ESTIMATING AND VALIDATING WIND TURBINES' CAPACITY FACTORS USING A SOLID MATHEMATICAL MODEL

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Abstract- This paper presents a mathematical model for estimating wind turbines' capacity factors (CFw) model. The model links climatic data (wind speed) with the wind turbine manufacturer data to produce an accurate estimate of the capacity factor. The proposed model has been developed and validated using real life manufacturing data. The developed model has been validated using Monte Carlo Simulation method. The statistical validation of the model is compared with the actual values, and the results obtained showed close match. The presented algorithm can be used as a robust guidance for the decision makers on which wind turbine fits best the site under study.

Keywords- Wind turbine, capacity factor, and Monte Carlo simulation.

I. INTRODUCTION

The world faces a tremendous increase in power supply demand. Renewable energy can offer a good solution for this problem in a clean way; thus keeping the environmental aspects, like pollution at minimum level. Renewable resources are sustainable and vary from wind, to solar, bio-energy, etc. Wind and solar energies represent the lowest risk and most established technologies due to their recent technical developments and financing options. Wind energy sources' importance comes from their ability to produce large power. Nowadays there are large number of manufacturers for wind turbines, each differs in its operating data, which make the decision of choosing the optimal turbine for certain location a challenging problem. Wind turbine's capacity factor is used as measuring aspect of how much certain device fits its installation location. To estimate wind turbines' capacity factors, the wind speed of the location under consideration is linked with the wind turbine manufacturing data. Several studies have been presented to support decision makers to select the appropriate turbine for the site under considerations [1,2]. In [3] the authors presented derivation for capacity factor formula based on stall power curve for wind turbines. The effect of varying both hub height and the rated wind speed on the overall value of capacity factor thus affecting the proper selection of wind turbine for certain location was presented in [4]. A techno-economic criterion for selecting appropriate wind turbine unit for certain location depending on the capacity factor value was presented in [5]. In this study, the authors recommended the using of Weibull distribution rather than Rayleigh distribution. In [6] the capacity factor and Weibull parameters for certain site used for the selection of rated wind speed. Authors in [7] presented the effect of both parts of capacity factor (rated region and unrated region) on the overall suitability of certain wind turbine for a given site. Authors of [8] presented the optimum selection of wind turbine based on capacity factor and investigated the effect of relation between ratios of wind turbine speed parameters and relative capacity factor. In [9] a comparison between the different approaches (arithmetic, root, and cubic) to get the mean wind speed is presented. Each mean method affected the value of capacity factor obtained using Weibull or Rayleigh distribution. The paper suggested using cubic mean method coupled with Weibull distribution for optimum wind turbine site matching. The mathematical derivation for capacity factor based on quadratic model for wind turbine has been presented in [10]. The authors in [11, 12] proposed a model for the estimation of the rated speed and capacity factor based on Weibull distribution; hence the final effect on optimal selection was presented. This paper presents a comprehensive cubic mathematical model to estimate the capacity factor of pitch controlled wind turbine using wind speed data of the considered location. The proposed mathematical model is explained in section II, followed by the model validation using Monte Carlo (MC) simulation in Section III. A case study is presented in Section IV followed by the main conclusions and appendixes.

II. MATHEMATICAL MODELING

The wind data used was get from NREL [13]. The climate data was provided on 10-min time intervals, so it was averaged first to get the hourly data needed using the following formula:

$$V_i = \frac{\sum_{j=1}^{n} V_j}{n} \quad (1)$$

Therefore the average hourly wind speed was obtained. The wind speed data was provided at altitude of 100m, provided that the power output from a turbine depends on the speed of wind at the hub height of the turbine. This imposes to project the provided wind speed to each and every hub height of the selected turbines understudy [11]. To do this projection, the following formula could be used:

$$V_t = V_i \left(\frac{Z}{Z_i}\right)^{x} \quad (2)$$

The value of power law exponent depends upon the roughness of the surface. For open land, $(x = \frac{1}{7})$, The published literature established a fact that wind speed data follows Weibull distribution. This fact was based using both Chi-Square and Kolmogorov – Smirnov goodness of fit test [11]. This section presents the derivation for a closed form equations to get the values of both parameters of Weibull distribution (k, λ). The Weibull distribution has a probability density function (pdf):

$$f_{v_t}(v_t;\, k, \lambda) = \frac{k}{\lambda^k}\; v_t^{k-1}\; e^{-\left(\frac{v_t}{\lambda}\right)^k} \quad (3)$$

