DESIGNING TS OBSERVERS FOR A CLASS OF NONLINEAR DYNAMICS OF THE BLOOD GLUCOSE FOR TYPE I DIABETES

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Abstract- This paper presents an approach to design observer for Takagi-Sugeno fuzzy model of the Blood glucose regulation based on the sector nonlinearity method. The Lyapunov stability criterion of the closed loop system and the incremental quadratic inequalities are outlined in the terms of linear matrix inequalities (LMI). Robustness against uncertainty and disturbance, consideration of the direct effect of insulin and glucagon in model, convergence of the observer error to zero, stability of the closed loop system are the main merits of the proposed observer. The simulation results show the promising performance of the proposed observer.

Index Terms- TS fuzzy model; TS observer; Type I Diabetes; the sector nonlinearity approach.

I. INTRODUCTION

The normal blood glucose concentration level in human is in a narrow range, 70–110 mg/dl, Different factors including food intake, rate of digestion and exercise can affect this concentration. Two pancreatic endocrine hormones, insulin and glucagon are responsible for regulating the blood glucose level. They are secreted from α and β cells of pancreas, respectively. These two hormones form two feedback loops in controlling the blood sugar level that function inversely diabetes, also called diabetes mellitus, is considered to be a disease of glucose–insulin regulatory system in which, the blood glucose level stays constantly high. If someone's glucose concentration level is constantly out of range, this person is considered to have blood glucose problem. Implementing tight glucose control in critically ill patients is the most important issue in diabetes management. The current medical treatments suggest three to four daily glucose measurements and an equivalent number of subcutaneous insulin injections [1]. An alternative approach is to deliver insulin using a closed-loop device like a pump, which works like an artificial pancreas [2][3]. This closed-loop device would include a glucose sensor imbedded under the skin and an insulin pump implanted in the abdomen. The sensor can measure blood glucose concentration and pass the information to a feedback control system that would calculate the necessary insulin delivery that use a TS fuzzy system to keep the patient under metabolic control. A signal will be sent to an insulin pump by the controller, to deliver the desired amount of insulin. The pump injects insulin through a catheter placed under the patient's skin. Since the latest generation of the implantable pumps allows different infusion rates of insulin, the feedback control system mimics the normal function of a pancreas more closely. [4]

A large class of nonlinear systems can be exactly represented by Takagi-Sugeno (TS) fuzzy models [5]. The TS fuzzy model consists of a rule base, with the consequent of each rule being a linear or affine model. A constructive approach to obtain a TS model given a dynamic nonlinear system is the sector nonlinearity approach. In order to analyze the stability of a TS fuzzy model, the direct LMI approach has been used. Stability conditions have been derived using. Several types of observers have been developed for continuous-time TS fuzzy systems such as: fuzzy Thau-Luenberger observers [6], [7], reduced-order observers [8], [9], and sliding-mode observers [10]. Most of the stability and design conditions rely on the feasibility of an associated system of linear matrix inequalities (LMIs), in general derived from the negative definiteness of a quadratic Lyapunov unction. However, in general, nonlinear systems are not globally observable or detectable, and therefore, the design of a global observer is not always possible. In this paper we want to control blood glucose concentration with a nonlinear control theory for reduced models with real data is proposed. A TS fuzzy model and LMI approach for a reduced model is constructed, and proof of its stability is also provided in order we generate suitable TS fuzzy model with the sector nonlinearity approach. We use LMI approach to design observers for TS systems obtained by the sector nonlinearity approach. For a fuzzy model, well established methods and algorithms exist to design fuzzy observers. At the same time we also determine the domain where the observer is valid where the estimation error converges to zero. Note that for the simplicity of the notation and computations, in this paper we only consider TS systems with a common measurement matrix. From this point of view, the membership functions may depend on the input as it is considered a measured variable.

The paper is organized as follows. Section I presents the TS models obtained by the sector nonlinearity approached and the stability conditions of our method is derived. Section II presents the TS models of blood glucose and In Section III, the observer design conditions are derived. Section IV illustrates the design methods on simulation result, and finally, Section V concludes the paper.

