PROPAGATION AND COLLISIONAL DYNAMICS OF ELECTROMAGNETIC SOLITON IN AN ANISOTROPIC FERROMAGNETIC NANOWIRE

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Abstract— Magnetization dynamics of an one-dimensional anisotropic ferromagnetic nanowire is investigated by solving the famous Landau-Lifshitz-Gilbert equation (LLG) for magnetization of the medium coupled with the Maxwell's equation for the nature of propagation of electromagnetic wave (EMW) in a ferromagnetic nanowire (FN). We made an uniform expansion of magnetization and magnetic field along the direction of EMW propagation in the framework of reductive perturbation method. The excitation of magnetization of the nanowire is restricted to the normal plane at the lowest order of perturbation and goes out of plane for higher orders. The dynamics of the nanowire is governed by the perturbed modified KdV equation (pMKDV). The pMKDV equation is solved by using the standard algebraic method, and the nature of the excitation is evolved in the form of soliton. Further the interaction of solitons is analyzed by using the Hirota bilinearization method. The analysis brings out the inelastic nature of collision due to intensity redistribution among the modes. The EMW propagation in the nanoscale magnetic medium has potential technological applications in optimizing the magnetic storage devices.

Index Terms— Landau-Lifshitz-Gilbert equation, Magnetization Dynamics, Electromagnetic soliton, Reductive perturbation method.

I. INTRODUCTION

Nowadays one of the most exciting areas in material science is the study of nanomaterials due to their potential applications in fields as diverse as optics, electronics [1], catalysis, magnetism, electrochemistry, information processing and storage [2, 3], blood flow and other biological applications [4, 5], etc. Nanoscale magnetic systems have been fabricated and studied extensively in different geometry such as arrays of wires, dots, rings and pillars due to their potential applications in high density magnetic storage media and sensor devices. Among these, a ferromagnetic nanowire exhibit unique and tunable magnetization properties due to their inherent shape anisotropy which is fabricated by the electrodeposition method. Current interest of research on the ferromagnetic nanowire is to develop the ultra high density magnetic recording at the range of Tera byte per inch^2 under the influence of electromagnetic wave propagation. Understanding the dynamical properties of such a system has been the main objective of the recent theoretical and experimental works. The magnetic property of nanowire depends on the size and shape of the individual wires, the material parameter and interwire interaction. The propagation of EMW in a magnetic media especially in ferromagnetic nanowire has become very useful in the context of magneto-optical recording for higher storage and fast reading. The phenomenon of propagation of EMW in a ferromagnet is not only interesting itself but also important in connection with its application to ferrite devices at microwave frequencies such as ferrite-loaded waveguides [6]. Nakata [7-9] and also Leblond [10-12] have been extensively studied the electromagnetic wave propagation in ferromagnets in the form of solitons. Further, the propagation of EM soliton in isotropic and anisotropic ferromagnetic media was studied by Daniel and Veerakumar [13-15]. Recently, Kavitha et al [16], studied the breather-like electromagnetic soliton in an antiferromagnetic medium with higher order interaction [17-19] and also the EM soliton interaction was studied [20, 21]. The ferromagnetic medium with different magnetic interactions was identified as an interesting class of nonlinear medium exhibiting soliton excitations of their magnetization in the classical continuum limit. This soliton excitation of magnetization is mainly due to the nonlinearity present in the ferromagnetic medium. The above studies were restricted to the bulk ferro/antiferromagnetic materials and ferromagnetic thin films. Recently, Leblond et al [22] have studied the linear and nonlinear interaction of an EM field with a charge free, isotropic ferromagnetic hollow nanotube governed by the integrable nonlinear Schrodinger equation (NLS) which exhibits bright and dark soliton. However, the ferromagnetic nanowires are uniform in cross section with lengths of

micrometer range and diameters in the range of nanometer and, hence, can be realized as infinite in length to an excellent approximation. The diameter range considered for the isolated nanowire prompts one to include exchange and dipolar contributions to the excitation energy. Recently, the spin excitations in ferromagnetic nanowires with exchange and dipolar contributions have been studied by Arias [23]. Also, Leblond studied the spin wave excitation in a cylindrical ferromagnetic nanowire [10].

