BENEFIT-FUNCTION OF TWO- IDENTICAL COLD STANDBY FUEL CELL TECHNOLOGY SYSTEM SUBJECT TO FAILURE OF FUEL CELL OPERATION DUE TO HIGH TEMPERATURE, PRESSURE OR RELATIVE HUMIDITY OF REACTANT GAS

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Abstract- The main objective of the Chemical Sources of Energy Programme is the development and applications of fuel cell technology which produces electricity, water and heat through reaction between hydrogen and oxygen/air. The fuel cell technology offers high conversion efficiency, modularity, compactness and noise-free operations. Fuel Cells are environmentally benign. Hydrogen is the primary fuel for fuel cells. Other fuels can also be used to produce hydrogen gas with the aid of reformers. Because of modular nature, fuel cells are ideally suited for de-centralized power generation and for automotive application. Prototypes of polymer electrolyte membrane or proton exchange membrane fuel cells (PEMFCs) and phosphoric acid fuel cells (PAFCs) have been developed in kW size in India. The applications of these prototypes have been demonstrated for power generation (PEMFC & PAFC) and transport sectors (PEMFC).A fuel cell-battery hybrid vehicles with indigenously developed PEMFC stack of 10kW had undergone field performance evaluation. Efforts made are expected to lead to the indigenous production and wider applications of fuel cell systems in the country. This programme focuses on development and demonstration of fuel cells, which produce electricity, water and heat through reaction between hydrogen and oxygen/air. Hydrogen is the primary fuel for fuel cells. Hydrogen for fuel cells can be produced by reformation of other fuels. It can also be produced from coal and biomass and by electrolysis of water. Renewable energy sources and nuclear energy can also be used for production of hydrogen. Because of their modular nature, fuel cells are ideally suited for distributed power generation. Small fuel cell power packs can be used for power generation by industrial and residential users. Fuel Cells are emerging as power sources for automobiles. A 10 kW PEMFC has been used in a prototype vehicle developed in India, in addition to a battery bank. Efforts are being made to develop indigenous technology for production of fuel cell systems in the country. The widespread use of fuel cells for power generation, transport and other applications is expected to reduce dependence on scarce fossil fuels and help in preserving the environment. In this paper we have taken failure of fuel cell operation due to high temperature, pressure or relative humidity of reactant gas. When the main unit fails then cold standby system becomes operative. Failure of fuel cell operation due to relative humidity of reactant gas cannot occur simultaneously in both the units and after failure the unit due to high temperature; pressure undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords- Cold Standby, Failure of Fuel Cell Operation Due to High Temperature, Pressure or Relative Humidity of Reactant Gas, First Come First Serve, MTSF, Availability, Busy period, Benefit -Function.

I. INTRODUCTION

A fuel cell is a device that converts the chemical energy from a fuel into electricity through a chemical reaction with oxygen or another oxidizing agent. Hydrogen produced from the steam methane reforming of natural gas is the most common fuel, but for greater efficiency hydrocarbons can be used directly such as natural gas and alcohols like methanol. Fuel cells are different from batteries in that they require a continuous source of fuel and oxygen/air to sustain the chemical reaction whereas in a battery the chemicals present in the battery react with each other to generate an electromotive force (emf). Fuel cells can produce electricity continuously for as long as these inputs are supplied.

The first fuel cells were invented in 1838. The first commercial use of fuel cells came more than a century later in NASA space programs to generate power for probes, satellites and space capsules. Since then, fuel cells have been used in many other applications. Fuel cells are used for primary and backup power for commercial, industrial and residential buildings and in remote or inaccessible areas. They are also used to power fuel-cell vehicles, including forklifts, automobiles, buses, boats, motorcycles and submarines.

There are many types of fuel cells, but they all consist of an anode, a cathode and an electrolyte that allows charges to move between the two sides of the fuel cell. Electrons are drawn from the anode to the cathode through an external circuit, producing direct current electricity. As the main difference among fuel cell types is the electrolyte, fuel cells are classified by the type of electrolyte they use followed by the difference in startup time ranging from 1 sec for PEMFC to 10 min for SOFC. Fuel cells come in a variety of sizes. Individual fuel cells produce relatively small electrical potentials, about 0.7 volts, so cells are "stacked", or placed in series, to increase the voltage and meet an application's requirements. In

addition to electricity, fuel cells produce water, heat and, depending on the fuel source, very small amounts of nitrogen dioxide and other emissions. The energy efficiency of a fuel cell is generally between 40–60%, or up to 85% efficient in cogeneration if waste heat is captured for use.

