DEVELOPMENT OF A PREDICTIVE MODEL FOR ATMOSPHERIC CO_2 CONCENTRATIONS UTILIZING KEELING CURVE DATA

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Abstract - The quantity of carbon dioxide (CO_2) in the atmosphere is increasing more quickly, which is heightening critical concerns about the mitigation of greenhouse gases, which are the primary contributors of anthropogenic climate change. This intensifying challenge has projected CO_2 emissions into a significant global issue, prompting strenuous efforts among atmospheric scientists, climate researchers, and engineers to devise strategies to manage and dwindle the rapid increase in CO_2 levels. Creating predictive models that can estimate atmospheric CO_2 concentrations and identify the sources of emissions is one well-known strategy that has recorded massive success. In this study, we employed a mathematical modelling approach to analyse the trends shown in Keeling Curve data. Our findings demonstrate that the proposed predictive model performs with high accuracy, effectively capturing the observed trends in the Keeling Curve. The predicted CO_2 concentrations exhibit strong alignment with empirical Keeling Curve data, and future projections for the years, 2022-2023 and long-term, 2026 and 2050 are consistent with forecasts reported by other established predictive models. We opined that this novel model represents a valuable tool for researchers and scientists engaged in climate monitoring and forecasting.

Keywords - Carbon Dioxide, Atmosphere, Greenhouse Effect, Keeling Curve, Global Warming

I. INTRODUCTION

The atmosphere forms one of the principal components of Earth's interrelated physical systems. The atmosphere consists of layers of gases, about 78% of the gases in Earth's atmosphere are nitrogen, 21% are oxygen, and 1% are other gases such as carbon dioxide, helium, argon, and so on (Schlesinger & Bernhardt, 2020). These gases are present in the troposphere, mesosphere, thermosphere, stratosphere, and exosphere in different compositions by features often determined by pressure and temperature(Hu & Guan, 2022). Atmosphere, with the help of these vital gases, surrounds the earth, accommodates the air we breathe, insulates the earth from ultraviolet (UV) radiation, and controls the extreme temperature between the day and night. Atmosphere, alongside its many layers and gases from a blanketlike structure that protect the earth from extreme temperature and safeguard life on Earth. One of the naturally occurring gases in the Earth's atmosphere that is crucial for life on Earth is carbon dioxide written as CO_2(Fu & Davis, 2022). It is one of significant greenhouse gases (GHGs) that impacts Earth's climate by trapping heat in the atmosphere, contributing to the greenhouse effect. It is released to the atmosphere through physical process such as respiration, decomposition of carbon containing complex organic matters. Carbon dioxide is a major player in the carbon cycle, a process by which carbon is exchanged between the atmosphere, land, forest and oceans. It is mostly naturally absorbed by forests, oceans and other carbon sinks since plants and other lower photosynthetic organisms make useof carbon dioxide to produce oxygen which are utilise by animals for growth and energy. However, human activities, majorlythrough industrialization and domestic energy production, have negatively impacted this balance, mostly due to the burning of fossil fuels such as coal, oil, and natural gas for energy generation, transportation, and industrial operations. This has accelerated the concentration of CO_2 since the mid-20th century. This trend becomes worse by massive deforestation for companies, residential houses, recreation centres, airports, and road construction, among others, which hinders the planet's ability to absorb CO_2. CO_2 levels were comparatively constant before the industrial revolution, at around 280 ppm (Eggleton, 2013; Etheridge et al., 1996). CO_2 levels have been reported to have risen above 400 parts per million (ppm) and are still rising at a alarming rate of about 1.5 parts per year, according to measurements taken from Mauna Loa lab Observatory in Hawaii, USA and other worldwide monitoring stations on May 9, 2013, which is the first time concentration hit such high rate for more than 3 million years, which is an increment of around 40% above what was recorded in 1890 (Day & Hall, 2016).

CO_2 is an essential and significant component of the Earth's atmosphere; human activity has prompted its concentration to rise above what is needed,posing a serious threat to the environment. One of which is global warming, as the name suggests, is the long-term heating of the Earth’s surface that has been observed since the pre-industrial period (between 1850 and 1900)(Eggleton, 2013; Zoltán et al., 2021). One of the leading causes of global warming is the accumulation of greenhouse gasses in the Earth’s atmosphere via anthropogenic emissions. Greenhouse gasses such as water vapour, carbon dioxide,

methane, nitrous oxide, and ozone are gasses with the property of absorbing and emitting electromagnetic radiation from the sun within the thermal infrared range (Williams, 2021). Global warming has been identified to be responsible forcascade effects on our environment, melting glaciers, polar ice caps, rising sea levels, altered precipitation patterns, and increases frequency and severity of extreme weather hazardous occurrences such as hurricanes, floods, and droughts, which can have severe and long-lasting negative impacts on our ecosystems, human health, and economies of nations. Increased CO_2 concentrations have been identified as the causes of oceanic acidification, leading to a damaging effect and extinction to some marine life by changing the chemistry of sea water to be more acidic, threatening the marine habitat, and destabilizing marine ecosystems.Besteiro et al. (2017)asserted that attaining the ideal circumstances for animal wellbeing requires a careful consideration of the CO_2 concentration inside livestock buildings, especially in cases of intensive production.