From the above motivation, in the present paper we investigate the propagation of EMW in an anisotropic ferromagnetic nanowire with external applied field and Gilbert damping. In Sec. II, we formulate the model and derive the dynamical equations to be solved. In Sec. III, the dynamics of magnetization and the EMW propagation are treated using reductive perturbation theory when the magnitude of damping is small and the spin component of EMW excites in the form of soliton. The effect of damping is also investigated extensively. Finally, the results are concluded in the conclusion section V.

II. MODEL AND DYNAMICAL EQUATION

We consider a long anisotropic ferromagnetic nanowire with spin-spin exchange coupling and uniaxial anisotropy subjected to an external magnetic field via EM wave propagation.

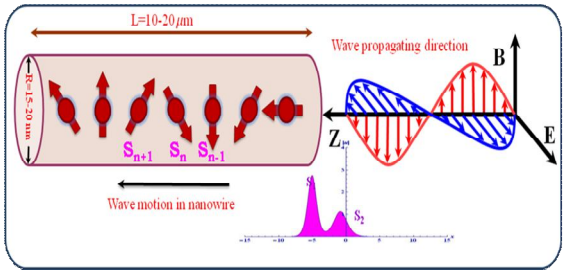


Fig. (1): Schematic representation of ferromagnetic nanowire

The nanowire is composed of free charges which are responsible for Gilbert damping and current density in the medium and the magnetization is along the symmetry axis of the medium. The Hamiltonian takes the form

$$H=\int\left[\frac{F}{M_s^2}\left(\frac{\partial\vec{S}}{\partial t}\right)^2+\frac{1}{2}\left(\frac{H_k}{M_s}-4\pi\right)(S^z)^2+\beta(\vec{S}\cdot\vec{B})\right]dz \quad (1)$$

In Eq. (1), S is a normalized magnetization vector or spin density defined by **S** = **M**/M, with the demand that the length of the spin vector does not change with time, i.e., S^2=1 and **M** is the magnetization of the medium, M is the saturated magnetization. Here, **S** is treated as a classical spin vector **S** = (S^x, S^y, S^z). We assume the ferromagnetic nanowire as a one dimensional spin chain and the width of the wire is much smaller than the characteristic length of the nanostructure but is large enough so that total spin in a cross section is large [24]. The first term in Eq. (1) represents the spin-spin exchange energy and the middle one associates the anisotropy. The last term is the Zeeman energy contribution through the EM wave.

Consequently, we consider that, in the nonlinear regime, the spin excitations in the nanowire are governed by the Landau-Lifshitz Gilbert equation,

$$\frac{\partial\vec{S}}{\partial t}=\vec{M}\times\vec{H}_{eff}+\alpha(\vec{M}\times\vec{M}\times\vec{H}_{eff}) \quad (2)$$

The dimensionless parameter α is the Gilbert damping parameter and H_{eff} represents the effective field due to the exchange interaction, the anisotropy field and the external field. For a ferromagnetic nanowire the effective field takes the form

$$H_{eff}=\frac{F}{M_s^2}\left(\frac{\partial^2\vec{S}}{\partial t^2}\right)+\left(\frac{H_k}{M_s}-4\pi\right)(S^z)\hat{e}_z+\beta(\vec{S}\cdot\vec{B}),$$

where F is the exchange constant, H_k is the energytic anisotropy coefficient and β = $g\mu_B$ denotes the applied external magnetic field. Here the $\hat{e}$ is the unit vector along *z* axis. The EM wave propagation in the nanowire is governed by the Maxwell's equation with charge density ρ and current density **J** written as [25]

$$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0},$$

$$\nabla\cdot\vec{B}=0,$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \quad (3)$$

