# COMMENTS ON THE ARTICLE "LONGITUDINAL INSTABILITIES IN EXIT FLOW FROM A SLIT" BY V. V. GOKHALE IN THE LIGHT OF RENEWED INTEREST IN THE OHNESORGE NUMBER

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**Abstract** - Recently there has been renewed interest in the dimensionless number known as the Ohnesorge number denoted by the letter Z, for example a discussion of the biography of Ohnesorge in [1]. This article is concerned with the setting down of a purely mathematical number 0.944261248574953 , which happens to be the critical value of the dimensionless number Z, consisting of a combination of the viscosity, density, surface tension and half slit width (in a rectangular geometry) for the onset of instability. The work was performed by me [2] as a first year graduate student at the University of Delaware as part of a class. Some questions had been raised about the algebra leading to the critical value of Z after the work was published [3]. I responded to the comments [4] but evidently not with sufficient impact. These questions are settled decisively here with an a posteriori analysis of the magnitude of terms in the associated boundary condition. The idea of such a method of solving problems is implied in the article but was not fully appreciated. Data from [5] is analyzed in the light of this analysis.

**Keywords** - Capillary Instability, Rheology, Surface Rheology, Surface Tension, Thin Film, Viscosity

## I. INTRODUCTION

Recently there has been renewed interest in a dimensionless number known as the Ohnesorge number. See for example [1]. I am compelled to write this article which was originally conceived as a letter to the editor to set forth a model of how problems in fluid Mechanics should be solved. I hope to see more productive ways for future studies on this topic and other problems in fluid mechanics.

I wrote the article because I had taken up the problem for Prof Mort Denn's "Stability of Reactions and Transport Processes" class as an original problem (not an assigned problem) in February 1977, less than 6 months after joining the graduate program in the Dept of Chemical Engg at the University of Delaware. I had to conceive the problem and also ensure that it had not been treated before. I placed it before Mort and he agreed it had not been treated before to the best of his knowledge. My plan in writing the article was to throw out terms from the basic equations in dimensional form first, and later justify the throwing out of terms based on dimensional analysis using the experimental and algebraic results obtained from the analysis. This was meant to contrast sharply from conventional treatments which do the dimensional analysis first. In my view my approach was more consistent with the spirit of science than the conventional approach which involved kowtowing to the presumed correct multidimensional equations.

The most productive way to use the article in the future is to include surface rheology constitutive equations which treat the surface tension $\sigma$ in the boundary conditions as a variable with its own constitutive equation from a surface rheology viewpoint. This can be used to handle derivatives of $\sigma$ with respect to t and x , y . Terms can be discarded with no other reason than algebraic simplicity and then shown to be insignificant via dimensional analysis from the results obtained. This use would be consistent with the plan of the article and with the logic of engineering science. The viscosity $\mu$ used in my article is a value appropriate for the kinematics of the flow, which with its alternate thinning and thickening, is more akin to the flow between 4 counter-rotating rollers than the usual measured number. It should be considered the parameter at the onset of instability i.e. the value derived from solving the equation $\frac{\mu}{\sqrt{\rho a \sigma}} = 0.944$ for a given slit width.

In the case of viscoelastic fluids any other constitutive equations may be used to plug in for $\tau_{yy}$ in the boundary condition as a first approximation. Neglect of any terms in the basic flow equation (4) being justified from estimating the magnitude of neglected terms a posteriori by dimensional analysis. This is the plan of the original article.

## II. ORIGINS OF USE OF THE OHNESORGE NUMBER

### A. Background

The following background about this number is relevant. Wilhelm Ohnesorge was a WW2, German minister in Hitler's government. The following news item from The New York Times dated February 4, 1962 gives the following: ")--Dr. Wilhelm Ohnesorge, Minister of Posts in Hitler's Cabinet, died Thursday. He was 89 years old." .

- The Ohnesorge number is related to the inverse of an older number called Laplace number (also called the Suratman number)
- Wilhelm Ohnesorge was Hitler's Post Office minister. He supervised Germany's attempt to make an A Bomb. He was an expert in radio and

cryptography.