II. TS FUZZY MODEL

Consider a nonlinear system of the form[5]

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= cx \end{aligned} \tag{1}$$

with f and g smooth nonlinear matrix functions, $x \in R^n$ the state vector, $x \in R^{n_u}$ the input vector, and $x \in R^{n_y}$ the measurement state vector, all variables assumed to be bounded on a compact set C_{xyu}.

Let $nl_j(.) \in \left[\underline{nl_j}, \overline{nl_j}\right]$ $j = 1,2,.....p$ be the set of bounded nonlinearities in f and g. Using the sector nonlinearity approach [5], an exact TS fuzzy representation of (1) can be obtained by constructing first the weighting functions

$$\begin{aligned} \omega_0^j(.) &= \frac{\overline{nl_j} - nl_j(.)}{\overline{nl_j} - \underline{nl_j}} \\ \omega_1^j(.) &= 1 - \omega_0^j(.) \quad j = 1,2,3..... \end{aligned} \tag{2}$$

z is the vector of scheduling variables , $z = T[x^T y^T u^T]^T$, T a constant matrix and defining the membership functions as [5]

$$h_i(z) = \prod_{j=1}^{p} \omega_{ij}^j(z_j) \tag{3}$$

where $i = 1,2,.....,2^p, ij \in \{0,1\}$. Note that these membership functions are normal, i.e. $h_i(z) \geq 0$ and $\sum_{i=1}^{r} h_i(z) = 1$ $,r = 2^p$, where r is the number of rules. Then, an exact representation of (1) is given as [9]:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} h_i(z)(A_i x + B_i u) \\ y &= C_i x \end{aligned} \tag{4}$$

with $i = 1,2,.....,2^p$, r the number of local linear models, A_i, B_i $i = 1,2,.....,r$ matrices of proper dimensions, and h_i , $i = 1,2,...p$ defined as in (3).

Controller and observer design for such systems often leads to establishing the negative definiteness of double summations of the form $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij}$ with Υ_{ij} , symmetric matrices of appropriate dimensions. In this paper we use the following relaxation [17]:

Theorem 1 : Let Υ_{ij} be matrices of proper dimensions. Then

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij} < 0 \tag{5}$$

holds if

$\Upsilon_{ij} > 0$ $i, j = 1,2,.....,r$ based on stability conditions for the dynamics of the estimation error, observer design for systems of the form (4) has been largely investigated. It is assumed that system (4) is local observable. In this paper, we extend the novel local asymptotic stability conditions reported in [6] to design observer.

It is assumed $M_z = \sum_{i=1}^{r} h_i(z) M_i$ $i = 1,2,.....,r$ are matrices . i and 0 denote the identity and zero matrices of proper dimensions, and H(x) denotes the Hermitian of the matrix X , where $H(x) = X + X^T$.

The conditions in [18] have been developed for the autonomous TS system

$$\dot{x} = \sum_{i=1}^{m} h_i(z) A_i x \tag{6}$$

with hidefined as in (3) and are formulated as equation (3).

Theorem 2 : if $H(P_z A_z) < 0$ has solution $P_i = P_i^T > 0$ $i = 1,2,....,r$ then there exists a domain D including the origin such that the TS model (6) is locally asymptotically stable [11].

INSULIN–GLUCOSE REGULATION DYNAMICAL MODEL

Until now, a wide range of models has been used to describe the insulin–glucose regulatory system dynamics in the human body. One of the pioneers in this task was Dr Richard Bergman, who developed the so-called 'Minimal Model. Bergman minimal model, which is a commonly referenced model in the literature, approximates the dynamic response of a diabetic patient's blood glucose concentration to the insulin injection using the following nonlinear differential equations [12,13]:

$$\begin{aligned} \dot{G} &= -P_1\left[G(t) - G_b\right] - X(t)G(t) + D(t) \\ \dot{X} &= -P_2 X(t) + P_3\left[I(t) - I_b\right] \\ \dot{I} &= -n\left[I(t) - I_b\right] - \gamma\left[G(t) - h\right]^+ t + u(t) \end{aligned} \tag{7}$$

Where the state and the parameters of the system are shown in table (1).