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}.$$

In Eq. (3), **E**, **D**, **H** and **B** are respectively the electric field, the electric induction, magnetic field and magnetic field induction. The set of coupled equations (2) and (3) describes the propagation of EMW in an anisotropic ferromagnetic medium. The constitutive equation for **E** and **D** is $\vec{D}=\varepsilon\vec{E}$, where ε is the scalar permittivity of a ferromagnetic nanowire. Whereas the constitutive equation for **H** and **B** is

$$\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}, \quad (4)$$

where μ_0 is the magnetic permeability in vacuum and **M** is the magnetization density of the nanowire. Using Eq. (3) and Eq. (4) with little algebra we obtain the equation for the propagation of EMW in a ferromagnetic nanowire. Assuming, the direction of wave propagation along z-direction, we can write

$$c^2\frac{\partial^2\vec{B}}{\partial z^2}-\frac{\partial^2\vec{B}}{\partial t^2}=\frac{1}{\varepsilon_0}\left[\frac{\partial^2\vec{S}}{\partial z^2}-\frac{\partial^2 S^z}{\partial z^2}k\right]. \quad (5)$$

where *c* is the velocity of the propagating EM wave. Now the set of coupled equations (2) and (5) completely describes the evolution of magnetization

and propagation of the EM wave in the ferromagnetic nanowire.

III. ELECTROMAGNETIC SOLITON AND PERTURBED MODIFIED KORTEWEG DE-VRIES EQUATION

Let us solve the EM wave propagation equation (5) and equation (2) for nonlinear spin excitations through the multiscale analysis or reductive perturbation method. The reductive perturbation method involves perturbation expansion for the magnetic induction **B** and spin density **S** about the uniform fields B_0 and S_0. The perturbation parameter ε must validate for the long-wave approximation and hence we use the length scales of the nanowire. Thus the perturbation parameter ε is given by

$$\varepsilon \sim \frac{l}{\lambda'} \sim \frac{\lambda'}{L} << 1,$$

and take the exchange length l as a reference length i.e., l_0 is of order ε_0 while the wavelength λ' is very large, supporting the long wave propagation. The long wave profile evolution is accounted for by introducing the slow stretching variables, $\xi = \varepsilon(z - vt)$ and $\tau = \varepsilon^3 t,$ where v is the velocity of the pulse written as $v = \sqrt{a/(1+a)}c,$ where $a = H_0 / M_0$. As the effect of damping is small compared to the exchange interaction and external magnetic field, we rescale the damping parameter as $\alpha = \varepsilon^2 \alpha$. Now we expand the magnetization **S** and magnetic induction **B** using the perturbation parameter ε as

$$\vec{B} = \vec{B}_0 + \varepsilon \vec{B}_1 + \varepsilon^2 \vec{B}_2 + \cdots, \qquad (6a)$$

$$\vec{S} = \vec{S}_0 + \varepsilon \vec{S}_1 + \varepsilon^2 \vec{S}_2 + \cdots. \qquad (6b)$$

The above perturbations depend on the slow space and time variable (ξ, τ) . We now substitute the above uniform perturbations (6a) and (6b) in equations (2) and (5) and solving the resultant equations order by order of the perturbation parameter $\varepsilon,$ we obtain

At the order of (ε^0)

$$B_0^x = kS_0^x, \qquad (7a)$$

$$B_0^y = kS_0^y, \qquad (7b)$$

$$B_0^z = 0, \quad S_0^z = 0, \qquad (7c)$$

where $k = [\varepsilon_0 (c^2 - v^2)]^{-1}$. For the next order ε^1, we get

$$B_1^x = kS_1^x, \qquad (8a)$$

$$B_1^y = kS_1^y, \qquad (8b)$$

$$B_1^z = 0, \qquad (8c)$$

and

$$\frac{\partial S_0^x}{\partial \xi} = \left(\frac{Bk + \beta}{vk} \right) S_0^y S_1^z, \qquad (8d)$$

$$\frac{\partial S_0^y}{\partial \xi} = \left(\frac{Bk + \beta}{vk} \right) S_0^x S_1^z. \qquad (8e)$$

