DESIGN TS FUZZY OBSERVER FOR A CLASS OF NONLINEAR HIV INFECTION DYNAMICS

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Abstract- This paper proposes an approach to design Takagi-Sugeno fuzzy observers for model of nonlinear dynamics of HIV infection by the method of the sector nonlinearity. The designing procedure method is based on a non-quadratic Lyapunov function. Based on LMI method, the problem can be formulated. The main merit of the presented observer and TS model is robustness against model uncertainty and disturbances, stability of the closed loop system, and convergence of the tracking and observer error to zero. Finally, the simulation result of a numerical example is given to illustrate the effectiveness of this method.

Index Terms- TS fuzzy model ; TS observer ; HIV Infection ; sector nonlinearity approach.

I. INTRODUCTION

HIV dynamic models, a set of ordinary differential equations (ODEs), have provided new understanding of the pathogenesis of HIV infection and the treatment effects of antiviral therapies. However, the estimation of the parameters for ODEs is very challenging due to the complexity of this nonlinear system. [1] proposes a comprehensive procedure to deal with this issue. HIV virus infects vital to the immune system, including T helper cell (CD4 + T), the Macrophage and dendritic cells. A large class of nonlinear systems can be exactly represented by Takagi-Sugeno (TS) fuzzy models [2]. The TS fuzzy model consists of a rule base, with the consequent of each rule being a linear or affine model. A constructive approach to obtain a TS model given a dynamic nonlinear system is the sector nonlinearity approach [3]. In order to analyze the stability of a TS fuzzy model, the direct Lyapunov approach has been used. Stability conditions have been derived using quadratic Lyapunov functions [4], [5], [6], piecewise continuous Lyapunov functions [7],[8], and more recently, nonquadratic Lyapunov functions [9],[10],[11].For discrete-time TS systems, nonquadratic Lyapunov functions have been successfully employed. However, their use for continuous-time fuzzy systems has been scarce, due to the fact that the analysis and synthesis using nonquadratic Lyapunov functions involve the derivatives of the membership functions, and without further assumptions on the membership functions, they lead to very conservative bounds. However, latest results [12] prove that such Lyapunov functions are extremely useful for establishing local stability of an equilibrium point, and for approximating the domain of attraction for fuzzy models obtained by the sector nonlinearity approach. Several types of observers have been developed for continuous-time TS fuzzy systems, among which: fuzzy Thau-Luenberger observers [4], [13], reduced-order observers [14], [15], and sliding-mode observers [16]. Most of the stability and design conditions rely on the feasibility of an associated system of linear matrix inequalities (LMIs), in general derived from the negative definiteness of a quadratic Lyapunov unction. Therefore, all these observers are globally valid. However, in general, nonlinear systems are not globally observable or detectable, and therefore, the design of a global observer is not always possible. Moreover, when using a quadratic Lyapunov unction, the conditions are conservative.

In this paper we want to control HIV illness with a Nonlinear control theory for reduced models with real data is proposed. A TS fuzzy model and Lyapunov function for a reduced model is constructed, and proof of its global stability is also provided in order we use TS fuzzy model with is the sector nonlinearity approach. We use non-quadratic Lyapunov functions to design local observers for TS systems obtained by the sector nonlinearity approach. For a fuzzy model, well-established methods and algorithms exist to design fuzzy observers. we, instead of designing a global observer, we design local observers. At the same time we also determine the domain where the observer is valid, i.e., where the estimation error converges to zero. Note that for the simplicity of the notation and computations, in this paper we only consider TS systems with a common measurement matrix. We distinguish two cases: 1) if the scheduling vector depends only on the measured variables, the design conditions can be formulated as LMIs; and 2) if the scheduling vector depends also on the states that have to be estimated, the approach has to be modified, and BMI conditions are obtain , the approach has to be modified. Since in this paper we only consider observer design, without state or output feedback control, the input vector is treated simply as a measured input (not necessarily control input). From

this point of view, the membership functions may depend on the input as it is considered a measured variable.

