 1: States reached at each sampling time during the process cycle

State	*ith* sampling Process status	Control mode/Inspection result
1	In-control	T^2 falls into the relaxing region zone
2	In-control	T^2 falls into the warning region zone
3	In-control	T^2 falls into the action region zone
4	Out-of-control	T^2 falls into the relaxing region zone
5	Out-of-control	T^2 falls into the warning region zone
6	Out-of-control	T^2 falls into the action region zone

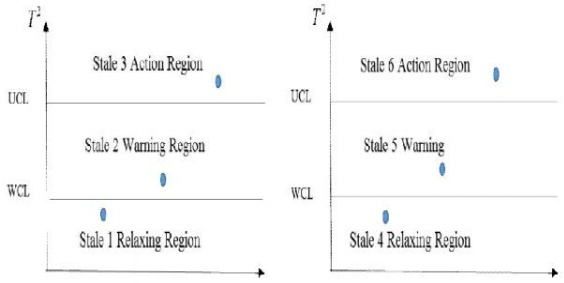


Fig. 2. Control region, In-control and Out-of-control

(T3) Let M be average time from the cycle start to the time the chart signals after the process shift. Thus,

$$\text{AATS} = M - (1/\lambda) \quad (9)$$

At each sampling time during the period M one of the five transient is reached, according to the status of the process (in or out-of-control), see Table 1. According to the elementary properties of Markov Chains by Chen [2], that

$$r'(I-Q)^{-1} \quad (10)$$

provides the mean number of transitions in each transient state before the true alarm signals, where

$$r' = (r_1, ..., r_5) \quad (11)$$

is the vector of starting probability such that

$$r_1 + ... + r_5 = 1 \quad (12)$$

I is the identity matrix of order 5, and Q is the transition matrix where the elements associated with the absorbing state have been deleted. Therefore, the process of the average number of samples and the corresponding sampling interval determines the period M. That is,

$$M = r'(I-Q)^{-1}t \quad (13)$$

where t is the vector of the sampling intervals corresponding to the five transient states used for next sampling, reasonable choices for r' and t' are $r' = (0,1,0,0,0)$ and $t' = (h_1, h_2, h_2, h_1, h_2)$ respectively.

Let t_0 be the average amount of time wasted searching for the assignable cause when the process is in-control, and $E[FA]$ be the expected number of false alarms per cycle given by

$$E[FA] = r'(I-Q)^{-1} f \quad (14)$$

where $f' = (0,0,1,0,0)$. Subsequently, the expected length of searching period due to false alarm is $E[FA]t_0$ and (V_0, V_1, C_0, C_1, S) as below,

V_0 The hourly profit earned when the process is operating in-control state

V_1 The hourly profit earned when the process is operating in out-of-control state

C_0 The average search cost if the given signal is false

C_1 The average cost to discover the assignable cause and adjust the process to in-control state

S The cost for each inspected item.

When State (T3) is arrived, the signal the chart produces is a false alarm. If the approximation T^2 falls into the action region at some sampling point while the process status is out-of-control, then the signal is true alarm and the absorbing state, State 6, is arrived. The transition probability matrix is given by

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} \\ 0 & 0 & 0 & P_{44} & P_{45} & P_{46} \\ 0 & 0 & 0 & P_{54} & P_{55} & P_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (15)$$

This can be applied by Chen [2], where P_{ij} denotes the transition probability that i is prior state and j is the current state. Therefore,