Finally at the order of ε^2, we get

$$\frac{\partial}{\partial \xi}\left(\frac{1}{k} B_2^x - S_2^x \right) = -\left(\frac{2v\varepsilon_0}{k} \right) \frac{\partial B_0^x}{\partial \tau}, \qquad (9a)$$

$$\frac{\partial}{\partial \xi}\left(\frac{1}{k} B_2^y - S_2^y \right) = -\left(\frac{2v\varepsilon_0}{k} \right) \frac{\partial B_0^y}{\partial \tau}, \qquad (9b)$$

$$B_2^z = 0, \qquad (9c)$$

and

$$-v \frac{\partial S_1^z}{\partial \xi} = A\left(M_0^x \frac{\partial^2 S_0^y}{\partial \xi^2} - M_0^y \frac{\partial^2 S_0^x}{\partial \xi^2} \right)$$
$$+\beta\left(B_0^y S_2^x + B_1^y S_1^x + B_2^y S_0^x - B_0^x S_2^y - B_1^x S_1^y - B_2^x S_0^y \right)$$
$$-\frac{\alpha\beta}{M_s}\left(S_0^x S_0^z S_0^x + S_0^y S_0^z S_0^y \right)$$
$$+\frac{\alpha\beta}{M_s}\left[S_0^x \left(B_0^x S_0^z - B_0^z S_0^x \right) - S_0^y \left(B_0^z S_0^y - B_0^y S_0^z \right) \right], \qquad (9d)$$

Similarly, the equations for the x and y components of $\mathbf{S_1}$ can be obtained by replacing the component of **B** in the above equation cyclically. Since the z-axis is the symmetry axis of the nanowire it is reasonable to study the evolution along the symmetry axis by solving the z-component of the LLG equation using the results obtained in the previous orders of the Maxwell's and LLG equations. Without loss of generality, we assume the transverse component of the magnetization in terms of a new variable ϕ , a function of slow variable (ξ, τ) . We use the unperturbed state of the magnetization **S** in a functional form $S_0^x = \cos\phi, \quad S_0^y = \sin\phi$ and $S_0^z = 0.$ (10)

It is interesting to note that in Eq. (9d) the contribution due to Gilbert damping is absent. This is because of the effect of damping which is compensated by the nonlinearity. In polar coordinate representation given in Eq. (10) and (8d) takes the form

$$S_1^z = -\frac{vk}{\beta - Bk} f, \qquad (11)$$

where $f = \frac{\partial \phi}{\partial \xi}$.

Then using new variable $\phi,$ we can write equation (9d) as

$$\mu \frac{\partial^2 \phi}{\partial \xi^2} = \sin\phi \frac{\partial}{\partial \tau} \int \cos\phi \, \partial\xi - \cos\phi \frac{\partial}{\partial \tau} \int \sin\phi \, \partial\xi \qquad (12)$$

Using Eqs. (9a), (9b), (10) and (11) in Eq. (9d), and differentiating with respect to ξ, after making some lengthy algebra and calculations Eq. (9d) can be made equivalent to the modified Korteweg-de Vries (MKDV) equation in the form

$$\frac{\partial f}{\partial \tau}+\frac{3}{2}\mu f^2\frac{\partial f}{\partial \tau}+\mu\frac{\partial^3 f}{\partial \tau^3}=0, \tag{13}$$

where

$$\mu=\left(\frac{V^2k}{\beta-Bk}-\frac{F}{M_s^2}\right)\left(\frac{k^2}{2V\varepsilon_0\beta}\right)$$

$$B=\left(\frac{H_k}{M_s}-4\pi\right).$$

The MKDV is completely integrable nonlinear evolution equation possessing N-soliton solution [26]. For instance the one soliton solution of MKDV equation can be written as

$$f=2a\,\mathrm{sech}\,a(\xi-\gamma\tau), \tag{14}$$

where $a^2=\gamma/\mu$, and γ is a constant. Knowing f and ϕ, one can calculate the components of magnetization as

$$S_0^{\,x}=1-2\,\mathrm{sech}^2 a(\xi-\gamma\tau), \tag{15}$$

$$S_0^{\,y}=2\tanh a(\xi-\gamma\tau)\,\mathrm{sech}\,a(\xi-\gamma\tau), \tag{16}$$

$$S_1^{\,z}=-\frac{vk}{\beta-Bk}2a\,\mathrm{sech}\,a(\xi-\gamma\tau). \tag{17}$$

The above equations (15), (16), (17) represent the excitation of magnetization components in the form of soliton modes as shown in Fig. (2). The figure depicts the lossless propagation of electromagnetic soliton modes in an anisotropic ferromagnetic nanowire.

