ON THE ERR AT THE BAND CRACK FRONT IN A RECTANGULAR PRISM

A. TURAN DINCEL

Yildiz Technical University, Fac. Chemistry and Metallurgy, Dep. Mathematical Engineering,
Davutpasa Campus , 34210 Esenler, Istanbul, Turkey

Abstract— In this study, the rectangular prism containing band-crack whose edges are parallel to the face planes of the prism is considered. It is assumed that uniformly distributed normal extensional forces act on the upper and lower planes. Mathematical modelling of the problem is made by using Three-dimensional Linear Theory of Elasticity. The boundary value problem is solved numerically by employing 3D FEM. The numerical results on the Energy Release Rate (ERR) are presented with different geometric parameters.

Keywords— Band crack, Energy Release Rate, 3D FEM.

I. INTRODUCTION

Fracture of structural members is one of the most important problems encountered in various engineering applications. In these problems, one of fracture mechanics parameters is Energy Release Rate (ERR) to describe the fracture behaviour on cracked structural members. There are many studies in the published literature on the calculation of the ERR, [1-8]. And there are few studies about the determination of the effects of geometrical parameters on the SIFs and the ERR, [9-10]. However, these studies are analyzed within the framework of the two-dimensional (2D) problem formulation.

In the present paper, we assume that the band crack is in the rectangular prism and we aim to determine the effects of geometrical parameters on the ERR at the band crack front. The corresponding boundary-value problem is modelled by the use of the equations and relations of the Three-dimensional Linear Theory of Elasticity and this problem is solved by employing FEM modelling.

II. FORMULATION OF THE PROBLEM

Consider a rectangular prism containing band crack. The geometry of the half-part prism is shown in Fig.1. Assume that the plate occupies the region $\{0 \le x_1 \le \ell,\ 0 \le x_2 \le h, 0 \le x_3 \le t\}$.

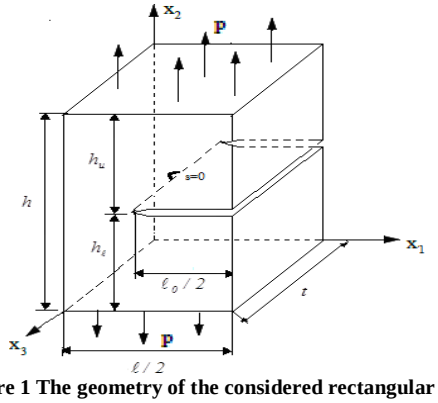


Figure 1 The geometry of the considered rectangular prism

The stress-strain state can be determined by the solution to the boundary value problem given below:

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0,\ \sigma_{ij} = C_{ijkn}\varepsilon_{kn},\ \varepsilon_{kn} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_n} + \frac{\partial u_n}{\partial x_k}\right),$$

$$u_2\big|_{x_1=0} = u_2\big|_{x_1=\ell} = 0,\ u_2\big|_{x_3=0} = u_2\big|_{x_3=t} = 0,\ \sigma_{2i}\big|_{x_2=0} = \sigma_{2i}\big|_{x_2=h} = p\delta_i^2,$$

$$\sigma_{1i}\big|_{x_1=0} = \sigma_{1i}\big|_{x_1=\ell} = 0, \sigma_{3i}\big|_{x_3=0} = \sigma_{3i}\big|_{x_3=t} = 0$$

$$\sigma_{2i}\big|_{x_2=h_{\ell\pm 0}} = 0 \qquad i;j;k;n=1,2,3 \quad (1)$$

$$(\ell/2-\ell_0/2)<x_1<(\ell/2+\ell_0/2)$$

$$0<x_3<t$$

This completes mathematical formulation. After using this determination, ERR at the crack front is found.

III. FEM MODELLING

We will analyze this problem by using the 3D FEM. For the FEM modeling of the boundary value problem, the functional

$$\Pi = \frac{1}{2}\iiint_{\Omega-\Omega'} (\sigma_{11}\varepsilon_{11} + 2\sigma_{12}\varepsilon_{12} + 2\sigma_{13}\varepsilon_{13} + 2\sigma_{23}\varepsilon_{23} + \sigma_{22}\varepsilon_{22} + \sigma_{33}\varepsilon_{33})d\Omega$$

$$-\int_0^t \int_0^\ell p\, u_2\big|_{x_2=h}\, dx_1 dx_3 - \int_0^t \int_0^\ell p\, u_2\big|_{x_2=0}\, dx_1 dx_3 \quad (2)$$

where

$$\Omega = \{0 \le x_1 \le \ell,\ 0 \le x_2 \le h,\ 0 \le x_3 \le t\},$$

$$\Omega' = \{(\ell-\ell_0)/2 < x_1 < (\ell+\ell_0)/2,\ x_2 = h_\ell - 0,\ 0 \le x_3 \le t\} \cup$$

$$\{(\ell-\ell_0)/2 < x_1 < (\ell+\ell_0)/2,\ x_2 = h_\ell + 0,\ 0 \le x_3 \le t\}$$

is used. Using the virtual work principle and employing the well known Ritz technique, FEM modeling for problem is obtained,[13]. In this case, the solution domain is divided into a finite number of brick elements with eight nodes (Fig.2).