The fuel cell market is growing, and Pike Research has estimated that the stationary fuel cell market will reach 50 GW by 2020.

II. AIRPLANES

Boeing researchers and industry partners throughout Europe conducted experimental flight tests in February 2008 of a manned airplane powered only by a fuel cell and lightweight batteries. The fuel cell demonstrator airplane, as it was called, used a proton exchange membrane (PEM) fuel cell/lithium-ion battery hybrid system to power an electric motor, which was coupled to a conventional propeller. In 2003, the world's first propeller-driven airplane to be powered entirely by a fuel cell was flown. The fuel cell was a unique FlatStack^{TM} stack design, which allowed the fuel cell to be integrated with the aerodynamic surfaces of the plane.

There have been several fuel-cell-powered unmanned aerial vehicles (UAV). A Horizon fuel cell UAV set the record distance flown for a small UAV in 2007. The military is especially interested in this application because of the low noise, low thermal signature and ability to attain high altitude. In 2009 the Naval Research Laboratory's (NRL's) Ion Tiger utilized a hydrogen-powered fuel cell and flew for 23 hours and 17 minutes. Fuel cells are also being used to provide auxiliary power in aircraft, replacing fossil fuel generators that were previously used to start the engines and power on board electrical needs. Fuel cells can help airplanes reduce CO 2 and other pollutant emissions and noise.

Submarines

The Type 212 submarines of the German and Italian navies use fuel cells to remain submerged for weeks without the need to surface.

The U212A is a non-nuclear submarine developed by German naval shipyard Howaldtswerke Deutsche Werft. The system consists of nine PEM fuel cells, providing between 30 kW and 50 kW each. The ship is silent giving it an advantage in the detection of other submarines. A naval paper has theorized about the possibility of a Nuclear-Fuel Cell Hybrid whereby the fuel cell is used when silent operations are required and then replenished from the Nuclear reactor (and water). In this paper we have taken failure of fuel cell operation due to high temperature, pressure or relative humidity of reactant gas. When the main operative unit fails then cold standby system becomes operative. Failure of fuel cell operation due to relative humidity of reactant gas cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of failure of fuel cell operation due to high temperature, pressure immediately. The repair is done on the basis of first fail first repaired.

III. ASSUMPTIONS

1. λ_1, λ_2 are constant failure rates for failure of fuel cell operation due to high temperature, pressure; relative humidity of reactant gas respectively. The CDF of repair time distribution of Type I and Type II are $G_1(t)$ and $G_2(t)$.
2. The failure of fuel cell operation due to relative humidity of reactant gas is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to the high temperature, pressure or relative humidity of reactant gas and works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

Notations

λ_1 , λ_2 - Failure of fuel cell operation due to high temperature, pressure; relative humidity of reactant gas respectively.

$G_1(t)$, $G_2(t)$ – repair time distribution Type –I or Type-II due to high temperature, pressure or relative humidity of reactant gas respectively.

p, q - probability of failure of fuel cell operation due to high temperature, pressure; relative humidity of reactant gas respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state at instant t

R_i (t) System having started from state i is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

Symbols for states of the System

Superscripts O, CS, TPF, RHF,

Operative, Cold Standby, Failure of fuel cell operation due to high temperature, pressure, Failure of fuel cell operation due to relative humidity of reactant gas respectively
Subscripts nrhf, rhf, tpf, ur, wr, uR

No failure of fuel cell operation due to relative humidity of reactant gas, Failure of fuel cell operation due to relative humidity of reactant gas, Failure of fuel cell operation due to high temperature, pressure of reactant gas, under repair, waiting for repair, under repair continued from previous state respectively
Up states – 0, 1, 2, 7, 8 ;
Down states – 3, 4, 5, 6
regeneration point – 0,1,2, 7, 8
States of the System

$0(O_{nrhf}, CS_{nrhf})$

One unit is operative and the other unit is cold standby and there is no Failure of fuel cell operation due to relative humidity of reactant gas in both the units.

$1(TPF_{tpf, ur}, O_{nrhf})$

The operating unit fails due to high temperature, pressure and is under repair immediately of very costly Type- I and standby unit starts operating with no Failure of fuel cell operation due to relative humidity of reactant gas

$2(RHF_{rhf, ur}, O_{nrhf})$

The operative unit fails due to failure of fuel cell operation due to relative humidity of reactant gas and undergoes repair of type II and the standby unit becomes operative with no failure of fuel cell operation due to relative humidity of reactant gas

$3(TPF_{tpf,uR}, RHF_{rhf, wr})$

The first unit fails due to failure of fuel cell operation due to high temperature, pressure and under Type-I repair is continued from state 1 and the other unit fails due to RHF resulting from failure of fuel cell operation due to relative humidity of reactant gas and is waiting for repair of very costly Type -II.