$$\begin{aligned} &P_{11} = a(\lambda e^{-\lambda h_1}); P_{12} = (b-a)(\lambda e^{-\lambda h_1}); \\ &P_{13} = (1-b)(\lambda e^{-\lambda h_1}); \\ &P_{14} = c(1-e^{-\lambda h_1}); P_{15} = (d-c)(1-e^{-\lambda h_1}); \\ &P_{16} = (1-d)(1-e^{-\lambda h_1}); \\ &P_{21} = P_{31} = a(\lambda e^{-\lambda h_2}); P_{22} = P_{32} = (b-a)(\lambda e^{-\lambda h_2}); \\ &P_{23} = P_{33} = (1-b)(\lambda e^{-\lambda h_2}); P_{24} = P_{34} = c(1-e^{-\lambda h_2}); \\ &P_{35} = P_{25} = (d-c)(1-e^{-\lambda h_2}); P_{26} = P_{36} = (1-d)(1-e^{-\lambda h_2}); \\ &P_{44} = P_{54} = c; P_{45} = P_{55} = d-c; P_{46} = P_{56} = 1-d. \end{aligned} \quad (16)$$

this paper used probability density function (pdf) of $\lambda e^{-\lambda h}$ and cumulative distribution function (cdf) of $1-e^{-\lambda h}$ when $h = h_1$ *or* h_2, Refer to Khrueasom and A. Pongpunponsak [14].

From Refer to (4), where $F_{k,\nu}(\alpha)$ is the $(1-\alpha)$ percentile point of F-distribution with k and $\nu = m+n-k-1$ degrees of freedom. Moreover,

$$UCL = C(m,n,k)F_{k,\nu}(\alpha), \nu = m+n-k-1 \quad (17)$$

$$C(m,n,k) = \frac{(m+n-2)k}{m+n-k-1}, \nu = m+n-k-1 \text{ for } n>1 \quad (18)$$

In this section, we used the concept and defined warning control limit from applied of the general model as below,

$$WCL = UCL - L\frac{\sqrt{C_{UV}}}{\sqrt{n}} \quad (19)$$

When

Given

UCL is the upper control limit

L is the distance of the control limits from the upper control limit line equal to 2.5

n is the sample size equal to 5

$\mathbf{C} = [C_{uv}]$ is the $k \times k$ matrix of pooled covariance from Refer to (3)

$$a = F_{k,\nu}\left(\frac{WCL}{C(m,n,k)}\right), b = F_{k,\nu}\left(\frac{UCL}{C(m,n,k)}\right)$$
$$c = F_{k,\nu,\tau}\left(\frac{WCL}{C(m,n,k)}\right), d = F_{k,\nu,\tau}\left(\frac{UCL}{C(m,n,k)}\right) \quad (20)$$

The notations $F_{k,\nu}$ represent the F-distribution function with k and ν degrees of freedom, and $F_{k,\nu,\tau}$ represents the non-central F-distribution function with k and ν degrees of freedom and non-centrality parameter τ. The non-centrality parameter τ is given by

$$\lambda = \frac{mn}{m+n}(\bar{\mathbf{F}} - \bar{\mathbf{G}})' \mathbf{C}^{-1} (\bar{\mathbf{F}} - \bar{\mathbf{G}}), \quad (21)$$

let $$\delta = \sqrt{(\bar{\mathbf{F}} - \bar{\mathbf{G}})' \mathbf{C}^{-1} (\bar{\mathbf{F}} - \bar{\mathbf{G}})}, \quad (22)$$

then τ can be rewritten by

$$\tau = \frac{mn}{m+n}\delta^2 \quad (23)$$

We see that δ is the Mahalanobis distance, used to measure a change in the process mean vector.

(T4) The time to identify and correct the assignable cause following an action signal is a constant t_1. As a result, the expected length of a process cycle can be aggregately represented as

$$E[T] = M + E[FA]t_0 + t_1 \quad (24)$$

If one define

$$E[C] = V_0(\frac{1}{\lambda}) + V_1(M - \frac{1}{\lambda}) - C_0 E[FA] - C_1 - SE[N] \quad (25)$$

where $E[N]$ is the average numbers of inspected items during process cycle (the expected number of sampling in a cycle period) and given by

$$E[N] = r'(I-Q)^{-1}\eta, \ \eta' = (n,n,n,n,n). \quad (26)$$

The model can be considered as a renewal-reward process; hence, the expected cost per hour E[H] can be expressed as

$$E[H] = \frac{E[C]}{E[T]} \quad (27)$$

$$E[H] = \frac{V_0(\frac{1}{\lambda}) + V_1(M - \frac{1}{\lambda}) - C_0 E[FA] - C_1 - sE[N]}{M + E[FA]t_0 + t_1} \quad (28)$$