In order to study the effect of damping, we rescale the damping parameter as $\alpha'=\varepsilon\alpha$ instead of $\alpha'=\varepsilon^2\alpha$. However at the order of ε^2, we obtain the equation in the presence of damping term as follows

$$\begin{aligned}
&-v\frac{\partial M_1^{\,z}}{\partial\xi}=A\left(M_0^{\,x}\frac{\partial^2 M_0^{\,y}}{\partial\xi^2}-M_0^{\,y}\frac{\partial^2 M_0^{\,x}}{\partial\xi^2}\right)\\
&+\beta\left(B_0^{\,y}S_2^{\,x}+B_1^{\,y}S_1^{\,x}+B_2^{\,y}S_0^{\,x}-B_0^{\,x}S_2^{\,y}-B_1^{\,x}S_1^{\,y}-B_2^{\,x}S_0^{\,y}\right)\\
&-\frac{\alpha\beta}{M_s}\left[\begin{array}{l}S_0^{\,x}\left(S_0^{\,x}S_1^{\,z}+S_1^{\,x}S_0^{\,z}\right)+S_0^{\,y}\left(S_0^{\,y}S_1^{\,z}+S_1^{\,y}S_0^{\,z}\right)\\+S_1^{\,x}S_0^{\,x}S_0^{\,z}+S_1^{\,y}S_0^{\,y}S_0^{\,z}\end{array}\right]\\
&+\frac{\alpha\beta}{M_s}\left[\begin{array}{l}S_0^{\,x}\left(B_0^{\,x}S_1^{\,z}+B_1^{\,x}S_0^{\,z}-B_0^{\,z}S_1^{\,x}-B_1^{\,z}S_0^{\,x}\right)\\-S_0^{\,y}\left(S_0^{\,y}B_1^{\,z}+S_1^{\,y}B_0^{\,z}-B_0^{\,y}S_1^{\,z}-B_1^{\,y}S_0^{\,z}\right)\\+S_1^{\,x}\left(B_0^{\,x}S_0^{\,z}-B_0^{\,z}S_0^{\,x}\right)-S_1^{\,y}\left(B_0^{\,z}S_0^{\,y}-B_0^{\,y}S_0^{\,z}\right)\end{array}\right]
\end{aligned} \tag{18}$$

On solving Eq. (19) as in above procedure after cumbersome and lengthy calculations we end up with the perturbed modified Korteweg-de Vries equation (pMKDV) with dissipation term as

$$\frac{\partial f}{\partial \tau}+\frac{3}{2}\mu f^2\frac{\partial f}{\partial \tau}+\mu\frac{\partial^3 f}{\partial \tau^3}+\eta\frac{\partial^2 f}{\partial \tau^2}=0, \tag{19}$$

where

$$\eta=\left(-\frac{\alpha\beta}{M_s}+\frac{\alpha\beta}{KM_s}\right)\left(\frac{VK}{\beta-BK}\right)\left(\frac{K^2}{2V\varepsilon_0\beta}\right)$$

$$\mu=\left(\frac{V^2K}{\beta-BK}-A\right)\left(\frac{K^2}{2V\varepsilon_0\beta}\right).$$

The pMKDV equation is solved using a standard technique that exhibit damped soliton due to the presence of damping. The two-soliton solution of pMKDV is analyzed in the subsequent section