$4(TPF_{tpf,uR}, TPF_{tpf,wr})$

The repair of the unit is failed due to TPF resulting from Failure of fuel cell operation due to high temperature; pressure is continued from state 1and the other unit failed due to TPF resulting from Failure of fuel cell operation due to high temperature, pressure is waiting for repair of Type-I.

$5(RHF_{rhf, uR}, RHF_{rhf, wr})$

The operating unit fails due to failure of fuel cell operation due to relative humidity of reactant gas and under repair of Type – I continue from the state 2 and the other unit fails also due to failure of fuel cell operation due to relative humidity of reactant gas is waiting for repair of Type- I.

$6(RHF_{rhf, uR}, TPF_{tpf,wr})$

The operative unit fails due to failure of fuel cell operation due to relative humidity of reactant gas and under repair continues from state 2 of Type –II and the other unit is failed due to TPF resulting from Failure of fuel cell operation due to high temperature, pressure and under very costly Type-I

$7(O_{nrhf}, TPF_{tpf,ur})$

The one unit is operative with no failure of fuel cell operation due to relative humidity of reactant gas and the other unit failed due to TPF resulting from failure of fuel cell operation due to high temperature, pressure is under repair of very costly Type-I

$8(O_{nrhf}, RHF_{rhf, ur})$

The one unit is operative with no failure of fuel cell operation due to relative humidity of reactant gas and the other unit is failed due to failure of fuel cell operation due to relative humidity of reactant gas is under repair of Type-II.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = p,\ p_{02} = q,$
$p_{10} = pG_1^*(\lambda_1) + qG_1^*(\lambda_2) = p_{70},$
$p_{20} = pG_2^*(\lambda_1) + qG_2^*(\lambda_2) = p_{80},$
$p_{11}^{(3)} = p(1 - G_1^*(\lambda_1)) = p_{14} = p_{71}^{(4)}, p_{28}^{(5)} = q(1 - G_2^*(\lambda_2)) = p_{25} = p_{82}^{(5)}$ (1)

We can easily verify that

$p_{01} + p_{02} = 1,$
$p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (= p_{13}) = 1,$
$p_{80} + p_{82}^{(5)} + p_{87}^{(6)} = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\, dt$

Mean Time To System Failure

$\emptyset_0(t) = Q_{01}(t)[s]\, \emptyset_1(t) + Q_{02}(t)[s]\, \emptyset_2(t)$
$\emptyset_1(t) = Q_{10}(t)[s]\, \emptyset_0(t) + Q_{13}(t) + Q_{14}(t)$
$\emptyset_2(t) = Q_{20}(t)[s]\, \emptyset_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing
Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$\emptyset_0^*(s) = N_1(s) / D_1(s)$ (6)

where

$N_1(s) = Q_{01}^*[\,Q_{13}^*(s) + Q_{14}^*(s)\,] + Q_{02}^*[\,Q_{25}^*(s) + Q_{26}^*(s)\,]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$

Making use of relations (1) & (2) it can be shown that $\phi_0^*(0) = 1$, which implies that $\phi_0(t)$ is a proper distribution.

$$MTSF = E[T] = \left.\frac{d}{ds}\,\phi_0^*(s)\right|_{s=0}$$

$$= (D_1'(0) - N_1'(0)) \,/\, D_1(0)$$

$$= (\mu_0 + p_{01}\mu_1 + p_{02}\mu_2) / (1 - p_{01}p_{10} - p_{02}p_{20})$$

where

$\mu_0 = \mu_{01} + \mu_{02}$,

$\mu_1 = \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)}$,

$\mu_2 = \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}$

IV. AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$, $M_1(t) = p\,G_1(t)\; e^{-(\lambda_1+\lambda_2)} = M_7(t)$

$M_2(t) = q\,G_2(t)\; e^{-(\lambda_1+\lambda_2)} = M_8(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$,

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{28}^{(5)}(t)[c]\,A_8(t) + q_{27}^{(6)}(t)]\,[c]A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) + [q_{71}^{(4)}(t)[c]\,A_1(t) + q_{78}^{(3)}(t)]\,[c]A_8(t)$

$A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t) + [q_{82}^{(5)}(t)[c]\,A_2(t) + q_{87}^{(6)}(t)]\,[c]A_7(t)$

(7-11)