Table I Defination of Parameter

parameter	Definition
+	positive reflection to glucose intake
G(t)	glucose concentration in the blood plasma (mg/dl)
X(t)	presents the insulin's effect on the net glucose disappearance
I(t);	insulin concentration in plasma at time t(μ U/ml)
G_b	basal pre-injection level of glucose (mg/dl)
I_b	basal pre-injection level of insulin(μ U/ml)

P_1	insulin-independent rate constant of glucose uptake in muscles and liver (1/min)
P_2	rate for decrease in tissue glucose uptake ability (1/min)
P_3	insulin-dependent increase in glucose uptake ability in tissue per unit of insulin concentration above the basal level $\left[\mu U/ml/min^2\right]$
n	first-order decay rate for insulin in blood (1/min)
h	threshold value of glucose above which the pancreatic β cells release insulin (mg/dl)
γ	rate of the pancreatic β cells' release of insulin after the glucose injection with glucose concentration above the threshold $\left[\mu U/ml/min^2/mg/dl\right]$

These two factors are part of the so-called 'Metabolic Portrait' for every person and are important indicatives of how glucose and insulin act inside that person's body. The term, $\gamma\left[G(t)-h\right]^+$;in the third equation of the model acts as an internal regulatory function that formulates the insulin secretion in the body, which does not exist in diabetic patients. It has been also argued in [14] that for diabetic subjects, the value of P_1 will be significantly reduced; therefore, it can be approximated as zero. The parameters of the model and their values are introduced in [14]. It is worth noting that all the values are calculated for a person of average weight and these are not constant numbers and vary from patient to patient, which makes the design of the controller a more challenging task. To show the complete dynamics of the glucose–insulin regulatory system, two other terms are considered in Equation (7). $D(t)$ shows the rate at which glucose is absorbed to the blood from the intestine, following food intake. Since in diabetic patients, the normal insulin regulatory system does not exist, this glucose absorption is considered as a disturbance for the system dynamics presented in (7). This disturbance can be modelled by a decaying exponential function of the following form [14]:

$$D(t)=A\exp(-bt), \quad b>0 \tag{8}$$

To derive the TS model of Blood glucose regulation, we consider nonlinear terms as a vector scheduling

$$Z_1=GX\in[75,110]$$

$$\eta_0^1(z)=(1-Z_1)/(110-75),\eta_1^1(z)=1-\eta_0^1(z)$$

$$\omega_1=\eta_0^1,\omega_2=\eta_1^1$$

III. OBSERVER DESIGN

In practice, if states are not available for measurement and feedback, an observer is needed. The objective is that the estimation error that is the difference between the system states and the observer states tends to zero, in other terms fuzzy observer is required to satisfy $x(t)-\hat{x}(t)\to 0$ when $t\to\infty$, where $\hat{x}(t)$ denotes the states vector estimated by the fuzzy observer .The observer rule is in the following from [15],[16]:

Observer rule i^{th} :

$$\text{if } \to Z_1(t) \text{ is } M_{i1} \text{ and} \ldots Z_p(t) \text{is} M_{ip}$$

$$\text{THEN}\begin{cases}\dot{\hat{x}}(t)=A\hat{x}(t)+B_iu+K_i\left(y(t)-\hat{y}(t)\right)\\ \hat{y}(t)=C_i\hat{x}(t), \quad i=1,2,\ldots.r\end{cases}$$

(9)

where the variables $z(t)$ are independent from the estimated state variables, K_i $i=1,2,.....,r$ are the observation gain matrices and $\hat{y}(t)$ is the estimated output. Then, the final outputs of the fuzzy observer are inferred as follows [9]:

$$\dot{\hat{x}}(t)=\sum_{i=1}^{r}h_i(z(t))(A\hat{x}(t)+B_iu(t)) \\ +\sum_{i=1}^{r}h_i(z(t))K_i\left(y(t)-\hat{y}(t)\right) \tag{10}$$

$$\hat{y}(t)=\sum_{i=1}^{r}h_i(z(t))C_i\hat{x}(t) \tag{11}$$

By substituting (4) and (10) into (11), we obtain [9],[7] :

$$\dot{\hat{x}}(t)=\sum_{i=1}^{r}h_i(z(t))(A\hat{x}(t)+B_iu(t)) \\ +\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))K_iC_i\left(x(t)-\hat{x}(t)\right) \tag{12}$$