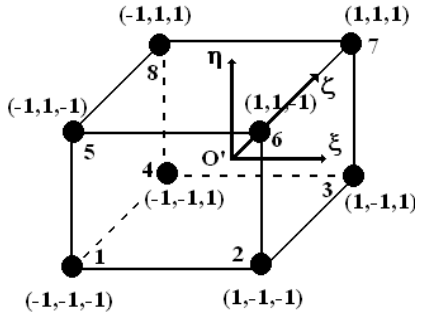


Figure 2. Finite elements and positions of the nodes

IV. NUMERICAL RESULTS

First we consider the validity of the algorithm and programmes which are composed by the authors . For this purpose, we consider a single edge crack problem to compare with reference solution obtained by [11-12] (Fig.3). In Fig. 3 is selected two different finite meshes: a) Mesh 1 consists of 20x20x20 brick elements b) Mesh 2 consists of 24x24x24 brick elements along Ox_1, Ox_2 and Ox_3 axes, respectively. Here, the arc length co-ordinate s is distance along the crack front from the center of crack ($s=0$) to the free surface ($s=t/2$). The regarding results are given in Fig 4 for the isotropic case. Fig. 4 shows the values of, $K_I/\sigma\sqrt{\Pi a}$ for various s/t under $\ell_0/\ell=0.5$, $t/\ell=0.75$, $h/\ell=0.875$, $h_u=h/2$. In Fig.4 is shown in comparisons with the results obtained by references [11-12] are shown.

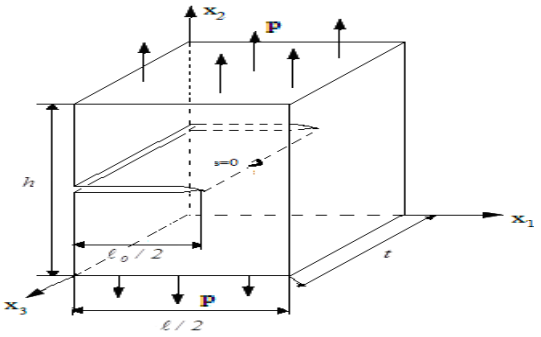


Figure 3 Single-edge crack rectangular prism

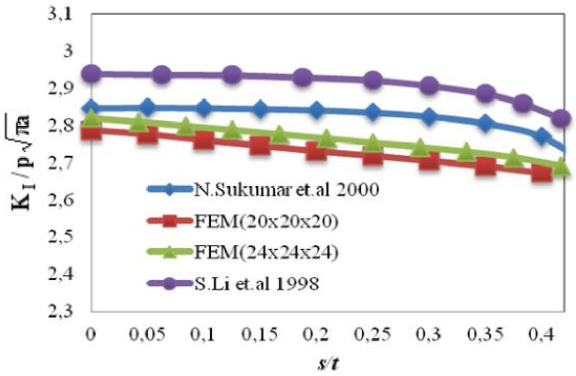


Figure 4. Comparison of stress SIF of Mode I for the single edge-crack problem

These results clearly shows the well agreement between each other and verifies the validity of the present FEM modelling and PC programs.

Now, let us consider band crack problem. Table 1 shows the influence of crack length, i.e. the influence of the $\ell_0/2\ell$ on the ERR. According to the known mechanical considerations, the values of ERR must decrease with decreasing crack length. This is also verified by Table 1.

Table 1. The influence of the crack length on the ERR.

s/t	$\ell_0/2\ell$		
	0.25	0.15	0.10
0	0.7825	0.5545	0.3873
0.05	0.7668	0.5441	0.3804
0.10	0.7205	0.5130	0.3598
0.15	0.6453	0.4620	0.3256
0.20	0.5454	0.3929	0.2784
0.25	0.4273	0.3096	0.2205
0.30	0.3009	0.2190	0.1565
0.35	0.1798	0.1309	0.0934
0.40	0.0798	0.0575	0.0407

Table 2 illustrates the influence of the parameter t/ℓ on the values of the ERR . The parameter t/ℓ shows the length of the prism along the Ox_3 axis. The numerical results in Table 2 show that the values of ERR increase with the length of the prism along the Ox_3 direction.

Table 2. The influence of the the length of the prism along the Ox_3 direction.on the ERR

s/t	t/ℓ		
	0.75	1	1.5
0	0.7825	1.4779	2.3004
0.05	0.7668	1.4494	2.2785
0.10	0.7205	1.3786	2.2108
0.15	0.6453	1.2607	2.0915
0.20	0.5454	1.0967	1.9111
0.25	0.4273	0.8911	1.6577
0.30	0.3009	0.6551	1.3218
0.35	0.1798	0.4106	0.9102
0.40	0.0798	0.1924	0.4719

CONCLUSION

Thus, in the present paper rectangular prism containing band crack is considered. It is assumed that uniformly distributed normal extensional forces act on the top and bottom of the planes. The mathematical formulation of the corresponding boundary value problems was studied within the scope of the Three-Dimensional Linear Theory of Elastisity. The investigation was focused on the effect of the geometrical parameters on the ERR. For the solution to the corresponding boundary-value problems, the Finite Elements Method (FEM) was utilizied.

According to the obtained numerical results, it is drawn that:

- The values of the ERR decrease with decreasing crack length,
- The values of the ERR increase with the length of the prism along the Ox_3 direction.
- The influence of parameters to ERR is more notable at the center of the crack

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