CO_2 emissions have been a global issue for several years, and efforts to proffer effective and consistent means to reduce this threat have seen limited success. The introduction of carbon capture and storage (CCS) technologies offers a promising solution by capturing CO_2 and repurposing it in various industries, including chemical, petrochemical, and fuel production. Captured CO_2 are used in processes like methanol synthesis and methane gas production, which enhances carbon utilization efficiency and contributes to reducing overall emissions (Kudapa, 2023).The adoption of renewable energy targets reduction of CO_2 emissions by replacing fossil fuels with cleaner form of energy alternatives like solar, wind, and hydropower. Alongside this, reforestation and afforestation could improve absorption of CO_2 from the atmosphere by increasing the number of trees that function as natural carbon sinks(Molokova, 2023; Tran et al., 2022). These approaches, if implemented on a large scale and combined with strong government policy support, could meaningfully lower atmospheric CO_2 levels, alleviate global warming, and promote environmental sustainability. Additionally, they could lead to a reduction in the reliance on fossil fuels, enhance biodiversity, and improve air quality, contributing to overall climate resilience(Hepburn et al., 2019).

Prediction using mathematical models has been very pivotal in forecasting patterns in real-life instances. Mathematics provides tools and models that can observe patterns, identify trends, and predict future outcomes. It is essential for making predictions about real-world occurrences and complex systems, from real-world instances such as weather patterns, price fluctuation, and financial markets to population increase and infectious disease dynamics, to become clearer to understand and predicted with accuracy using mathematical modeling. These models simulate the behavior of systems across time using formulas and algorithms, facilitating well-informed strategic planning and decision-making. Several mathematical models have been deployed in environmental research to predict atmospheric carbon dioxide concentrations, which helps in evaluating the possible effects of climate change and suggests likely measures to be taken to avert any unwanted mishaps. Prediction using a mathematical model converts data into useful insights that society often deploys to foresee opportunities and difficulties in a variety of domains.Ahn et al. (2022)deployed a hybrid of Bayesian neural networks and mass balance equations to model enhanced indoor air quality to accurately predict indoor CO_2 concentration levels in residential units.Kuga et al.(2022)deployed computer-simulated persons (CSP), which help in the design of advanced ventilation in residential buildings by precisely predicting the amounts of carbon dioxide breathed around the human body. A hybrid of cell-DEVS (Discrete Event System) and deep learning was used to predict carbon dioxide dispersion indoors, which eventually helps in minimizing energy consumption and optimizing air quality (Khalil & Wainer, 2024). The concentration of CO_2, which has been identified as an integral part of greenhouse gases in Ujung Berung district, Bandung City, West Java, Indonesia was predicted by (Ernwati et al., 2022)using the support vector regression (SVR) model. Ravi Kumar et al.(2022) asserted the superiority of the autoregressive integrated moving average (ARIMA) model in terms of accuracy and reliability in the forecasting of carbon dioxide concentration when compared with other available techniques. Niu et al. (2023) projected that by the year 2050, atmospheric CO_2 concentration could reach approximately 507 parts per million (ppm), an increment that is predicted to drive the global temperature to temperatures by about 2 degrees Celsius. These changes will not only threaten biodiversity but also increase the frequency of extreme weather events and pose serious challenges to coastal communities worldwide.

Various forms of model has been proposed and deployed to predicted the concentration of CO_2 in the atmosphere and since prediction models are designed to forecast the nature or status of instance based on historical data, and the relationship or patterns presents within the data, and such relationship or identified patters are used to make informed predictions about the future or unknown event, Ravi Kumar et al.(2022) used ARIMA model and predicted that carbon dioxide concentration levels will rise to 685 parts per million by the year 2103, while the gray GM (1,1) model proposed by (Zheng & Chen, 2023)reported that CO_2 concentrations attained the same benchmark of 685 ppm a little bit later, in 2132. The goodness-of-fit for the gray GM (1,1) model was

0.99989, while the ARIMA model achieved a higher goodness-of-fit of 0.99998, indicating that the ARIMA model provided more accurate predictions compared to the gray prediction model. These two contrasting projections are enough to illustratethe unpredictability of forecasting models; many studies might use the same datasets but might obtain different results, an indication that predictive models will always be relevant in the field of mathematics.

In this research, we propose a straightforward predictive model, based on the assumption that the existing trajectory of CO_2 emissions remains unchanged and that there is a linear relationship between time and atmospheric CO_2 levels. This model facilitates better understanding of the general trends in CO_2 levels. However, it is imperative to note that this assumption is inherently simplistic and may not fully capture the complexities of the real-world dynamics influencing atmospheric CO_2.