That can be writen as

$$\dot{\hat{x}}(t)=\sum_{i=1}^{r}h_i(z(t))h_j(z(t))[(A_i-K_iC_i)\hat{x}(t) \\ +B_iU_i(t)+K_iC_ix(t)] \tag{13}$$

The design of the fuzzy observer is to determine the local gains K_i using stability conditions proposed in [17], such that the estimation error tends to zero. Hence, the observer dynamics is stable if there exists positive definite matrices $P_{01},P_{02},\ldots..,P_{0r}$ and matrices $K_1,K_2,\ldots..,K_r$. Such that the following is satisfied [16]

$$P_{0i}>0, \quad i=1,2,\ldots.,r \tag{14}$$

$$\sum_{p=1}^{r}\varnothing P_{0p}+\left(G_{jj}^T+P_{oi}G_{jj}\right)<0 \quad i,j=1,2,..,r \tag{15}$$

$$\left\{\frac{G_{jk}+G_{kj}}{2}\right\}^T P_{0i}+P_{0i}\left\{\frac{G_{jk}+G_{kj}}{2}\right\}<2, \tag{16}$$

$$\forall i,j,k\in\{1,2,\ldots.,r\}\,s.t. \quad j<k$$

$$\begin{bmatrix}1 & x^T(0)\\ x(0) & P^{-1}\end{bmatrix}\geq 0 \quad for\ i=1,\ldots,r \tag{17}$$

$$\begin{bmatrix}\varnothing_pP_{0i} & w_{ijpl}^T\\ w_{ijpl} & \varnothing_pI\end{bmatrix}\geq 0, \quad \forall i,j,p\in\{1,2,\ldots.,r\} \tag{18}$$

Where $G_{jk}=A_j-K_jC_k$, $G_{jj}=A_j-K_jC_j$ and

$w_{ijpl} = \xi_{pl}(A_i - K_i C_j)$. These inequalities can be recast in terms of LMIs by the following changes of Variables [16]:

$$P_{0i} = X_{0i}^{-1} \quad \forall \in \{1,2,\ldots,r\}$$

$$X_{0i} = \alpha_{ij} X_{0j} \; s.t \; \alpha_{ij} = \frac{1}{\alpha_{ji}} \; \forall i, j \in \{1,2,\ldots,r\} \; , i \neq j \qquad (19)$$

$$K_i = \beta_{ij} K_j \; s.t \; \beta_{ij} = \frac{1}{\beta_{ji}} \; \forall i, j \in \{1,2,\ldots,r\} \; , i \neq j$$

$$N_i = K_j C_i X_{oi} \qquad \forall i \in \{1,2,\ldots,r\}$$

The coefficients, α_{ij} , β_{ij} and $\varnothing_p$ for $i, j, p = 1,2,...,r$ and $i \neq j$,can be chosen heuristically according to the considered application. α_{ij} and β_{ij} must be different from 1 (for $i = j$, $\alpha_{ij} = \beta_{ij} = 1$). However, selection of ξ_{pl} is obtained from $h_i(z(t))$. Subsequently, we will consider that the premises variables do not depend on the estimated states $\hat{x}(t)$ [18, 19].

IV. SIMULATION RESULT

In this section, the proposed observer is applied on the model of the Blood Glucose for Type I Diabetes that mentioned in equation (7). The parameters of the mentioned model are shown in table (2).