To get the closed form for a pdf parameters, maximum likelihood estimation could be used for estimating the parameters. The likelihood estimation has the formula [6]:

$$L(\theta|x_1, \dots, x_n) = \prod_{i=1}^{n} f(x_i|\theta) \quad (4)$$

In practice it is more convenient to work with the logarithm of the likelihood function, to get the log-likelihood:

$$\ln L(\theta, x) = \sum_{i=1}^{n} \ln f(x, \theta) \quad (5)$$

To maximize the previous function the first derivative of the function to be equated to zero:

$$\frac{\delta}{\delta x} \ln L(\theta, x) = \sum_{i=1}^{n} \frac{\delta}{\delta x} \ln f(x, \theta) = 0 \quad (6)$$

To find the values of Weibull distribution parameters k, λ.

This will be done using the maximum likelihood estimation (MLE). To get the MLE, Equation (5) is applied on equation (3):

$$L_1(k, \lambda) = \prod_{i=1}^{n} f_{v_t}(v_i) = \prod_{i=1}^{n} \frac{k}{\lambda^k}\; v_t^{k-1}\; e^{-\left(\frac{v_t}{\lambda}\right)^k} \quad (7)$$

Taking the log of both sides of (7)

$$\ln L1(k, \lambda) = n \ln k - nk \ln \lambda + (k-1) \sum_{i=1}^{n} \ln v_i - \lambda^{-k} \sum_{i=1}^{n} v_i^k \quad (8)$$

Differentiate equation (8) w.r.t.

$$\frac{\delta}{\delta k} \ln L_1(k, \lambda) = \frac{n}{k} - n \ln \lambda + \sum_{i=1}^{n} \ln v_i - \lambda^{-k} \sum_{i=1}^{n} v_i^k \ln v_i + (\lambda^{-k} \ln \lambda) \sum_{i=1}^{n} v_i^k \quad (9)$$

λ

Therefore the closed form for will be as follows:

$$\lambda = \left[\frac{1}{n} \sum_{i=1}^{n} v_i^k\right]^{\frac{1}{k}} \quad (10)$$

Substituting from (10) in (9), the following equation

$$\frac{\delta}{\delta k} \ln L_1(k, \lambda) = \frac{n}{k} + \sum_{i=1}^{n} \ln v_i - \frac{n \sum_{i=1}^{n} v_i^k \ln v_i}{\sum_{i=1}^{n} v_i^k} = 0 \quad (11)$$

By using Newton Raphson with equation (11) to solve for *k*

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (12)$$

Hence, the derivative becomes:

$$f'(k) = -\frac{n}{k^2} - n\left(\frac{\left(\sum_{i=1}^{n} v_i^k\right)\left(\sum_{i=1}^{n} v_i^k (\ln v_i)^2\right)}{\left(\sum_{i=1}^{n} v_i^k\right)^2} - \frac{\left(\sum_{i=1}^{n} v_i^k \ln v_i\right)^2}{\left(\sum_{i=1}^{n} v_i^k\right)^2}\right) \quad (13)$$

After converting the wind speed data to fit a Weibull distribution, The following parameters of the wind turbine (*Vci*), (*Vr*), (*Vco*), and (Pr) are used to get the output power from the turbine (PWt). The previous parameters are fixed for a particular wind turbine. Fig. 1 shows a curve describing the wind turbine output power [14]:

The maximum power from the wind data will be *PWm = T Pr*. The above curve can be formulated as follows:

$$PW(v_t) = \begin{cases} P_r\,(a v_t^3 - b), & for\ V_{ci} < v_t \le V_r \\ P_r, & for\ V_r < v_t \le V_{co} \\ 0, & otherwise. \end{cases} \quad (14)$$

Where $a = \frac{1}{(V_r^3 - V_{ci}^3)}$ And $b = \frac{V_{ci}^3}{(V_r^3 - V_{ci}^3)}$

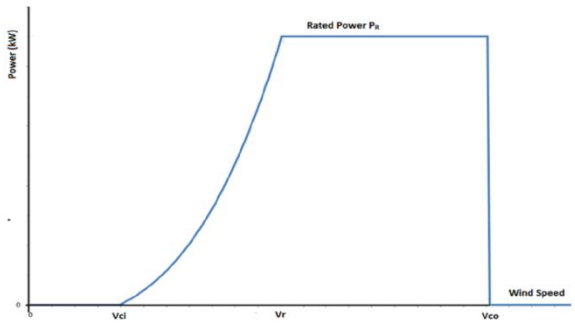


Fig. 1. Wind turbine model

The following equations provide a relation between the wind turbine parameters and the Weibull distribution parameters. The relation between input wind power and the output electrical power is called the capacity factor (CF). Capacity factor is the quotient of average power (P_{avg}) by the rated power (P_R). The average power produced by wind turbine is the integration of the turbine power curve multiplied by the probability density function.

CF derivation was done as follows:

$$CF = \frac{P_{avg}}{P_R} \quad (15)$$

$$P_{avg} = \int_0^{\infty} PW(v_t)\, f(v_t)\, dv_t \quad (16)$$

The turbine power curve is described in eqn. (14), the probability density function is described in eqn. (3), Using eqn. (3), (14), and (15) in eqn. (16) [15]:

$$\begin{aligned} CF_W &= \frac{1}{P_R}\int_0^{\infty} PW(v_t)\, f(v_t)\, dv \\ CF_W &= \frac{T}{V_r^3 - V_{ci}^3}\int_{V_{ci}}^{V_r} (v_t^3) \times \frac{k}{\lambda^k}\ v_t^{k-1}\ e^{-\left(\frac{v_t}{\lambda}\right)^k} dv_t \\ &\quad - \frac{TV_{ci}^3}{V_r^3 - V_{ci}^3}\int_{V_{ci}}^{V_r} \frac{k}{\lambda^k}\ v_t^{k-1}\ e^{-\left(\frac{v_t}{\lambda}\right)^k} dv_t \\ &\quad + \int_{V_r}^{V_{co}} T\frac{k}{\lambda^k}\ v_t^{k-1}\ e^{-\left(\frac{v_t}{\lambda}\right)^k} dv_t \end{aligned} \quad (17)$$

Then the following formula was derived:

$$\begin{aligned} CF_W = \frac{3\lambda^3 aT}{k}\Bigg\{\gamma\left(\frac{3}{k},\left(\frac{V_r}{\lambda}\right)^k\right) \\ - \gamma\left(\frac{3}{k},\left(\frac{V_{ci}}{\lambda}\right)^k\right)\Bigg\} \\ - Te^{-\left(\frac{V_{co}}{\lambda}\right)^k} \end{aligned} \quad (18)$$

III. MODEL VALIDATION

The developed capacity factor closed form equation is used to estimate the capacity factor of a given location. To ensure the degree of accuracy of the obtained capacity factors' values, statistical evaluation of the capacity factor has been done using Monte Carlo estimation (MCE).

A. Monte Carlo Estimation (MCE)

This technique aims to approximate the expectations of a function as follows [16]:

$$Ef(x) = \frac{C_{MCE}}{n}\sum_{i=1}^{n} f(x_i) \quad (19)$$

Where C_{MCE} equals the difference between the upper and lower bounds of the integration.