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \qquad (12)$$

where

$N_2(s) = \hat{M}_0(1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\hat{q}_{82}^{(5)}(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)}\hat{q}_{18}^{(3)})] + \hat{q}_{01}[\hat{M}_1(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{M}_7 + \hat{q}_{78}^{(3)}\hat{M}_8) + \hat{q}_{18}^{(3)}(\hat{M}_7\hat{q}_{87}^{(6)} - \hat{M}_8) - \hat{q}_{82}^{(5)}(\hat{M}_1(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(-\hat{M}_2(\hat{q}_{78}^{(3)} + \hat{M}_7\hat{q}_{28}^{(5)}) - \hat{q}_{18}^{(3)}(\hat{M}_2 + \hat{M}_7\hat{q}_{27}^{(6)}))\}] \hat{q}_{02}[\hat{M}_2(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{M}_7 + \hat{q}_{78}^{(3)}\hat{M}_8) + \hat{q}_{28}^{(5)}(\hat{M}_7\hat{q}_{87}^{(6)} + \hat{M}_8) - \hat{q}_{71}^{(4)}(\hat{M}_1(-\hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} + \hat{q}_{87}^{(6)}) + \hat{q}_{17}^{(4)}(\hat{M}_2 + \hat{q}_{28}^{(5)}\hat{M}_8) - \hat{q}_{18}^{(3)}(-\hat{M}_2\hat{q}_{87}^{(6)} + \hat{M}_8\hat{q}_{27}^{(6)}))\}] \hat{q}_{18}^{(3)}(\hat{M}_2 + \hat{M}_7\hat{q}_{27}^{(6)})\}]$

$D_2(s) = (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)}(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\hat{q}_{82}^{(5)}(\hat{q}_{17}^{(4)}\hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)})] + \hat{q}_{01}[-\hat{q}_{10}(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{70}\hat{q}_{87}^{(6)} - \hat{q}_{80}) - \hat{q}_{82}^{(5)}(-\hat{q}_{10}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{q}_{20}(\hat{q}_{78}^{(3)} - \hat{q}_{70}\hat{q}_{28}^{(5)}) + \hat{q}_{18}^{(3)}(\hat{q}_{20} + \hat{q}_{70}\hat{q}_{27}^{(6)})\}] \hat{q}_{02}[-\hat{q}_{20}(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) - \hat{q}_{27}^{(6)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\hat{q}_{80}) - \hat{q}_{28}^{(5)}(\hat{q}_{70}\hat{q}_{87}^{(6)} + \hat{q}_{80}) - \hat{q}_{71}^{(4)}(\hat{q}_{10}(\hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)}(\hat{q}_{20} - \hat{q}_{28}^{(5)}\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{20}\hat{q}_{87}^{(6)} + \hat{q}_{80}\hat{q}_{27}^{(6)})\}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (13)$$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^x A_0(x)\,dx$ So that

$$\bar{\lambda}_u(s) = \frac{\bar{A}_0(s)}{s} = \frac{N_2(s)}{sD_2(s)} \qquad (14)$$

The expected down time of the system in (0,t] is

$\lambda_{dt}(t) = t - \lambda_u(t)$ So that

$$\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s) \qquad (15)$$

The expected busy period of the server when there is Failure of fuel cell operation due to relative humidity of reactant gas or Failure of fuel cell operation due to high temperature, pressure in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{18}^{(3)}(t)[c]\,R_8(t) + q_{17}^{(4)}(t)[c]R_7(t)$

$R_2(t) = S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t)\,R_8(t) + q_{27}^{(6)}(t)][c]R_7(t)$

$R_7(t) = S_7(t) + q_{70}(t)[c]R_0(t) + Q_{71}^{(4)}(t)\,R_1(t) + q_{78}^{(3)}(t)][c]R_8(t)$

$R_8(t) = S_8(t) + q_{80}(t)[c]R_0(t) + Q_{82}^{(5)}(t)\,R_2(t) + q_{87}^{(6)}(t)][c]R_7(t)$ (16-20)

Taking Laplace Transform of eq. (16-20) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s) / D_2(s)$ (21)

where

$N_3(s) = \hat{q}_{01}[\hat{S}_1(1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{S}_7 + \hat{q}_{78}^{(3)} \hat{S}_8) + \hat{q}_{18}^{(3)}(\hat{S}_7 \hat{q}_{87}^{(6)} \hat{S}_8)] - \hat{q}_{01} \hat{q}_{82}^{(5)}(\hat{S}_1 \hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{S}_2 \hat{q}_{78}^{(3)} + \hat{S}_7 \hat{q}_{28}^{(5)}) - \hat{q}_{18}^{(3)}(\hat{S}_2 + \hat{S}_7 \hat{q}_{27}^{(6)})] + \hat{q}_{02}[\hat{S}_2(1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{S}_7 + \hat{q}_{78}^{(3)} \hat{S}_8) + \hat{q}_{28}^{(5)}(\hat{S}_7 \hat{q}_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02} \hat{q}_{71}^{(4)}(\hat{S}_1(- \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} \hat{q}_{87}^{(6)} \hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)} \hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2 \hat{q}_{87}^{(6)} + \hat{S}_8 \hat{q}_{27}^{(6)})]$

and

$D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (22)