The paper is organized as follows. Section I presents the TS models obtained by the sector nonlinearity approached and the stability conditions our method relies on. In Section II presents the TS models of HIV illness and In Section III, the observer design conditions are derived, for the two cases: when the scheduling vector does not depend on the states to be estimated and when it does. The resulting conditions are summarized as LMI and BMI problems, respectively. Section IV illustrates the design methods on examples, and finally, Section V concludes the paper.

II. TS FUZZY MODEL

Consider a nonlinear system of the form [2]

$$\dot{x} = f(x,u)$$
$$y = cx \qquad (1)$$

With f and g smooth nonlinear matrix functions, $x \in R^n$ the state vector, $x \in R^{n_u}$ the input vector, and $x \in R^{n_y}$ the measurement vector, all variables assumed to be bounded on a compact set C_{xyu}.

Let $nl_j(.) \in \left[\underline{nl_j}, \overline{nl_j}\right]$ $j = 1,2,.....p$ be the set of bounded nonlinearities in f and g. Using the sector nonlinearity approach [2], an exact TS fuzzy representation of (1) can be obtained by constructing first the weighting functions

$$\omega_0^j(.) = \frac{\overline{nl_j} - nl_j(.)}{\overline{nl_j} - \underline{nl_j}} \qquad (2)$$
$$\omega_1^j(.) = 1 - \omega_0^j(.) \quad j = 1,2,3.....$$

z is the vector of scheduling variables, $z = T[x^T y^T u^T]^T$, T a constant matrix and defining the membership functions as [5]

$$h_i(z) = \prod_{j=1}^{p} \omega_{ij}^j(z_j) \qquad (3)$$

where $i = 1,2,.....,2^p, ij \in \{0,1\}$. Note that these membership functions are normal, i.e. $h_i(z) \geq 0$ and $\sum_{i=1}^{r} h_i(z) = 1$ $,r = 2^p$, where r is the number of rules.

Then, an exact representation of (1) is given as [2]:

$$\dot{x} = \sum_{i=1}^{r} h_i(z)(A_i x + B_i u)$$
$$y = C_i x \qquad (4)$$

with $i = 1,2,.....,2^p$, r the number of local linear models, A_i, B_i $i = 1,2,.....,r$ matrices of proper dimensions, and h_i, $i = 1,2,...p$ defined as in (3).

Controller and observer design for such systems often leads to establishing the negative definiteness of double summations of the form $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij}$ with Υ_{ij}, symmetric matrices of appropriate dimensions. In this paper we use the following relaxation [17]:

Theorem 1 : Let Υ_{ij} be matrices of proper dimensions. Then

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij} < 0 \qquad (5)$$

holds.if

$$\Upsilon_{ij} > 0 \quad i,j = 1,2,.....,r \qquad (6)$$

Based on stability conditions for the dynamics of the estimation error, observer design for systems of the form (4) has been largely investigated.. In this paper, we extend the novel local asymptotic stability conditions reported in [4] to design observer.

It is assumed $M_z = \sum_{i=1}^{r} h_i(z) M_i$ $i = 1,2,.....,r$ are matrices . i and 0 denote the identity and zero matrices of proper dimensions, and $H(x)$ denotes the Hermitian of the matrix X, where $H(x) = X + X^T$. The conditions in [12] have been developed for the autonomous TS system

$$\dot{x} = \sum_{i=1}^{m} h_i(z) A_i x \qquad (7)$$

with h_i defined as in (3) and are formulated as equation (3).

Theorem 2 : if $H(P_z A_z) < 0$ has solution $P_i = P_i^T > 0$ $i = 1,2,....,r$ then there exists a domain D including the origin such that the TS model (7) is locally asymptotically stable [11].

III. TS MODEL OF HIV ILLNESS

HIV-1 infection model is considered here, allows to describe the dynamics of infection (CD4 + T cell count and viral load), with special attention to initial conditions, as well as medication. It has four main blocks: the commitment, (PK)1 model, (PD)2 model, and the dynamics of the virus. This model shows in below Figure.

