TIME-PERIODIC EXCITATION OF A SELF-SUSTAINED BUOYANT JET

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Abstract - We apply external time-periodic excitation to a self-sustained jet in the buoyancy-dominated regime where the Richardson number is on the order of unity. We examine the jet response in the shear layer and find evidence of a global shear mode in the form of period-1 oscillations. Above a critical Reynolds number and below a critical Richardson number, those oscillations become period-2 with a strong sub harmonic component. We find evidence of the torus-breakdown and intermittency routes to chaos, depending on the strength and winding number of the excitation. These findings pave the way for further studies on the forced synchronization of self-sustained buoyant jets.

Keywords - Chaos; Nonlinear dynamics; Intermittency; Oscillation

I. INTRODUCTION

Buoyant jets are present in many engineering applications and natural phenomena. Sometimes, they can become hydro dynamically self-sustained if they develop a global shear mode due to the presence of local absolute instability [1]. This causes the jet to oscillate at a well-defined natural frequency. The response of momentum-dominated self-sustained jets to external excitation at different frequencies has been looked at in the past [2,3], but the same cannot be said for buoyancy-dominated self-sustained jets whose Richardson number is on the order of unity. In particular, the way in which such buoyant jets transition from order to chaos is still an open question, but has important implications for flow control, process optimization, and low-order model development. In this paper, we investigate how a self-sustained buoyant jet responds to external excitation, with a focus on its transition to chaos via different universal routes found in nonlinear dynamical systems.

II. EXPERIMENTAL METHODS

We produce a buoyant jet in the laboratory using a stainless-steel convergent nozzle. We orient the jet upwards and use helium gas as the jet fluid. We measure the jet response with a constant-temperature hot-wire probe positioned in the shear layer. This position is selected to maximize the signal-to-noise ratio, so that even weak jet fluctuations can be detected. We process the data using a variety of techniques, including spectral analysis and phase space reconstruction. We identify different states/attractors of the jet for different strengths and frequencies of the external excitation, which is applied via a loudspeaker mounted downstream of the jet outlet.

III. RESULTS AND DISCUSSION

Figure 1 shows the forced synchronization diagram of the jet. Without any excitation, the jet is buoyancy-dominated, oscillating in a period-2 state with a strong sub harmonic mode. The Reynolds number based on the jet diameter is 630, while the Richardson number is 0.18. When excitation is applied even at a low amplitude, the jet transitions to a quasi periodic state with two incommensurate modes. This occurs for most values of the excitation frequency relative to the natural jet frequency (winding number). However, for some specific combinations of the excitation strength and frequency, we find that the jet transitions to chaos via the intermittency route as well as the torus-breakdown route, i.e. the quasi periodic route. Both these routes to chaos have been well studied in nonlinear dynamics and are generally considered to be universal [4].

The fact that we also find it here in a buoyant jet suggests that this hydrodynamic system could be studied via a nonlinear dynamical systems approach. As the excitation strength increases, we find that the jet eventually transitions to a synchronous state of lock-in where the natural global mode becomes dominated by the excitation mode.

This transition to lock-in does not always occur via chaos. For example, in Fig. 2, we show a sample case where the jet starts off from an unexcited self-sustained state, transitions to quasi periodicity, then to intermittency, and finally to lock-in – as the excitation strength increases for a fixed excitation frequency. This reveals the complexity present in even such a simple flow system, making the case for broader applications of a nonlinear approach to flow analysis based on synchronization concepts [5].

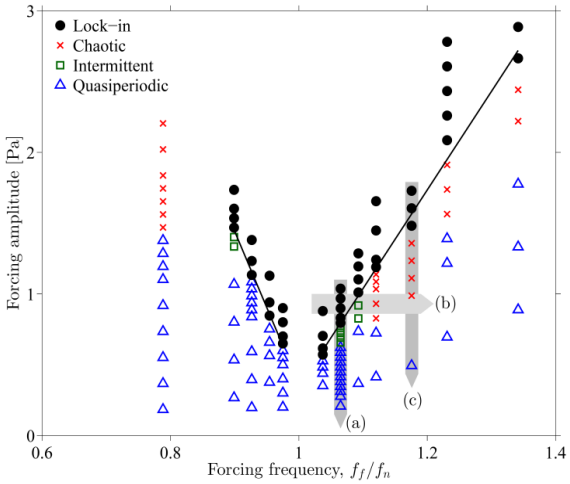

Fig 1: Forced synchronization diagram showing the different states of the self-sustained buoyant jet under external excitation at different amplitudes and frequencies.The Reynolds number based on the jet diameter is 630, while the Richardson number is 0.18.