For instance, our model does not account for potential anomalies that could arise from unforeseen events, such as economic downturns, natural disasters, advanced carbon capture and sequestration technologies, carbon free energy innovations or significant policy shifts aimed at reducing carbon emissions. This could dramaticallychange the course of CO_2 concentrations, potentially disrupting the upward trend observed in the Keeling Curve. Therefore, the model provides a foundational understanding of the trend in Keeling curve data under the imposed assumptions.

By analysing the overall trend of the Keeling Curve from 1980 to 2021, we employ simple model to predict the concentration of atmospheric CO_2 for the years 2022-2023, 2030 and 2050. Although our predictions focus on these specific years, the model can be extended to forecast CO_2 levels beyond stated years, allowing for a broader exploration of future atmospheric conditions.

This approach provides valuable insights into potential future scenarios based on current CO_2 emissions trajectories, while also highlighting the necessity for continuous evaluation and adjustment of the model in light of emerging data and technological advancements. Ultimately, the simplicity of our model enables a clearer understanding of CO_2 trends while also serving as a starting point for more complex modelling efforts that could incorporate additional variables and factors influencing atmospheric carbon dynamics.

II. DATA SOURCE AND ITS FEATURES

The industrial carbon dioxide data utilized in this study were derived from the Keeling Curve analysis, based on data compiled by the United Nations for the period spanning 1980 to 2020.

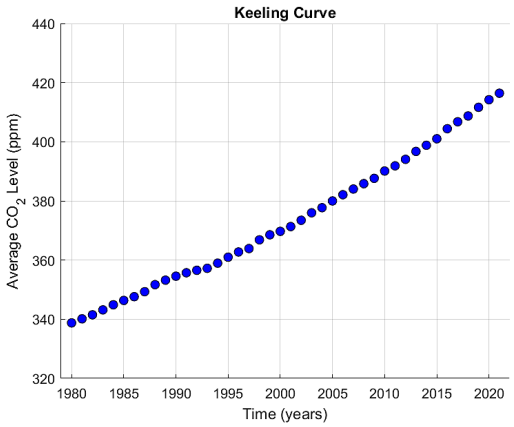


Figure 1: Industrial CO_2 Keeling Curve for the year 1980 to 2021.

Notice that the trend of the Keeling Curve in these past 41 years is almost homogeneous since the concentration in the Earth's atmosphere is increasing and follows a pattern. Modelling the anthropogenic emissions in the Earth's atmosphere can be extremely challenging, as there are multiple variables to be taken into consideration.

By analysing the trend observed in the Keeling Curve, we develop a mathematical model that can capture the atmospheric CO_2 concentrations. The Keeling Curve, which was first proposed by Charles David Keeling in 1958 at Hawaii's Mauna Loa Observatory, shows how atmospheric carbon dioxide levels are continuously monitored. Figure 1 illustrates the Keeling Curve from 1980 to 2021, revealing noteworthy patterns and insights into the trending behaviour of atmospheric CO_2 during the period under review.A consistent upward trajectory is evident from the Keeling Curve, exhibitingan ongoing increase in CO_2 emissions and highlighting the intensifying impact of human activities and natural occurrences on the global carbon cycle. In the period of four decades, atmospheric CO_2 concentrations rose from approximately 338 parts per million (ppm) in 1980 to over 415 ppm in 2021, resulting in an average annual increase of around 1.5 ppm per year during the 1980s. This rate has accelerated to approximately 2.5 ppm per year in recent decades.Observations from Keeling data also reveals seasonal oscillations within the data, which can be linked to disparities in photosynthesis, respiration, and other natural processes that alter with the seasonssuch as spring, summer, and winter. Despite these seasonal fluctuations, the overall data presented in Figure 1 provides convincing evidence of a continuous increase in atmospheric CO_2 levels, underscoring the imperative need for strategies to mitigate greenhouse gas emissions into atmosphere that can lead to massiveimpacts on climate change.

III. MATHEMATICAL FORMULATION

3.1. Modelling using linear function

Linear regression is a straightforward approach that assumes a linear relationship between time and CO_2 levels so we define a function $M: \mathbb{R}_{\geq 1980} \to \mathbb{R}$ such that for every $T \in \mathbb{R}_{\geq 1980}$, $M(T)$ denotestheapproximate average CO_2 level (in ppm) in the Earth's atmosphere at time T(in years).We assume that the Keeling curve exhibits a feature similar to that a linear function. Consequently, we model the Keeling curve using a linear function. Based on our assumptionfor $T \in \mathbb{R}_{\geq 1980}$

$$M(T) = kT + l \quad (1)$$

where $M(T)$ represents the concentration of CO_2, T represents the time (in year) and l is the intercept. we determine value of k using the relation in equation (2) since:

$$\Delta M(T) = M(T) - M(T-1) \quad (2)$$

Simplifying equation (1) we have

$$\Delta M(T) = k \quad (3)$$

$\Delta M(T)$ is the approximate change in the average CO_2level in the year T ; the equation above implies that k is a single number thatapproximates the change in the average CO_2level in any possible year between 1981 and 2021.To find the value of k from the data of the Keeling curve, we estimate the $\Delta M(T)$ for each year under consideration and present the solution as shown in Table 1.