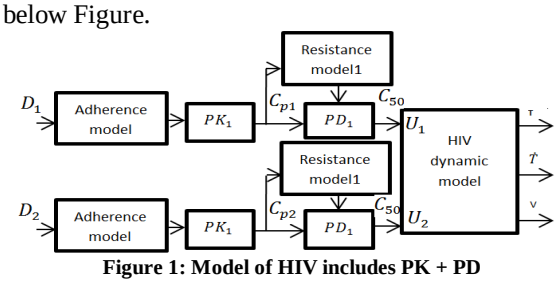


Figure 1: Model of HIV includes PK + PD

1 Pharmacokinetics
2 Pharmacodynamics

Consider the dynamic model of infection, HIV-1(8): Dynamic model of HIV illness consists three states that Shown in the formula (7), which are as follows [18]

$$\begin{cases} \dot{T} = s - dT - (1-u_1)\beta T\upsilon \\ \dot{T}^* = (1-u_1)\beta T\upsilon - \mu T^* \\ \dot{\upsilon} = (1-u_1)KT^* - c\upsilon \end{cases} \quad (8)$$

Where T is healthy cells, $T*$ shows infected cells, υ presents the free virus in the blood.

Table I. Parameter values.

Parameters	Description	Unit of measurement
s	Rate of healthy CD4 + T cell regeneration	$cellmm^{-3}day^{-1}$
d	Rate of healthy CD4 + T cell death	day^{-1}
β	Rate of healthy CD4 + T cell death	$particle^{-1}\ mm^3day^{-1}$
μ	The average rate of healthy CD4 + T cell death	day^{-1}
K	The coefficient of the amount of virus	$particle\ cell^{-1}day^{-1}$
C	The average death rate of the virus	day^{-1}
u_1	RTI effect	----------
u_2	PI effect	----------

Let consider non-linear terms m and $(T_2)_{jl+n_y}$, $l=1,\ldots n_u$, by M_{jl}, $l=1,\ldots,n+2n_u$.

Then we have [19]

$$\dot{P}_z = \sum_{i=1}^{r} h_i(z) \sum_{j=1}^{p}\sum_{k=1}^{r} h_k(z) \times \sum_{l=1}^{n+2n_u} q_{jl}\ M_{jl}\left(P_{g_1(i,j)} - P_{g\,2(i,j)}\right) \quad (9)$$

Knowing the bounds $|q_{jl}| < \lambda_{jl}$, $l=1,\ldots,n+2n_u$ and using the property that $Y + q_{jl}X \le 0$, if

$$\begin{aligned} Y\ &+\lambda_{jl}X\ \le 0 \\ Y\ &-\lambda_{jl}X\ \le 0 \end{aligned} \quad (10)$$

LMI constraints can be formulated, as follows [19]:

Theorem 3 : if there exists matrices $P_i = P_i^T > 0$ and $L_i, i=1,2,\ldots..,r$ such that

$$\begin{aligned} &\Upsilon_{ij}^m < 0 \quad i,j=1,2,\ldots,r \\ &\frac{2}{r-1}\Upsilon_{ii}^m + \Upsilon_{ij}^m + \Upsilon_{ji}^m < 0 \quad i,j=1,2,\ldots,r \\ &m=1,\ldots,2^{p(n+2n_u)}\ i \ne j \end{aligned} \quad (11)$$

With

$$\Upsilon_{ij}^m = P_iA_j - L_jC + A_j^TP_i - C^TL_j^T + \sum_{k=1}^{p}\sum_{l=1}^{n+2n_u}(-1)^{d_{kl}^m}\lambda_{kl}M_{kl}\left(P_{g_1(i,k)} - P_{g\,2(i,k)}\right) \quad (12)$$

hold where d_{kl}^m are obtained from the binary representation of $M-1 = d_{pn+2n_u}^m + d_{pn+2n_u-1}^m \times 2 + \ldots + d_{11}^m \times 2^{n+2n_u}$ and $g_1(i,k), g_2(i,k)$ are defined as in (17), then the estimation error of the observer (10) tend to zero exponentially for any trajectory of x satifying (4) and $\bigcap_{k,l}\left\{(x,u,\dot{u}) : |q_{jl}| \le \lambda_{kl}\right\}$.

Proof: Follows immediately from Theorem 2 of [12].

