SOLITARY EVOLUTION OF BLOOD PRESSURE WAVES IN ARTERIES

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Abstract - Perturbed soliton excitations represent one of the central topics of nonlinear-wave dynamics. We demonstrate the blood flow in nonlinear pressure pulse waves of thin wall elastic tube filled with an incompressible nonviscous fluid. The nonlinear behavior of the pressure pulse wave equation is governing the blood flow model in large blood vessels with the help of mass conservation and momentum theorem in tube wall of the arterial system. The propagation of nonlinear pressure pulse waves in such a medium is studied using long wave approximation technique and the amplitude of nonlinear waves is examined. The transient phenomena of blood flow through the arterial system are replicated by the nonlinear Schrödinger type equation (NLS). A direct multiple scale perturbation analysis is carried out for the NLS equation. This perturbation analysis brings out perturbed solitons to represent the pressure pulse waves in arterial system and found the interesting classes of soliton solutions. We show that such nonlinear model can lead to the existence of solitons moving along the arteries in the blood flow. The obtained results are explored with the experimental measurements for saphenous artery in the dog. Finally, the blood flow in arteries are well explored using the behavior of the perturbed soliton excitations which may have potential applications in clinical diagnosing and also some results will be helpful in discovering the diseases of the arteries.

I. INTRODUCTION

The blood pressure and flow pulses are generated by the intermittent ejection of blood from the left ventricle of the heart. Experimental studies have revealed that these pulses propagate along the different branches of the mammalian arteries with characteristic of nonlinear modulation [1].

We know that the propagation of the blood wave is accompanied by an increase in amplitude with the creation of steep front for the blood pressure pulses. The pulse wave character of the blood wave is soliton like and it suggests a possible interpretation in terms of soliton [2]. The mathematical modeling of such elastic tubes and the waves flowing inside has been of interest during past years [3]. Many models have been proposed that the blood pressure waves are not far away from soliton type evolution [4]. In arterial mechanics, the propagation of pressure pulses in fluid filled distensible tubes has been studied by several researchers [5, 6]. The physiological applications are concerned by most of the workers on wave propagation in compliant tubes and have considered small amplitude waves ignoring the nonlinear effects and focused on the dispersive character of waves. Such problems have been investigated, especially with regard to their applications to biological problems involving pulse propagation in large blood vessels [7]. However, when the nonlinear terms arising from the constitute equations and kinematical solutions are introduced, one has to consider either finite amplitude, or small-but-finite amplitude waves, depending on the orders of nonlinearity. The propagation of finite amplitude waves in fluid filled elastic tubes has been examined, for instance by using the method of characteristics in studying the shock formation [8, 9]. On the other hand, the propagation of small-but-finite amplitude waves in distensible tubes has been investigated [10, 11, and 12] by employing various asymptotic methods. In all these works, depending on the balance between nonlinearity, dispersion and dissipation, the Korteweg de Vries (KdV), Burgers or KdV - Burgers equations are obtained as the evolution equations. As it is well known, when the nonlinear effects are small, the system of equations that describe the physical phenomenon admit a harmonic wave solution with constant amplitude. If the amplitude of the wave is small but finite, the nonlinear terms cannot be neglected, and the nonlinearity gives rise to the variation of amplitude both in space and time variables. Then the amplitude varies slowly over a period of oscillations. A stretching transformation allows decomposing of the system into a rapidly varying part associated with the oscillation and a slowly varying part such as the amplitude. A formal solution can be given in the form of an asymptotic expansion, and an equation determining the modulation of the first-order amplitude can be derived. For instance, the Korteweg-de vries (KdV) equation is the simplest representative equation describing the self-modulation of one-dimensional pressure pulse waves in dispersive media. It exhibits a balance between nonlinearity and dispersion. The problem of self-modulation of small but- finite amplitude waves in fluid-filled distensible tubes was