Year	Change in CO_2 concentration	Year	Change in CO_2 concentration
1980	-	2001	1.61
1981	1.36	2002	2.13
1982	1.36	2003	2.53
1983	1.67	2004	1.72
1984	1.72	2005	2.28
1985	1.48	2006	2.11
1986	1.26	2007	1.93
1987	1.70	2008	1.81
1988	2.38	2009	2.46
1989	1.51	2010	1.75
1990	1.35	2011	2.21
1991	1.15	2012	2.68
1992	0.84	2013	2.68
1993	0.67	2014	2.07
1994	1.75	2015	2.20
1995	2.01	2016	3.4
1996	1.77	2017	2.35
1997	1.14	2018	1.96
1998	2.96	2019	2019
1999	1.70	2020	2020
2000	1.17	2021	2.20

Table 1: Annual average CO_2 level in the Earth's atmosphere between the years 1980 and 2021, grand annual change in the average level in the Earth's atmosphere between the years 1981 and 2021

The variations in the data shown in Table 1 offer limited insights into the distribution properties of the CO_2 level dataset. To have a clearer understanding of data distribution, we plotted the graph of the data in Table 1 to visually examine the atmospheric CO_2 concentrations. The resulting histogram, is depicted in Figure 2, and it reveals a distinct negatively skewed distribution. The majority of the data points clustered on the right side of the histogram, indicating that most CO_2 concentrations fall within a higher range. In other word, there are relatively fewer data points on the left side of the distribution, which extend towards lower values. This pattern suggests that while the atmospheric CO_2 levels have generally increased over time, there are fewer instances of significantly lower concentrations, reflecting a trend towards higher overall levels of CO_2 in the atmosphere. This visualization not only helps illustrate the central tendency of the data but also highlights the extent of variability within the dataset. The negative skew indicates that the higher concentrations of CO_2 are more frequent, potentially pointing to the increasing impact of human activities on atmospheric composition. By analysing the

histogram, we can infer that the data are massively influenced by specific events or trends, such as industrial emissions and seasonal variations, that contribute to increased CO_2 levels. Comprehending the distribution of CO_2 concentrations is essential in analysis and modelling, as it assists researchers to identify patterns, trends, and potential anomalies in the data A negatively shewed histogram is characterized by Mean < Median < Mode, as the mean is pulled by the values to the left and might be inappropriate to use the mean of such a distribution as the centre of the data; thus, this makes the medium a better measure of central tendency (Rousselet & Wilcox, 2018).However, based on the assertion median is more reliable measure of central tendency in our data. As such we compute the value of median to be 1.81 and set the value for k: such that $k = 1.81$ so that;

$$\Delta M(T) = k = 1.81 \tag{4}$$

From the Keeling curve data, we have

$$M(1980) = 338.76$$

$$1.81.1980 + l = 338.76$$

$$l = -3245.04$$

We then write the equation of line representing the distribution as shown in equation (5).

$$M(T) = 1.81T - 3245.04 \tag{5}$$

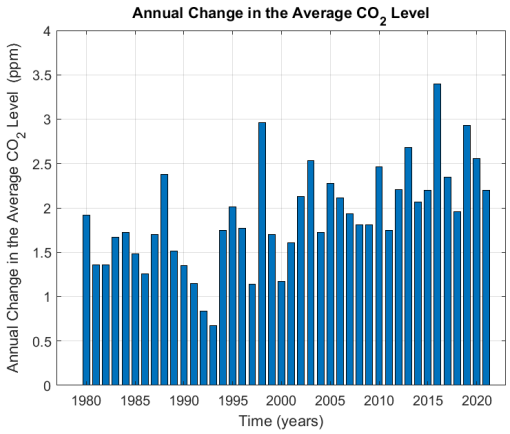


Figure2: Bar Chart of the Annual Change the in the average CO_2 level in the Earth's atmosphere between the year 1981 and 2021.

3.2 Modelling using Least Square approximation

Given a collection of data points (x_i, y_i), a least square approximation for (x_i, y_i) is a process of finding some line ℓ that minimizes the sum of the square of the vertical distances between all points in (x_i, y_i) and the line ℓ. Suchline ℓ is often called the of best fit for (x_i, y_i).