Under the assumption that failure mechanism is exponential distribution, the mean time to failure (MTTF) is corresponding with (T1),

$$\text{MTTF} = 1/\lambda. \quad (29)$$

III. RESULTS AND DISCUSSION

This study proposed, the numerical example and sensitivity analyses are conducted to study the effect of model parameters in the solution of economic design of the nonparametric multivariate chart. Using the genetic algorithms (GA) with MATLAB, 7.6.0 (R2009a) software [8].The Math WorksTM, 2009, the solution procedure is carried out to obtain the optimal values of $\left(n_1^*, n_2^*, h_1^*, h_2^*\right)$ which will be subsequently used to minimize in Refer to (27).

D. Genetic Algorithms (GA)

The GA is the stochastic and optimization search technique of natural selection and natural genetics. The GA solves problems used the approach to the process of Darwinian evolution. In recent years, many studies have been devoted to the GA solves problems of economic-statistical, engineering, mathematics, production processes, etc. Current, GA models were introduced and investigated by Holland [9]. The solution procedures of GA (e.g., Charongrattanasakul and Pongpullponsak [10]; C. Y. Chou, C. H. Chen, and C. H. Chen [11]; C. Y. Chou, J. C. Cheng and W. T. Lai [12]; S. N. Lin, C. Y. Chou, S. L. Wang and H. R. Liu [13]; Khrueasom and Pongpullponsak [14]) in this study are briefly described below,

Step 1. Initial Population: The procedure starts at randomly generating 100 solutions reach the constraint condition of individual test parameter. Meanwhile, the constraint condition represented for individual test parameter is set as below,

$5 \le n_1 \le n_2 \le 25,\ 0.1 \ge h_1 \ge h_1 \ge 5$

Step 2. Evaluation: This step, is evaluated through the fitness function. Each solution used the expected cost per hour E [H] in Refer to (27).

Step 3. Selection: The selected function chooses parents (survivors) for the next generation based on their scaled values of the fitness scaling function. The four individual solutions are selected randomly and the best is chosen (For the first generation the chromosome with the lowest cost is selected to replace the highest cost chromosome).

Step 4. Crossover: In this step by step 3, generate new chromosomes for the next generation and a pair of parents (survivors) are selected randomly as shown in this example, parents (survivors) used for crossover operations to produce new chromosomes (or children) for the next generation. This research used crossover rate 0.8 as below,

$D_1 = 0.8R + 0.2M, D_2 = 0.2R + 0.8M$

where D_1 is the first new chromosome, D_2 is the second new chromosome, and R and M are the parent (survivors) chromosomes. If 30 parents (survivors) are randomly selected, then there are 60 children that will be produced. Thus, the population size increases to 90 (i.e., 30 parents (survivors) + 60 children) in this step.

Step 5. Mutation: Mutation function is the small change of genes in chromosomes in the population, suppose that the mutation rate is 0.1, which is also determined by L_{16} orthogonal array experiment. In this example, we have 90 solutions and we can randomly select 9 chromosomes $\left(i.e., 90 \times 0.1 = 9\right)$ to mutate some parameters (or genes) in this step.

Step 6. Stopping Criteria: Repeat Step 2 to Step 5 until the stopping criteria is found. In this example, we use ‘‘50 generations or greater than” as our stopping criteria.

The initial values of the necessary parameters are given in Table 2, where eight independent parameters which will be tested in the sensitivity analysis and their corresponding level plans are illustrated. Accordingly, the effect of model parameters on the solution of economic design of the NMKST chart can be investigated by conducting numerical example and sensitivity analysis.