Table II. Parameter values.

parameter	value
P_1	0.0317
P_2	0.0123
P_3	4.92×10^{-6}
γ	0.0039
n	0.2659
h	79.0353
G_b	70
I_b	7
G_o	291.2
I_o	346.8

The matrices of the TS uzzy model is derived as follow.

$$A_1 = \begin{pmatrix} -0.0317 & -75 & 0 \\ 0 & -0.0123 & 4.92\times10^{-6} \\ 0.0039 & 0 & -0.2659 \end{pmatrix} B_1 = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.0317 & -110 & 0 \\ 0 & -0.0123 & 4.92\times10^{-6} \\ 0.0039 & 0 & -0.2659 \end{pmatrix} B_2 = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

In this systeme, it is considered Initial conditions $G(0) = 160$ this means Blood glucose level for one person is 160 and Insulin's value that is the input of this systeme is 10 vial and observer gain is $K = 50$. It is assumed that $x_1(t)$ is the glucose concentration in the blood plasma, $x_2(t)$ is the insulin's effect on the net glucose disappearance, $x_3(t)$ is the insulin concentration in plasma at time.

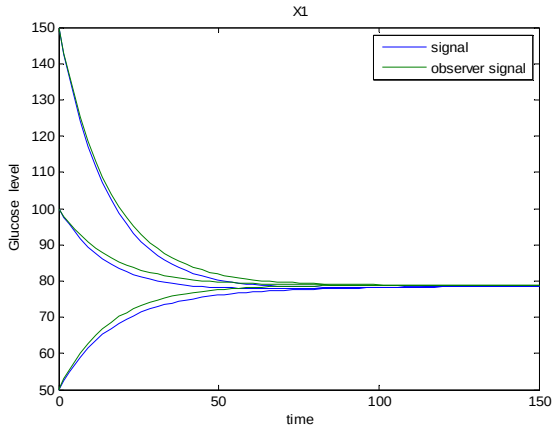


Figure 1: Glucose level of the model and the observer

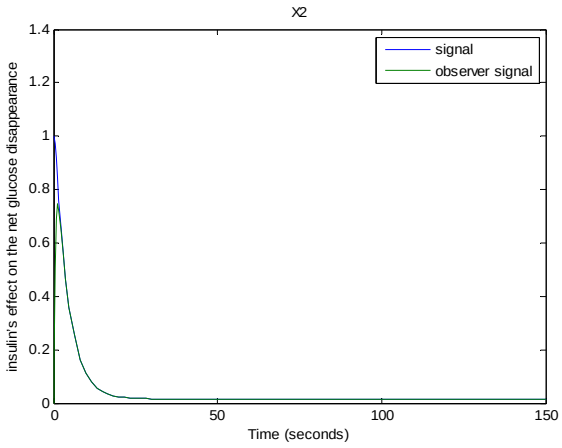


Figure 2: insulin's effect of the model and the observer

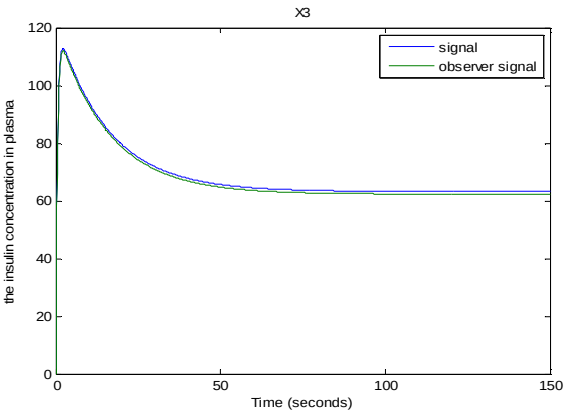


Figure 3: the insulin concentration in plasma

Based on the figures (1-3), the proposed observer has promising performance. These show that the observer error converge to zero, and the stability of the closed loop system is guaranteed .

CONCLUSIONS

This paper presented an observer design method for TS fuzzy systems obtained by the sector nonlinearity method. In designing procedure, it has been considered that the measurement matrix is common for all the rules bases of the TS fuzzy system. Stability of the closed loop system, convergence of the observer error to zero are the main advantages of the proposed method. The simulation results show that proposed TS model and observer has promising performance.

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