Remark: Note that the condition derived concerns x, since the local observability of the system depends on the actual states, not on the estimated states. It is also possible to derive conditions on $\hat{x}$, although those attract a condition on e.

if one uses instead of (10) the property that for any symmetrical positive definite matrix S ,

$$Y + q_{jl}X \le Y + \frac{1}{2}(\lambda_{jl}^2 + XS^{-1}X)$$

then the results can be formulated as:

Theorem 4: If there exists matrices $P_i = P_i^T > 0$ and $L_i, i=1,2,\ldots..,r$ $S_{kl}, k=1,2,\ldots,p, l=1,2,\ldots,n+2n_u$ such that [19]

$$\begin{aligned} &\Upsilon_{ij}^m < 0 \quad i,j=1,2,\ldots,r \\ &\frac{2}{r-1}\Upsilon_{ii}^m + \Upsilon_{ij}^m + \Upsilon_{ji}^m < 0 \quad i,j=1,2,\ldots,r \\ &m=1,\ldots,2^{p(n+2n_u)}\ i \ne j \end{aligned} \quad (13)$$

hold with

$$\Upsilon_{ij}^m = P_iA_j - L_jC + A_j^TP_i - C^TL_j^T + \frac{1}{2}\sum_{k=1}^{p}\sum_{l=1}^{n+2n_u}\lambda_{kl}^2S_{ku} \quad (14)$$

and $\bar{P}_{(i,j)} = \bar{P}_{g1(i,j)} - \bar{P}_{g2(i,j)}$ where $(*)$ denotes the symmetric term and $g_1(i,k), g_2(i,k)$ are defined as in (16), then the estimation error e of the observer (10) tends to zero exponentially for any trajectory of x satisfying (4) in the outermost Lyapunov level contained in $\cap_{k,l}\left\{(x,u,\dot{u}) : |q_{kl} \le \lambda_{kl}|\right\}$.

Proof: Follows immediately [4].

Remark: In general, the observer will not be valid on the whole domain C_{xyu} where the TS system is defined. In order to find estimates of the region where the observer is valid, the bounds λ_{kl} may be treated as variables and maximized. However, if λ_{kl} are treated as variables, a BMI problem is obtained.

IV. SIMULATION RESULT

In this section, we illustrate the proposed observer design on one simulation examples with Measured scheduling vector approach.

Where $x_1(t)$ denotes amount (quantity) of healthy CD4+ T cells, $x_2(t)$ denotes the infected CD4+ T cells, $x_3(t)$ denotes the viral load, and the positive constants S, D , and $x_3(t)$ and μ_2 denote the system parameters for an HIV dynamical model described as the rate of production of the healthy cells, the death rate of the health calls, the infection rate of healthy cells CD4 by virus HIV, the death rate of infected cells, the rate of production of free virus, and the death rate of free virus, respectively.

According to the proposed method, the obtained values of the matrix are as follows.

$$A_1=\begin{pmatrix}-0.8 & 0 & -0.94\\ 0 & -0.8 & 0.94\\ 0 & 1 & -0.01\end{pmatrix} B_1=\begin{pmatrix}0.725 & 0 & 0\\ -0.725 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$A_2=\begin{pmatrix}-0.8 & 0 & -0.94\\ 0 & -0.8 & 0.94\\ 0 & 1 & -0.01\end{pmatrix} B_2=\begin{pmatrix}0.725 & 0 & 0\\ -0.725 & 0 & 0\\ 0 & -0.27 & 0\end{pmatrix}$$

$$A_3=\begin{pmatrix}-0.8 & 0 & -0.94\\ 0 & -0.8 & 0.94\\ 0 & 1 & -0.01\end{pmatrix} B_3=\begin{pmatrix}0.83 & 0 & 0\\ -0.83 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$A_4=\begin{pmatrix}-0.8 & 0 & -0.94\\ 0 & -0.8 & 0.94\\ 0 & 1 & -0.01\end{pmatrix} B_4=\begin{pmatrix}0.83 & 0 & 0\\ -0.83 & 0 & 0\\ 0 & -0.27 & 0\end{pmatrix}$$