E. Numerical examples and Sensitivity Analysis

Table 2: Eight model parameters and their level plan

Model Parameter	Level 1	Level 2
V_0	250	500
V_1	50	100
t_0	2.5	5
t_1	1	10
C_0	250	500
C_1	50	500
s	5	10
λ	0.01	0.05

Table 3: Model parameter assignment in the L_{16} orthogonal array and the corresponding solution

Trial	Model Parameter							
	V_0	V_1	t_0	t_1	C_0	C_1	s	λ
1	250	50	2.5	1	250	50	5	0.01
2	500	100	5	10	250	50	5	0.05
3	250	100	5	10	500	500	5	0.05
4	500	50	2.5	1	500	500	5	0.01
5	250	50	2.5	1	250	50	10	0.05
6	500	100	5	10	250	50	10	0.01
7	250	100	5	10	500	500	10	0.01
8	500	50	2.5	1	500	500	10	0.05
9	500	50	2.5	10	250	500	10	0.01
10	250	100	5	1	250	500	10	0.05
11	500	100	5	1	500	50	10	0.05
12	250	50	2.5	10	500	50	10	0.01
13	500	50	2.5	10	250	500	5	0.05
14	250	100	5	1	250	500	5	0.01
15	500	100	5	1	500	50	5	0.01
16	250	50	2.5	10	500	50	5	0.05

	Solution				
	n_1	n_2	h_1	h_2	$E[H]$
1	16.836	18.225	4.375	2.142	103.375
2	9.933	14.924	3.848	2.553	112.251
3	16.599	18.634	2.244	1.586	131.945
4	18.297	21.629	1.185	0.709	110.663
5	6.322	24.168	1.524	1.366	127.538
6	5.642	12.075	1.611	0.826	118.023
7	16.300	24.806	3.763	2.630	141.123
8	7.781	21.717	2.725	1.592	109.412
9	22.652	24.989	3.556	1.085	118.686
10	5.923	19.094	1.494	0.598	118.420
11	13.533	18.282	2.911	2.121	112.661
12	18.254	24.455	4.994	3.186	104.156
13	15.160	23.116	1.618	1.516	139.190
14	20.477	23.686	2.794	1.628	110.060
15	10.552	19.050	2.928	0.858	121.836
16	14.332	16.718	1.201	0.377	100.277

Table 4: Model parameter assignment in the orthogonal array and the corresponding solution

Model Parameter	Trial 16
V_0	250
V_1	50
t_0	2.5
t_1	10
C_0	500
C_1	50
s	5
λ	0.05

Table 5: Optimal values for four variables and the optimal value of the total hourly costs

Variable	Integrated model (NMKST)
n_1	14.332
n_2	16.718
h_1	1.201
h_2	0.337
$E[H]$	100.277

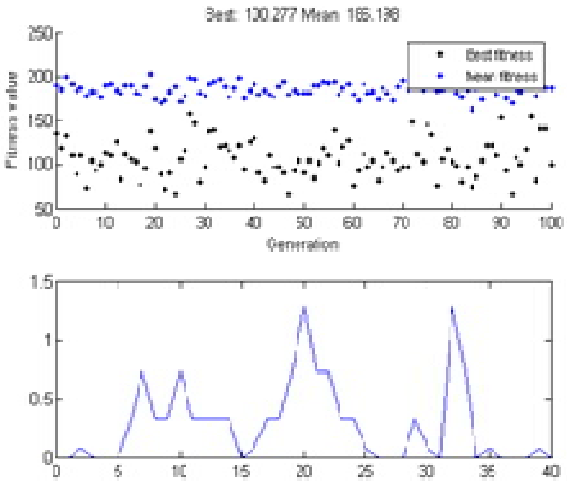


Fig. 3. Optimal values by GA

After calculation, the best value parameters by L_{16} orthogonal array of NMKST be obtained as shown in Table 4, from Table5 and Fig. 3, the optimal values of the policy variables estimated and the minimize E [H] are found to be, GA are $n_1^* = 14.332$, $n_2^* = 16.718$,$h_1^* = 1.201$,$h_2^* = 0.337$ and the corresponding hourly cost is $E[H] = 100.277$