The expected period of the system under Failure of fuel cell operation due to relative humidity of reactant gas failure or failure of fuel cell operation due to high temperature, pressure is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that

$\overline{\lambda_{rv}}(s) = \frac{\overline{R}_0(s)}{s}$

The expected number of visits by the repairman for repairing the identical units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)]$
$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{18}^{(3)}(t)[s] H_8(t) + Q_{17}^{(4)}(t)] [s]H_7(t)$,
$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t) [s] H_8(t) + Q_{27}^{(6)}(t)] [c]H_7(t)$
$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t) [s] H_1(t) + Q_{78}^{(3)}(t)] [c]H_8(t)$
$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t) [s] H_2(t) + Q_{87}^{(6)}(t)] [c]H_7(t)$

(23-27)

Taking Laplace Transform of eq. (23-27) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (28)

In the long run ,

$H_0 = N_4(0) / D_3'(0)$ (29)

V. BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under Failure of fuel cell operation due to relative humidity of reactant gas failure or failure due to high temperature, pressure, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure due to Failure of fuel cell operation due to relative humidity of reactant gas failure or Failure of fuel cell operation due to high temperature, pressure for repairing the units in (0,t]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t)$
$= \lim_{s\to 0}(s^2 C(s))$
$= K_1A_0 - K_2R_0 - K_3H_0$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under repair of type- I or type- II

K_3 - cost per visit by the repairman for units repair.

CONCLUSION

After studying the system , we have analyzed graphically that when the failure rate due to Failure of fuel cell operation due to relative humidity of reactant gas or Failure of fuel cell operation due to high temperature, pressure increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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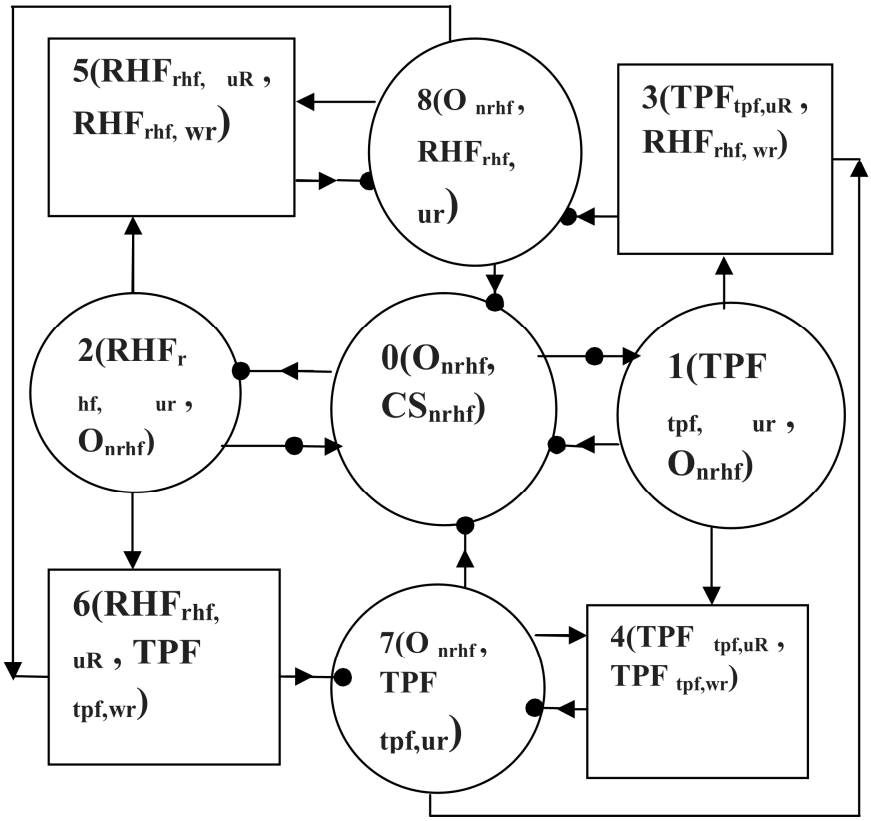

Fig. The State Space Diagram

up state **down state**

- **Regeration point**

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