The simplest form of this function is as derived in equation (5) above and it has a general form as shown in equation (6).

$$f(x) = ax + b \tag{6}$$

The parameters a and b are the parameters that needed to be computed from the data, since the least square minimizes the sum of the square difference between the observed data value and the predicted value.The simplest form of this function is a linear model, so for $(a,b) \in \mathbb{R}^2$ we have;

$$E(a,b) = \sum_{i=1981}^{2021} \left[y_i - (ax_i + b) \right]^2 \tag{7}$$

E is a bivariate polynomial function and differentiable, it follows from multivariable calculus that the partial derivatives of E at the point where the absolute minimum occurs must be zero. Knowing that $(k,l) \in \square^2$ such that

$$E_a(k,l) = \frac{\partial E(a,b)}{\partial a}\bigg|_{(k,l)} = 0$$

and

$$E_b(k,l) = \frac{\partial E(a,b)}{\partial b}\bigg|_{(k,l)} = 0$$

Using the first equality, we have: $E_k(k,l) = 0$

From equation (7)

$$E_k(k,l) = \sum_{i=1981}^{2021} 2\left[y_i - (kx_i + l)(-\mathrm{x_i})\right] = 0$$

$$E_k(k,l) = 2\sum_{i=1981}^{2021} \left[(y_i - (kx_i + l))(-\mathrm{x_i})\right] = 0$$

$$= \sum_{i=1981}^{2021} \left[-x_i y_i + kx_i^2 + bl\right] = 0$$

$$E_k(k,l) = k\left(\sum_{i=1981}^{2021} x_i^2\right) + l\left(\sum_{i=1981}^{2021} x_i\right) = \sum_{i=1981}^{2021} x_i y_i \qquad (8)$$

Using the second equality, we have

$$E_l(k,l) = 0$$

$$E_l(k,l) = \sum_{i=1981}^{2021} 2\left[y_i - (kx_i + l)(-1)\right] = 0$$

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$$= 2\sum_{i=1981}^{2021} \left[-x_i y_i + kx_i + bl\right] = 0$$

$$= k\left(\sum_{i=1981}^{2021} x_i\right) + l\left(\sum_{i=1981}^{2021} 1\right) = \sum_{i=1981}^{2021} y_i$$

$$E_l(k,l) = k\left(\sum_{i=1981}^{2021} x_i\right) + l(41) = \sum_{i=1981}^{2021} y_i \qquad (9)$$

Solving for k and l using (8) and (9) we have:

$$k = \frac{41\left(\sum_{1981}^{2021} x_i y_i\right) - \left(\sum_{1981}^{2021} x_i\right)\left(\sum_{1981}^{2021} y_i\right)}{41\left(\sum_{1981}^{2021} x_i^2\right) - \left(\sum_{1981}^{2021} x_i\right)^2}$$

and

$$l = \frac{\left(\sum_{1981}^{2021} x_i^2\right).\left(\sum_{1981}^{2021} y_i\right) - \left(\sum_{1981}^{2021} x_i y_i\right).\left(\sum_{1981}^{2021} x_i\right)}{41.\left(\sum_{1981}^{2021} x_i^2\right) - \left(\sum_{1981}^{2021} x_i\right)^2}$$

From the Keeling curvedata, we compute the value of:

$\sum_{1981}^{2021} x_i = 82041$, $\sum_{1981}^{2021} y_i = 77.65$, $\sum_{1981}^{2021} y_i x_i = 155551.3$ and $\sum_{1981}^{2021} x_i^2 = 82041$, substitute it in equations (8) and (9) we have $k = 0.0302526$ and $l = -58.6416$, Hence, the line of best fit for the Keeling curve

$$h(x) = 0.0302526x - 58.6416 \quad (10)$$

Now the visualization of line of best fit is shown in Figure 3.

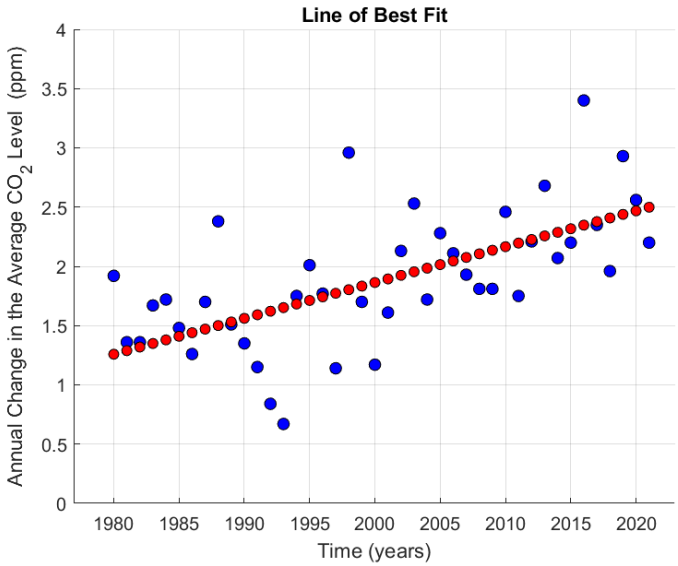


Figure 3: Line of Best Fit plotted against the scatter plot of the annual change in the average level in the Earth's atmosphere between the years 1981 and 2021.