Applying equation (19) on equation (17), the following equations results:

$$\begin{aligned} E[CF_W] &= (E[CF_W])_1 + (E[CF_W])_2 \\ &= \frac{T(V_{ci} - V_r)}{n}\sum_{t=1}^{n}\left\{(av_t^3 - b) \times \frac{k}{\lambda^k}\ v_t^{k-1}\ e^{-\left(\frac{v_t}{\lambda}\right)^k}\right\} \\ &\quad + \frac{T(V_r - V_{co})}{n}\sum_{t=1}^{n}\left\{\frac{k}{\lambda^k}\ v_t^{k-1}\ e^{-\left(\frac{v_t}{\lambda}\right)^k}\right\} \end{aligned} \quad (20)$$

Equation (20), has been used to estimate the expected capacity factor by running MCE for large number of trials and averaging the expected output.

B. Convergence Diagnosis

Statistical convergence diagnostics was used to insure that the estimation were reliable and has minimal deviation from the exact answer. This was done using *Estimated Error (EE)*. The estimated error formula is as follows:

$$EE|_{MC} \approx \frac{\sigma}{\sqrt{N}} \quad (21)$$

IV. SIMULATION RESULTS

In this section, published wind data was selected as a simulation data [13]. The algorithm presented in section II was applied using this data to estimate the capacity factor of (site ID 11348) [13]. The rated power of the chosen turbine is21.4 kW and the cut in speed, rated speed and cutoff speed are Vci = 4.47, Vr = 8.84 , Vco = 22.3 m/s respectively. The calculated Weibull parameters $K\lambda$ for this turbine are presented in table II.

TABLE I
WEIBULL DISTRIBUTION PARAMETERS

Season	k	λ
Winter	1.299	7.825
Spring	1.669	6.483
Summer	1.379	5.610
Autumn	1.370	3.351

Then, the (CF_W) value was calculated for the different seasons of the year using the main definition as given in equation (18). Finally the using equation (20) $CF_{W(MCE)}$ was computed employing Monte Carlo Estimation. The results obtained are presented in table II and a comparison between the actual and estimated values is depicted in figure 2.

TABLE II
CAPACITY FACTOR VALUES FOR WINTER SEASON

Season	Winter	Spring	Summer	Autumn
Capacity Factor	0.398	0.316	0.255	0.071
Estimated (MCE)	0.397	0.306	0.256	0.070

The differences between the actual and estimated values are given in Table III. It's clear from this table that the results of the proposed mathematical model and Monte Carlo estimation match to very good degree.

TABLE III
ERROR CALCULATION FOR DIFFERENT SEASONS

	Winter	Spring	Summer	Autum
Error	$5.95x10^{-6}$	$7.01x10^{-6}$	$0.11x10^{-6}$	$1.49x10^{-6}$

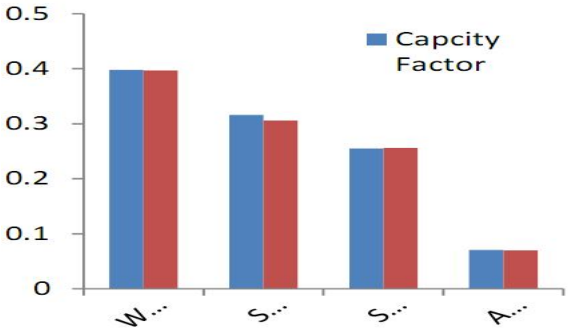


Fig. 2. Comparison between capacity factors of different seasons

CONCLUSION

This paper presented a novel capacity factor model for pitch controlled wind turbines based on cubic wind turbine model. The model linked the input wind speed with the wind turbine manufacturing data. Then the model was validated statistically with MCE and MCMC. Comparing the model estimated values with the exact values gives a low error values, thus showing the model accuracy. Also, the statistical nature of the model suites well the statistical nature of the input wind speed. The presented results shows that however some turbines have almost similar operating data, the CFw presented can choose optimally the best for the site installation, thus harvesting the maximum allowable power.

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