$$A_5=\begin{pmatrix}-0.8 & 0 & -1\\ 0 & -0.8 & 1\\ 0 & 1 & -0.01\end{pmatrix} B_5=\begin{pmatrix}0.725 & 0 & 0\\ -0.725 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$A_6=\begin{pmatrix}-0.8 & 0 & -1\\ 0 & -0.8 & 1\\ 0 & 1 & -0.01\end{pmatrix} B_6=\begin{pmatrix}0.725 & 0 & 0\\ -0.725 & 0 & 0\\ 0 & -0.27 & 0\end{pmatrix}$$

$$A_7=\begin{pmatrix}-0.8 & 0 & -1\\ 0 & -0.8 & 1\\ 0 & 1 & -0.01\end{pmatrix} B_7=\begin{pmatrix}0.83 & 0 & 0\\ -0.83 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$A_8=\begin{pmatrix}-0.8 & 0 & -1\\ 0 & -0.8 & 1\\ 0 & 1 & -0.01\end{pmatrix} B_8=\begin{pmatrix}0.83 & 0 & 0\\ -0.83 & 0 & 0\\ 0 & -0.27 & 0\end{pmatrix}$$

Here, the assumptions are $x_1(t) \in$ [0.94 1], $x_2(t) \in$ [0.2 0.27] and $x_3(t) \in$ [0.8 0.83] and observer gain is $K=10$. The system parameters in this example are set as follows: $S = 1, d = 0{:}8$, $\beta = 1$, $k = 1$ and $\mu_1 = 0.8$ $\mu_2 = 0.01078$

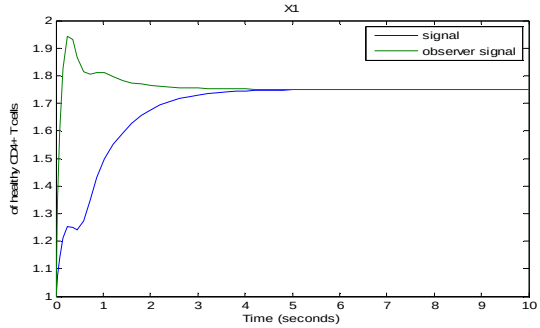


Figure 2: healthy CD4+ T cells with use observer

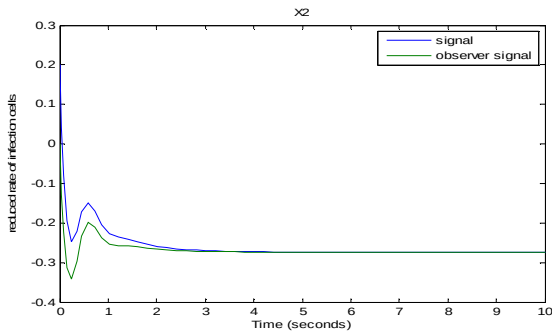


Figure 3: infected cells with use observer

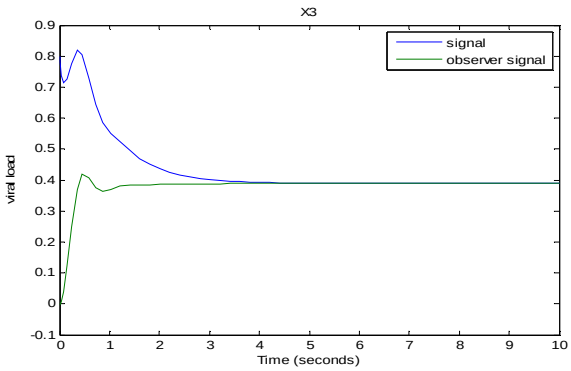


Figure 4: free virus in the blood with use observer

The above figures show the promising performance of the proposed observer.

CONCLUSIONS

This paper plans the local observer design method for TS fuzzy systems obtained by the sector nonlinearity approach. The main advantages of this designing procedure are presentation a general framework for the designing of non-quadratic observer and controller for TS fuzzy models, convergence of the observer error to zero, the common measurement matrix for all the

rules. Finally, applying the proposed observer on the HIV model shows promising performance.

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