From Figure 3, it can be observed that the function that describes the line of best fit is an increasing function that captures the trend in the keeling curve, and using the results from the linear least squares approximation is defined as shown in equation (10). Based on these facts we can then assert that:

$$\Delta M(T) = M(T) - M(T-1) \quad (11)$$

using slope and point relation we have;

$$M(T) - M(T-1) = 0.0302526\mathrm{T} - 58.6416$$

$$M(T) = M(T-1) + 0.0302526\mathrm{T} - 58.6416 \quad (12)$$

Using the Keeling curve data $M(1980) = 338.76$

We evaluate the values using the Old Model and the New Model, then compared them with the actual CO_2 concentrations recorded in the Keeling Curve dataset, as presented in Table 2. To evaluate the accuracy of each method, we compute the absolute errors for both Old models, allowing for a direct comparison of their performance in reflecting the trend in the Keeling curve data. The method applied using equation (5) is referred to as the 'Old Model,' while the method applied using equation (12) is referred to as the 'New Model.'From the absoluteerror's analysis, we assess the precision of each model in replicating the observed CO_2 levels over the study period. The Old Model, though providing reasonable approximations, consistently exhibits higher absolute errors compared to the New Model. The New Model, developed with an enhanced mathematical framework, demonstrates significantly lower absolute errors and a closer alignment with the actual Keeling Curve data. The improvement in accuracy highlights the superiority of the New Model in effectively capturing both short-term fluctuations and long-term trends in atmospheric

CO_2 concentrations.The results of this comparison reinforce the reliability of the New Model as a more precise tool for forecasting future CO_2 levels. This evaluation also emphasizes the importance of refining predictive models to better reflect complex atmospheric processes, particularly as climate data becomes increasingly essential for understanding and mitigating the impacts of global climate change.

IV. RESULT PRESENTATION AND DISCUSSION

Year	CO_2 Level (Old Model)	CO_2 Level (Old Model)	Actual CO_2 level	Abs Error old model	Abs Error new Model
1980	338.76	338.76	338.76	0	0
1981	340.57	340.0488	340.12	0.45	0.07120
1982	342.38	341.3679	341..48	0.9	0.11215
1983	344.19	342.7172	343.15	1.04	0.43284
1984	346	344.0967	344.87	1.13	0.77328
1985	347.81	345.5065	346.35	1.46	0.84347
1986	349.62	346.9466	347.61	2.01	0.66341
1987	351.43	348.4169	349.31	2.12	0.89309
1988	353.24	349.9175	351.69	1.55	1.77252
1989	355.05	351.4483	353.2	1.85	1.75170
1990	356.86	353.0094	354.55	2.31	1.5406
1991	358.67	354.6007	355.7	2.97	1.0993
1992	360.48	356.2223	356.54	3.94	0.3177
1993	362.29	357.8741	357.21	5.08	0.6641
1994	364.1	359.5562	358.96	5.14	0.5962
1995	365.91	361.2685	360.97	4.94	0.2985
1996	367.72	363.0111	362.74	4.98	0.2711
1997	369.53	364.7840	363.88	5.65	0.9040
1998	371.34	366.5871	366.84	4.5	0.2529
1999	373.15	368.4204	368.54	4.61	0.1196
2000	374.96	370.2840	369.71	5.25	0.5740
2001	376.77	372.1779	371.32	5.45	0.8579
2002	378.58	374.1020	373.45	5.13	0.6520
2003	380.39	376.0563	375.441	4.41	0.0763
2004	382.2	378.0409	377.7	4.5	0.3409
2005	384.01	380.0558	379.98	4.03	0.0758
2006	385.82	382.1009	382.09	3.73	0.0109
2007	387.63	384.1763	384.02	3.61	0.1563
2008	389.44	386.2819	385.83	3.61	0.4519
2009	391.25	388.4178	387.64	3.61	0.7778
2010	393.06	390.5839	390.10	2.96	0.4839
2011	394.87	392.7803	391.85	3.02	0.9303
2012	396.68	395.0069	394.06	2.62	0.9469
2013	398.49	397.2638	396.74	1.75	0.5238
2014	400.30	399.5509	398.813	1.49	0.7409
2015	402.11	401.8683	401.01	1.1	0.8583
2016	403.92	404.2160	404.41	0.49	0.1940
2017	405.73	406.5939	406.76	1.03	0.1661
2018	407.54	409.0020	408.72	1.18	0.2820
2019	409.35	411.4404	411.65	2.3	0.2096
2020	411.16	413.9091	414.21	3.05	0.3009
2021	412.97	413.4080	416.41	3.44	0.0020

Table 2 presents the predicted values from both the Old Method and New Method, alongside the actual data from the Keeling Curve for comparison using absolute error estimation.

To begin, we conduct a performance analysis of the Old Model over the period from 1980 to 2021. Based on the data in Table 2, the model demonstrates a maximum absolute approximation error of approximately 5.65 ppm, observed in 1997. The average approximation error across the entire time frame, by contrast, is about 2.9617 ppm. Despite the model's inherent structural limitations, these results suggest that its overall predictive accuracy is relatively satisfactory. While the approximations are not exceptionally precise, the simplicity of the model's architecture warrants recognition of its

reasonable performance. For a graphical representation of the Old Model's performance during this period, a comparison with the Keeling Curve is provided in Figure (4).