IV. COLLISIONAL DYNAMICS OF EM SOLITON IN FN

The multisoliton solution of nonlinear evolution equations is obtained by various methods such as Backlund transformation method, Lax pair and Hirota bilinearization method [27, 28]. However, the Hirota method [29] is very straight forward among these which gives N-soliton solutions for a variety of nonlinear equations. Recently Kavitha et al, have investigated the soliton-soliton interaction with exchange of energy in Heisenberg ferromagnetic spin chain and spin ladders [21, 30]. In this section, the bilinear form and two soliton solutions of pMKDV equation (19) is derived. The Eq. (19) can be transformed into bilinear form using dependent variable rational transformation as follows

$$f=\frac{G(\xi,\tau)}{F(\xi,\tau)}. \tag{20}$$

Using this transformation, Eq. (19) is converted into a set of bilinear equations with the aid of Hirota's bilinear operator as

$$\left.\begin{array}{l}\left(D_\tau+\eta D_\xi^{\,2}+\mu D_\xi^{\,3}\right)(G\cdot F)=0,\\ 2D_\xi^{\,2}(F\cdot F)=G^2,\\ D_\xi(G\cdot F)=0,\\ D_\xi(F\cdot F)=0.\end{array}\right\} \tag{21}$$

The above set of equations can be solved by introducing the following power series expansions for G and F:

$$G=\varepsilon g_1+\varepsilon^3 g_3+\cdots, \tag{22}$$

$$F=1+\varepsilon^2 f_2+\varepsilon^4 f_4+\cdots. \tag{23}$$

where ε is the formal expansion parameter. Upon substituting the above series (22, 23) in Eq. (21), and collecting the same powers, ε can be solved recursively to obtain the forms of g and f. After deducting the explicit forms of g and f, interaction of solitons can be obtained.

A. One soliton solution

After restricting the power series expansion (22) and (23) as

$$G = \varepsilon g_1,$$

$$F = 1 + \varepsilon^2 f_2,$$

and substituting in Eq. (21) and solving the equations recursively, one can write down the explicit one-soliton solution as

$$f = 4\frac{k_1^2 \varepsilon e^{\eta_1}}{4k_1^2 + \varepsilon^2 e^{2\eta_1}}, \qquad (24)$$

where $\eta_1 = k_1\xi + \omega_1\tau + \delta_1$. Eq. (24) exhibits in the form of one soliton solution.

B. Two soliton solution

In order to obtain the two soliton solution of pMKDV (20) equation, we restrict the power series as follows

$$G = \varepsilon g_1 + \varepsilon^3 g_3, \qquad (25a)$$

$$F = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4, \qquad (25b)$$

again solving the partial differential equations recursively to obtain the explicit form of two soliton solution as follows

$$f = \frac{G(\xi,\tau)}{F(\xi,\tau)}, \qquad (26)$$

Where

$$g_1 = e^{\eta_1} + e^{\eta_2},$$

$$f_2 = \alpha_1 e^{2\eta_1} + \alpha_2 e^{\eta_1+\eta_2} + \alpha_3 e^{2\eta_2},$$

$$\alpha_1 = \frac{1}{16k_1^2}, \quad \alpha_2 = \frac{1}{2(k_1+k_2)^2}, \quad \alpha_3 = \frac{1}{16k_1^2}.$$

$$g_3 = \beta_1 e^{3\eta_1} + \beta_2 e^{2\eta_1+\eta_2} + \beta_3 e^{\eta_1+2\eta_2} + \beta_4 e^{3\eta_2},$$

$$\beta_1 = \frac{\left(\omega_1\alpha_1 + 3k_1^2\eta\alpha_1 + 9\mu k_1^3\alpha_1\right)}{\left(3\omega_1 + 9k_1^2\eta + 27\mu k_1^3\right)},$$