Ranks

		N	Mean Rank	Sum of Ranks
EH1 - n1	Negative Ranks	0[a]	.00	.00
	Positive Ranks	8[b]	4.50	36.00
	Ties	0[c]		
	Total	8		
EH2 - n2	Negative Ranks	0[d]	.00	.00
	Positive Ranks	8[e]	4.50	36.00
	Ties	0[f]		
	Total	8		
EH3 - h1	Negative Ranks	0[g]	.00	.00
	Positive Ranks	8[h]	4.50	36.00
	Ties	0[i]		
	Total	8		
EH4 - h2	Negative Ranks	0[j]	.00	.00
	Positive Ranks	8[k]	4.50	36.00
	Ties	0[l]		
	Total	8		

Fig. 4. Wilcoxon Signed Ranks

Test Statistics[b]

	EH1 - n1	EH2 - n2	EH3 - h1	EH4 - h2
Z	-2.521[a]	-2.521[a]	-2.521[a]	-2.521[a]
Asymp. Sig. (2 tailed)	.012	.012	.012	.012

a. Based on negative ranks.
b. Wilcoxon Signed Ranks Test

Fig. 5. Wilcoxon Signed Ranks Test Statistics

Table 6: Nonparametric regression for four variables models (n_1, n_2, h_1, h_2)

Parameter	Estimate	T	P-values
Constant (n_1)	34.894	36*	0.012
$E[H]$	-0.182		
Constant (n_2)	11.346	36*	0.012
$E[H]$	-0.077		
Constant (h_1)	10.026	36*	0.012
$E[H]$	-0.062		
Constant (h_2)	6.517	36*	0.012
$E[H]$	-0.043		

*The Wilcoxon T statistics: reject H_0 when T (calculate) > T (critical)

Table 7: Numerical results of two integrated models

Variable	Integrated model (NMKST)	Integrated model (VP MEWMA)
n_1^*	16.644	27
n_2^*	19.067	33
h_1^*	3.809	2
h_2^*	2.205	1
$E[H]$	100.277	499.892

* VP MEWMA from Nakto and Pongpullponsak [3]

Table 6 illustrates the output of the nonparametric linear regression applied in order to fit the regression line. According to Brown and Mood [15] , Wilcoxon Matched–Pair Test is proposed by Corder and Foreman [16] which is used in hypothesis testing at 0.05 significance level by the statistical software SPSS 15.0 [17] (Fig. 4-5). It is noticed that the sign of the coefficient parameter of the constant is often estimated by assuming that the hourly cost E[H] is negative, which is consistent with the principle of nonparametric statistical hypothesis testing.

We use nonparametric linear regression test, the numerical results of the optimal values of the policy variables which minimize E[H] are found to be $n_1^* = 16.644$, $n_2^* = 19.067$,$h_1^* = 3.809$,$h_2^* = 2.205$ and the corresponding hourly cost is $E[H] = 100.277$

Finally, from Table 7 the comparison between two policies of the integrated model (NMKST) and (VP MEWMA) models shows that the model proposed in this study the NMKST has a better economic when it is distribution-free.

CONCLUSION

This study proposes the method used in searching the appropriate products for the NMKST control chart that is distribution-free and control chart is suitable for detection of little changes. In the same inning, little changes in the process can lead to incredible damage. For this reason, those factories produce non-restricted goods using appropriate control charts. It would cut or remove unnecessary cost. Conclusively, these two control charts shown in Table 7 are suitable for using in various types of factories and products; however, this selection of appropriate control chart is recommended in order to cut or remove unnecessary cost.

In Table 7, the optimal values of the policy variables are minimized E [H], this provides evidence that it is distributed-free the integrated NMKST model which will have an above economic behavior is better than the VP MEWMA models in case of distribution-free or unknown distribution. However, the model in this paper is suitable in mentioned case. The proposed will be improve for other cases in the future work in order to make better result. Therefore this problem could be an interesting issue for further study.

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