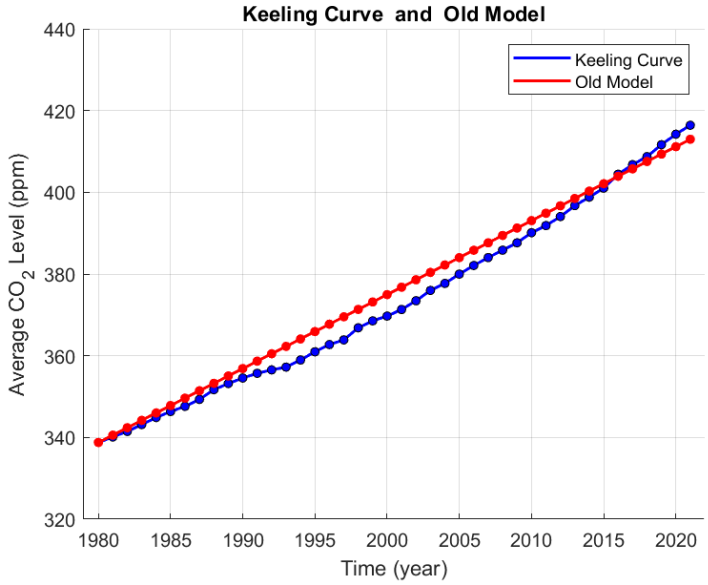


Figure 4: Comparison of Old Model to the Keeling Curve andfor the years 1980 to 2021.

Let us now assess the overall performance of the New Model for the period from 1980 to 2021. As indicated in Table 2, the New Model exhibits a maximum absolute estimation error of approximately 1.7725 ppm, which occurred in 1988. The average approximation error across the entire period is around 0.5474 ppm. These results clearly demonstrate that the New Model significantly outperforms the Old Model in capturing the trend of the Keeling Curve dataset.To further quantify this improvement, the median ratio of estimation errors between the Old Model and the New Model is approximately 6.3203. This indicates that, on average, the New Model is at least six times more accurate than the Old Model. The median was chosen over the mean to account for the presence of outliers, particularly in 2006 and 2021, as shown in Table 2, providing a more accurate representation of the model's typical performance.For a visual comparison of the New Model's performance relative to both the Old Model and the Keeling Curve, refer to the graphical representations in Figure 5. These illustrations highlight the superior accuracy of the New Model in closely tracking observed CO_2 levels.

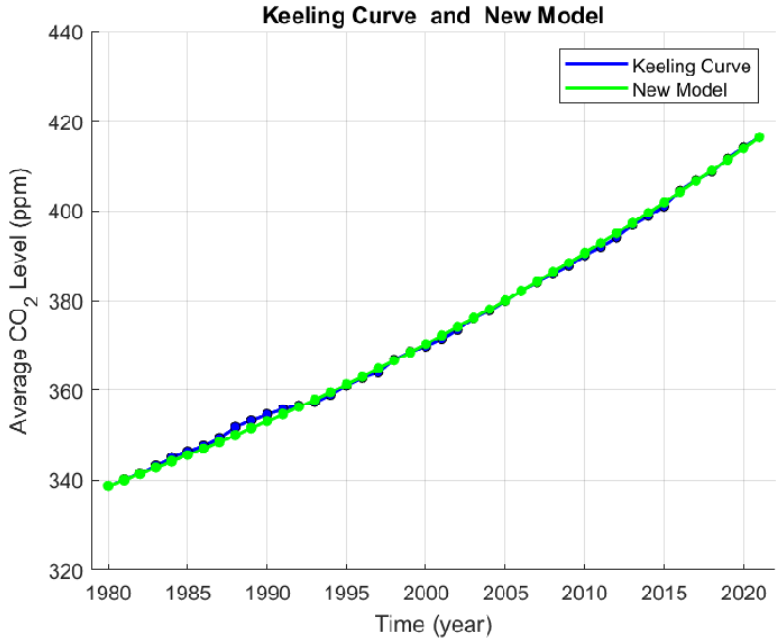


Figure 5a: Comparison of New Model to the Keeling Curve and for the years 1980 to 2021.