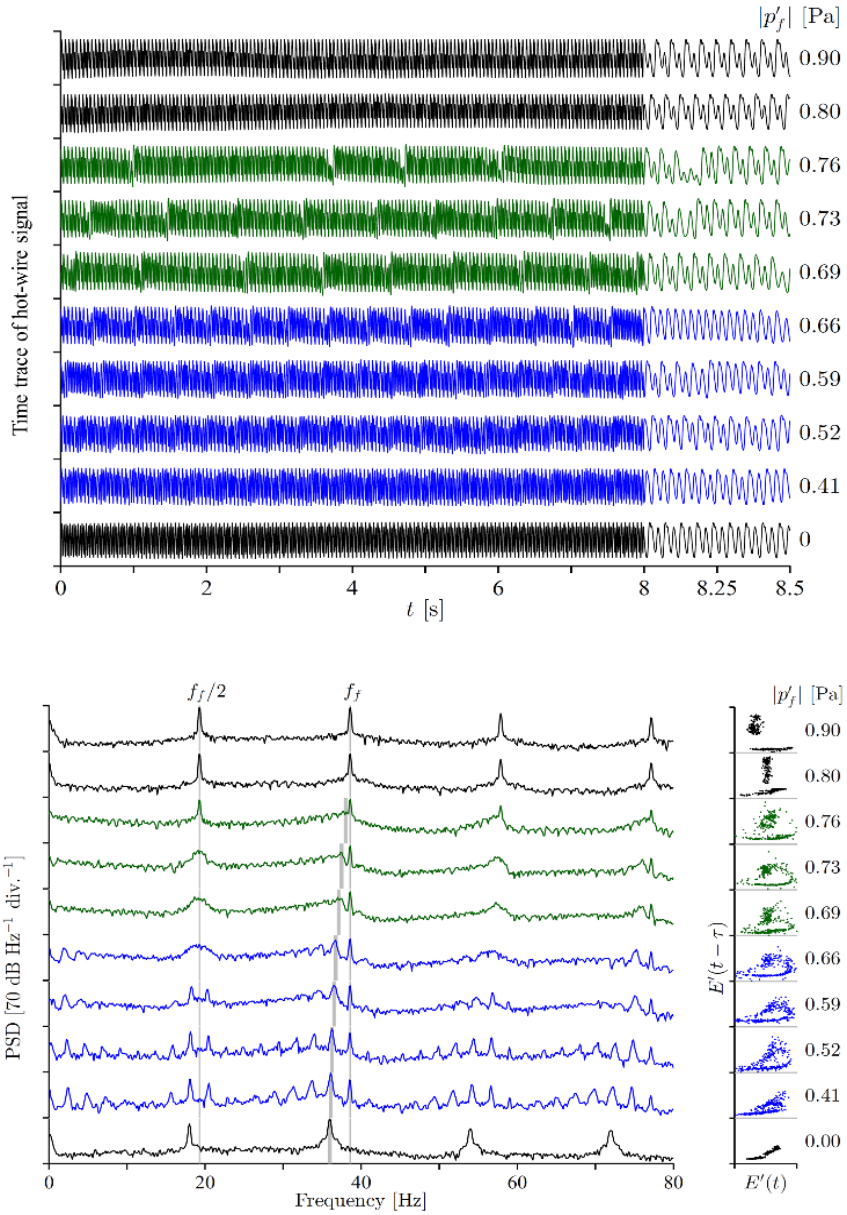

Fig. 2: Sample case showing transition to lock-in via intermittency without chaos. The top panel shows the time trace of the hot-wire signal. The bottom-left panel shows the power spectral density, while the bottom-right panel shows the Poincare map extracted via phase space reconstruction using Takens' embedding theorem. All the data are shown for increasing excitation strengths. The jet is excited at a frequency of 38.5 Hz, which corresponds to f_f/f_n = 1.065 in Fig. 1.

IV. CONCLUSION

We have shown that when a buoyancy-dominated jet is excited externally at a frequency away from its natural frequency, it can respond in various complicated ways. For example, it can transition to chaos via two classic routes: the intermittency route and the torus-breakdown route. Furthermore, it can also transition to lock-in, with or without passing through a chaotic regime, depending on the path taken in the excitation strength—frequency space. Future work will be directed at identifying the exact type of intermittency observed in the system, as well as other possible routes to chaos.

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