- After the war he was allowed to go free and died in 1962.ฟ้าก
- The man Wolfgang Ohnesorge after whom the number is named was a minor German official and died in 1976.
- I made my discovery of the MATHEMATICAL NUMBER or constant 0.944261248574953 in 1977..
- The Germans had clearly intended in 1937 that Americans should take the opportunity provided to refer to this number as the Ohnesorge Number so that people would assume it was named after the Post Office minister.
- This can be seen from the fact that the article by "W. von Ohnesorge" in [5] merely calls the Y-Axis number a dimensionless number without naming it as 1/Laplace number as any normal student would.
- From 1937 onwards several American government scientists used the term "Ohnesorge Number" for this number. This activity continued after America entered the war in 1942 and even later after the war.
- I had no idea about all this and had used the same name for the number as given in textbooks on polymers.
- However, immediately after I had made the discovery in 1977, an article appeared in the magazine Scientific American about German A bomb efforts including a photo of Ohnesorge, Hitler and others in a meeting.
- After I found that, I called a meeting with Mort Denn and Ken Bischoff. No funding application resulted from this. Mort asked me to include an acknowledgement to an NSF grant for the $60.00 needed by the shop to fabricate a slit which I had attached to an existing apparatus being used by a senior working on a summer project for Art Metzner. This was used to observe and photograph the longitudinal striations observed with a corn syrup-water solution and reported on in [2].

**B. Discussion of equations and boundary conditions Repeating my analysis in [2]:**

The position of the surface is assumed to be given by $y = a + \xi(t)$, where $\xi$ is the perturbation from the normal jet surface. For the purpose of writing boundary conditions we may assume that the surface is approximately at $y = \pm a$. Then boundary conditions are at $y \cong \pm a$ ; $\tau_{xy} = 0$ , $\tau_{yy} - p = \sigma(\partial^2\xi/\partial x^2)$, where $\sigma$ is the surface tension. The flow inside the slit is assumed to have a flat velocity profile due to turbulence or shear thinning rheology. The velocity profile is also flat in the emerging sheet as a result its thickness is the same as the slit. The coordinate system is assumed to be moving at the same speed as thesheet of fluid. The following stability analysis is also applicable if the disturbance on the surface is at right angles to the direction considered here i.e. it appears as a series of waves in the direction of the flow which may lead to the breakup of the sheet. The most important feature of the model is that it correctly anticipates the qualitative feature that the sheet is more unstablewhen it is thinner. This fact will be utilized later to analyze data reported in [5].Ohnesorge, in [5], attempted to correlate observed breakup of cylindrical jets with the Reynolds' number. While this analysis is in a sheet geometry, it correctly captures qualitative features which can lead to breakup of thin sheets or cylinders.

$$\rho\frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) \quad ,$$

$$\rho\frac{\partial v_y}{\partial t} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) \qquad (1)$$

and continuity requires

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad (2)$$

In terms of a stream function $\psi$ defined by

$$v_x = -\frac{\partial \psi}{\partial y} \ , \ v_y = \frac{\partial \psi}{\partial x} \qquad (3)$$

The Navier-Stokes equations reduce to

$$\nabla^2\bar{\psi} = \frac{\nu}{s}\nabla^4\bar{\psi} \qquad (4)$$

in the Laplace transform domain.

Here $\bar{\psi}(s) = \mathscr{L}\{\psi(t)\}$ and the boundary conditions become at $y \cong \pm a$ :

$$\frac{\partial^2\bar{\psi}}{\partial y^2} = \frac{\partial^2\bar{\psi}}{\partial x^2} \ , \ \frac{\partial^2\bar{\psi}}{\partial y\,\partial x} = \frac{\sigma}{2\mu s}\frac{\partial^3\bar{\psi}}{\partial x^3} \qquad (5)$$

In the second boundary condition in equation (5) in Gokhale(1979) the right hand side is printed as $\frac{\sigma}{2\mu s}\frac{\partial^3\bar{\psi}}{\partial x^2}$. Here it has been corrected to $\frac{\sigma}{2\mu s}\frac{\partial^3\bar{\psi}}{\partial x^3}$ . Of course since the boundary conditions are homogeneous both types of disturbances viz. sinuous and varicose give rise to the same characteristic equation as will be seen in what follows.