$$\beta_2 = \frac{\begin{pmatrix}(\alpha_2-\alpha_1)\omega_2 + 2\omega_1\alpha_1 + \eta\left[\alpha_2\left(k_1^2+k_2^2\right) + k_1k_2\left(3\alpha_2+2\alpha_1\right)\right] \\ +\eta\alpha_1\left(4k_1^2-k_2^2\right) + \mu\left(-4\alpha_2k_1k_2\left(k_1+k_2\right) - 3\alpha_2k_2^3\right) \\ +\mu\left[8k_1^3\alpha_1 + 4k_1^2k_2 + 6k_1k_2^2\alpha_1 - \alpha_1k_2^2\right]\end{pmatrix}}{2\omega_1 + \omega_2 + \eta\left(2k_1+k_2\right)^2 + \mu\left(2k_1+k_2\right)^3},$$

$$\beta_3 = \frac{\begin{pmatrix}2\omega_2\alpha_3 + (\alpha_2-\alpha_3)\omega_1 + \eta\left[\alpha_3\left(k_2^2-k_1^2\right) + \alpha_2k_1^2 + 2\alpha_2k_1k_2\right] \\ +\eta\left[2\alpha_3k_1k_2 + \alpha_2\left(k_1+k_2\right)k_2\right] + \mu\left[4\alpha_2k_1k_2^2 + 4k_1k_2^2\alpha_3\right] \\ +\mu\left[6k_1^2k_2\alpha_3 - 25k_1^3\alpha_3 - 3k_1^3\alpha_2 - 8\alpha_2k_1^2k_2 - 4\alpha_2k_1k_2^2\right]\end{pmatrix}}{\omega_1 + 2\omega_2 + \eta\left(k_1+2k_2\right)^2 + \mu\left(k_1+2k_2\right)^3},$$

$$\beta_4 = \frac{\left(\omega_2\alpha_3 + 4\eta k_2^3\alpha_3 + \eta k_2^2\alpha_3 + \mu k_2^3\alpha_3\right)}{3\omega_2 + 9\eta k_2^2 + 27\mu k_2^3}.$$

$$f_4 = \gamma_1 e^{4\eta_1} + \gamma_2 e^{2\eta_1+2\eta_2} + \gamma_3 e^{3\eta_1+\eta_2} + \gamma_4 e^{\eta_1+3\eta_2} + \gamma_5 e^{4\eta_2},$$

$$\gamma_1 = \frac{(1/2)\beta_1}{16k_1^2},$$

$$\gamma_2 = \frac{[(1/2)(\beta_2+\beta_3) - 8\alpha_1\alpha_3(k_2^2+k_1^2) + 8\alpha_1\alpha_3k_1k_2]}{4(k_1+k_2)^2},$$

$$\gamma_3 = \frac{[(1/2)(\beta_2+\beta_1) - \alpha_1\alpha_3[(k_2^2+k_1^2) + 4k_1^2] + 4\alpha_1\alpha_3(k_1+k_2)k_1]}{(3k_1+k_2)^2},$$

$$\gamma_4 = \frac{[(1/2)(\beta_3+\beta_4) - \alpha_2\alpha_3[(k_2^2+k_1^2) + 4k_2^2] + 4\alpha_2\alpha_3(k_1+k_2)k_2]}{(k_1+3k_2)^2},$$

$$\gamma_5 = \frac{(1/2)\beta_4}{16k_2^2}.$$

The solution (26) of equation (19) describes the general inelastic scattering process of two solitary waves with different velocities and frequencies. The nature of two soliton interaction of Eq. (26) is depicted in Fig. (3), for the choice of parameters k_1=0.98, k_2=1.1, F = 10^{-9}, H = 1, β = 4. The analysis reveals that there occurs amplitude suppression during collision. From which it is seen that the amplitude of solitons is suppressed after collision. The EM solitons suffer an inelastic collision with abrupt change in the amplitudes of both the EM solitons and the phase of two EM solitons also suffers. In Fig. (3), the EM solitons S_1 and S_2 show the amplitude suppression due to the presence of large damping. Fig. (4) is a typical intensity profile of EM soliton before and after collision. For the damping α=0.01, the two EM soliton suffers in amplitude after the interaction. However, the amplitude of EM soliton is further suppressed by increasing the damping α as shown in Fig. (4). These EM soliton collisions found immense applications in the optical systems. Also the soliton-soliton interaction and its stability can be used to carry logical informations for communication purpose and can be exploited for quantum computations.