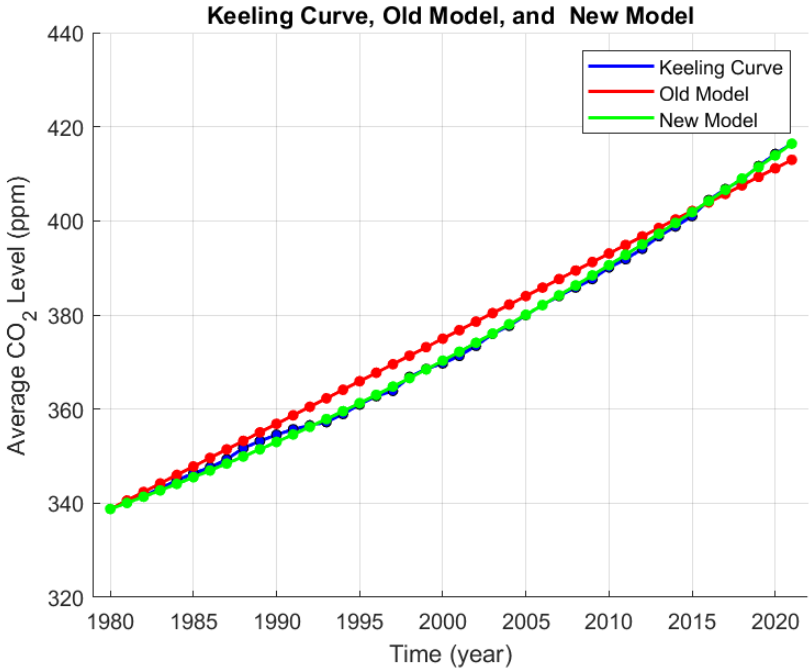


Figure 5b: Comparison of New Model to the Old Model, and Keeling Curve and for the years 1980 to 2021.

Finally, given that this study is conducted in the year 2024, and consequently, we know CO_2 levels for the years 2022 and 2023, respectively, by testing the predictive power of our old model and new modelswith the Keeling curve value for the years 2022 and 2023.

Year	CO_2 Level (Old Model)	CO_2 Level (Old Model)	Actual CO_2 level	Abs Error old model	Abs Error new Model
2021	414.78	418.9371138	418.53	3.75	0.4071138
2023	416.59	421.4965236	421.08	4.49	0.4165236

Table 3 shows the predictions using the old model, new model, and actual CO_2 level from

Keeling curve data for the years 2022 and 2023.

From Table 3, it is evident that our previous model, despite being relatively outdated, still manages to predict future average CO_2 concentrations with a reasonable degree of accuracy. However, while the old model demonstrates a level of utility, it is clearly outclassed by the new model in terms of precision and reliability. The new model achieves an absolute error of less than 0.5 ppm, a significant improvement over the old model. In fact, the prediction accuracy of the new model is nearly nine times better than the old model when evaluated against the observed and projected CO_2 levels for both 2026 and 2050.These findings underscore the improvements in the modelling techniques and the importance of continual refinement in predictive tools for atmospheric science. The improved performance of the new model is likely due to its incorporation of more complex procedures for computing the slope and improved algorithms for capturing non-linear trends in CO_2 concentration data. As a result, the new model stands as a more reliable tool for forecasting future atmospheric CO_2 levels, which is critical for understanding the trajectory of climate change and informing dwindle strategies.Based on current trends in human activities related to industrial emissions, deforestation, and fossil fuel consumption, our model predicts that if the global attitude and behaviours toward environmental management remain largely unchanged from the patterns observed between 1980 and 2021, the average CO_2 concentration in Earth's atmosphere will continue to rise steadily.

Our model forecasts that the average atmospheric CO_2 levels will reach approximately 429.3563 ppm by the year 2026 and 502.0361 ppm by 2050. These projections align well with other climate models that similarly predict continued increasesingreenhouse gases concentrations if significant reductions in emissions are not achieved.These forecasts have significant ramifications since exceeding 500 parts per million of atmospheric CO_2 may cause global temperatures to rise over critical levels, worsening the already dire effects of climate change. Elevations of this kind are probably going to cause more intense weather, an increase in sea level, and ecological disturbances. As a result, these projections send a strong message to businesses, governments, and the general public around the world about the necessity of acting more quickly to reduce CO_2 emissions and

shift to sustainable practices. The new model's precision increases even more its potential as a useful tool for climate researchers and decision-makers to estimate future CO_2 trends and assess how well environmental measures are working.

V. CONCLUSION

In this study, we model the average atmospheric CO_2 concentration based on the observed trends in the Keeling Curve, driven by the increasing and linear trends exhibited in the data. To capture and model these characteristics, we employed simple mathematical models, resulting in the development of two potential models for estimating average CO_2 levels: the Old Model and the New Model. Both models are demonstrated to be effective representations of atmospheric CO_2 levels, with the New Model providing particularly accurate approximations.Our results indicate that, if the environmental trends observed between 1980 and 2021 continue, the New Model offers a reliable method for predicting future CO_2 concentrations with a high degree of precision. The New Model not only aligns well with historical data but also proven effective in future projections. Specifically, our predictions for 2026 and 2050 estimate CO_2 concentrations of 429.3563 ppm and 502.0361 ppm, respectively. These projections are consistent with other notable CO_2 prediction models, such as those proposed by Hofmann et al. (2009) and Niu et al. (2023), further validating the accuracy and relevance of the New Model for forecasting future atmospheric CO_2 levels.

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