Let $\bar{\psi}(x,y) = F(y)\exp(-ikx)$ . Substituting this expression in (4) we get

$$F^{(iv)}(y) - \left(\frac{s}{\nu} + 2k^2\right)F''(y) + k^2\left(k^2 + \frac{s}{\nu}\right)F(y) = 0$$

Hence

$$F(y) = C_1\exp(ky) + C_2\exp(-ky) + C_3\exp\left(\sqrt{(s/\nu) + k^2}\,y\right) + C_4\exp\left(-\sqrt{(s/\nu) + k^2}\,y\right)$$

and

$$\bar{\psi}(x,y) = \Big[C_1\exp(ky) + C_2\exp(-ky) + C_3\exp\left(\sqrt{(s/\nu) + k^2}\,y\right) +$$

$C_4\exp(-\sqrt{(s/\nu)+k^2}y)\Big]\exp(-ikx)$ (6)

After application of the boundary conditions (5), the following characteristic equation results for both sinuous and varicose disturbances

$4\mu k\sqrt{(s/\nu)+k^2}\cosh(ka)\sinh\left(\sqrt{(s/\nu)+k^2}a\right) - \frac{\sigma k}{\nu}\cosh(ka)\cosh\left(\sqrt{(s/\nu)+k^2}a\right) - 2\mu\big((s/\nu)+2k2\sinh ka\cosh sv+k2a=0$ (7)

.Brown in [3]wrongly states that this equation is in error. This has been pointed out to not be the case in [4]. Brown in [3] also questioned setting the pressure to zero in the boundary condition (5). However, as pointed out in [4], $\frac{1}{\lambda} \ll \frac{2}{a}$ since wave lengths are much larger than a the half slit width. For the case considered in [2],$a = 0.05, 0.1$ cm, $\frac{2}{a} = 40, 20$(1/cm) while $\frac{1}{\lambda}$ is on the order of 2 to 5 (1/cm). Hence the pressure term p may be ignored completely or $p = 0$ as assumed in the analysis. The assumption $p = 0$ isthus consistent with the result. In practical situations the half slit width a is even smaller than 0.05 cm.

Equation (7) can be written as

$\bar{s}\tanh\bar{s} = \frac{1}{4Z^2} + \frac{(\bar{s}^2+\bar{k}^2)}{2\bar{k}}\tanh\bar{k}$ (8)

where

$\bar{s} = \sqrt{(s/\nu)a^2 + (ka)^2}$ , $\bar{k} = ka$ and $\frac{\mu}{\sqrt{\rho a\sigma}} = Z$

The maximum possible value of $\left(\frac{1}{4Z^2}\right)$ in (8) is 0.28 and the corresponding minimum value of Z for which unstable solutions exist is 0.945 and hence if Z is less than 0.945 no instability is possible. These are the values reported in [2]. To a greater number of decimals the minimum value of Z for the occurrence of instability,$Z_c$ is 0.944261248574953 .

## III. ANALYSIS OF THE DATA FROM REFERENCE [5]

Reference [5] by Ohnesorge contains data on onset of instability in a cylindrical geometry. The minimum value of Z , at which instability occurs is reported for 8 different materials. Assuming that the previous analysis applies in the cylindrical geometry as well and replacing the half slit width by the diameter of the tubes used, it is possible to find the relative viscosity of these 8 materials from the observed critical (minimum)$Z_{cobserved}$ . In terms of the diameter ( $= 2a$ ), the value of $Z_c$ is 0.66 . The relative viscosity or the ratio $\frac{\mu_{effective}}{\mu} = \frac{0.66}{Z_{cobserved}}$ . Results from these observations are reported in Table 1. The number after the material name refers to number of observations.

| Material | $Z_{cobserved}$ | $\frac{\mu_{effective}}{\mu}$ |
|---|---|---|
| Water 4 | 0.060810063 | 10.9786043507183 |
| Aniline 2 | 0.158817426 | 4.20362950382543 |
| Gasoil 2 | 0.256660777 | 2.60113612227347 |
| Glycerine A 2 | 0.427414932 | 1.5619707413141 |
| Glycerine B 1 | 0.740818221 | 0.901178723021041 |
| Oil A 1 | 0.818730753 | 0.8154202289273 |
| Oil B 2 | 4.48168907 | 0.148963841004327 |
| Ricinus 1 | 5.754602676 | 0.116013156023494 |

**Table 1**

Since composition data for the water, Glycerine A and Glycerine B samples is not provided no conclusions regarding the variation of $\frac{\mu_{effective}}{\mu}$ with composition in glycerine-water solutions can be made. Since currently there is much interest, further observations will enable mixing rules to be obtained for the variation of $\frac{\mu_{effective}}{\mu}$ with chemical composition.

## IV. CONCLUSION

Longitudinal instabilities in exit flow from a slit are analyzed and a stability criterion is obtained in terms of the mathematical number 0.944261248574953 . Data on instabilitiesobserved in [5] are explained in terms of an effective viscosity. Suggestions are made for using further data to generate mixing rules for calculating and correlating effective viscosities for predictive purposes

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