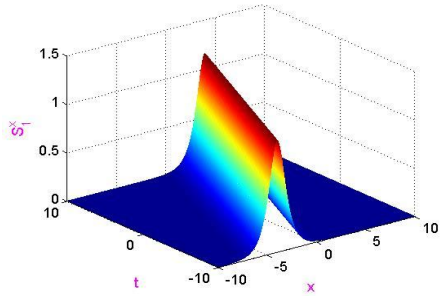


Fig. (2): The evolution of magnetization component S_1^x of Eq. (17).

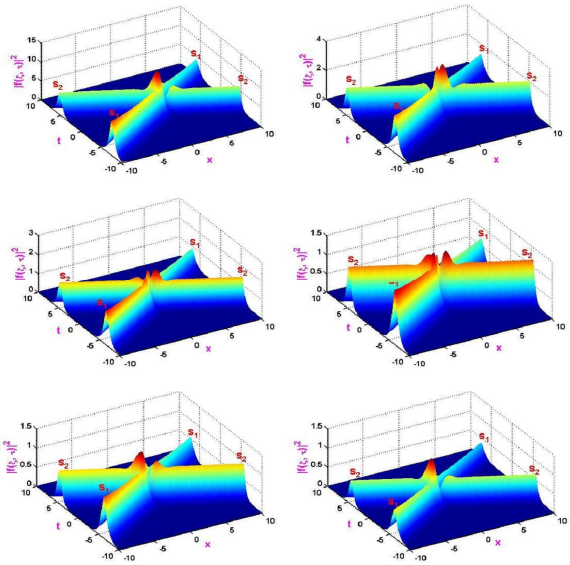


Fig. (3): Inelastic nature of electromagnetic soliton collision of equation (26) for various α (0.01, 0.1, 0.5, 1, 5 and 10).

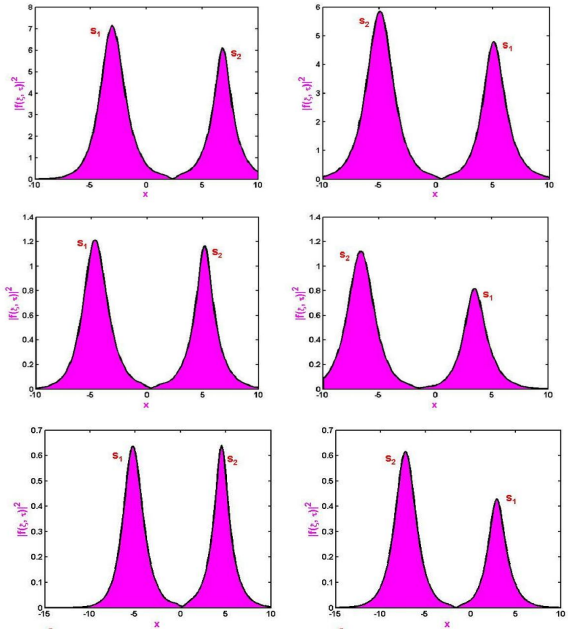


Fig. (4): Intensity profile before (t=-3) and after (t=3) inelastic electromagnetic soliton collision of equation (26) for various values of α (0.01, 0.1 and 10).

CONCLUSION

In summary, we investigated the propagation of an electromagnetic wave in an anisotropic ferromagnetic nanowire. The dynamics of magnetization governs the coupled LLG equation and Maxwell's equation. These coupled nonlinear evolution equations are further reduced to MKDV and pMKDV equations using the standard reductive perturbation technique. The reductive perturbation method is a great tool to study the interaction of EMW and magnetic medium. The propagation of EMW in a ferromagnetic nanowire excites in the form of soliton modes. Also, we have analyzed the inelastic collision of EM solitons through Hirota bilinearization method. The collisional dynamics demonstrate the suppression of electromagnetic soliton by increasing the magnitude of damping. These results may have potential technological applications in magnetoelectronics and magnetic memory devices.

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