considered [13], which showed that, for a linear elastic tube wall model, the nonlinear self-modulation of pressure waves is governed by the KdV equation. The dynamics of an inviscous fluid and of a prestressed elastic tube was studied for the amplitude modulation of nonlinear waves and obtained the dissipative KdV equation as the governing equation [14]. Mathematical model with such characteristics has been recently developed [15]. The models appearing in the literature describing blood flow in arteries are represented by linear equations [16, 17] and there has been increasingly strong evidence that nonlinear effects have to be taken into the arterial system. The dynamics of the blood waves are governed by the KdV, modified KdV, nonlinear Schrödinger equation (NLS) and Boussinesq type equations. The solitons propagating along the large arteries have been investigated based on a model system composed of blood flow in a thin walled elastic tube [18], and the effects of blood viscosity have been studied [19], with other approximations which led to nonlinear Schrödinger type equations to describe waves in fluid filled elastic tube [20]. The dynamics can be governed by the nonlinear approach for investigating the Korteweg-de vries equations [21] or even to lattice equations such as the Toda lattice [22]. The solitons propagating through the large arteries have been investigated based on the model composed of blood flow in a thin elastic tube, with the nonlinear Schrödinger (NLS) type equations to describe pressure pulse waves and more recently, in surgical planning [8]. In this paper we investigate some interesting features of nonlinear blood pressure pulse waves by employing the prestressed thin elastic tube, which assumed to be a model for large arteries. The paper is organized as follows: In Section II, we describe the model and derive the equation of motion for such a medium studied using the reductive perturbation method. The model equation shows that the amplitude modulation of the waves which is governed by the nonlinear Schrödinger (NLS) equation interms of amplitude and velocity. The governing equation is devoted to the analytical investigation through multiple scale perturbation analysis and derives the different conditions for perturbed solitary wave solution. The results are concluded in Section III and IV.

II. GOVERNING EQUATIONS OF THE MODEL

We have chosen a model that assumes the blood vessel as a cylindrical tube having the nonlinear elastic walls. The unit base vectors are e_r, e_θ, e_z in cylindrical polar coordinates in the basic configuration as shown in Fig. 1

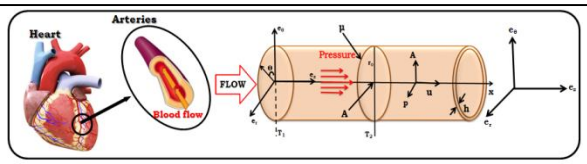


FIG 1: Schematic diagram of elastic tube of large arteries with propagation of blood pressure pulse waves along x- direction.

The laws of hydrodynamics governing the transport of inviscid incompressible fluid are the conservation of mass and momentum equations is given by [1],

$$\frac{\partial u}{\partial t}+A\frac{\partial U}{\partial x}+u\frac{\partial A}{\partial x}=0,$$
$$\frac{\partial A}{\partial t}+A\frac{\partial A}{\partial x}+\frac{\partial p}{\partial x}=0, \qquad (1)$$

The approximation of uniform blood velocity u (z, t) and pressure P (z, t) for all points on each cross-section A (z, t) of the artery. The internal pressure p is expressed as follows

$$p=\frac{2}{1+u}\frac{\partial^2 u}{\partial t^2}+\frac{u(1+\alpha u)}{(1+u)^2}, \qquad (2)$$

where α is nonlinear wall coefficient of elastic tube. We examine the propagation of pulse waves in such medium using long wave approximation technique and the dispersion relation and group velocity are obtained as,

$$\omega^2=\frac{k^2}{1+2k^2}, \quad \text{and} \quad v=\frac{k}{\omega}\frac{k}{(1+2k^2)^2}.$$

Now we introduce the following stretched variables

$$\xi=\varepsilon(x-vt), \quad \tau=\varepsilon^2 t, \qquad (3)$$

where ϵ is a small parameter, and v is the group velocity. Now, we can expand the field quantities into an asymptotic series of ϵ as

$$u=1+\sum_{n=1}^{\alpha}\varepsilon^n u^{(n)}(\xi,\tau,x,t),$$
$$p=\sum_{n=1}^{\alpha}\varepsilon^n p^{(n)}(\xi,\tau,x,t),$$
$$A=\sum_{n=1}^{\alpha}\varepsilon^n A^{(n)}(\xi,\tau,x,t), \qquad (4)$$

Introducing the expansion Eq. (3 & 4) into Eq. (1) & (2) and setting the like powers of ϵ equal to zero.

At $0(\varepsilon^1 e^1)$ order equations:

$$u^{(1,1)}=\frac{k}{\omega}A^{(1,1)},$$
$$A^{(1,1)}=\frac{k}{\omega}p^{(1,1)},$$
$$p^{(1,1)}=(1-\omega^2)u^{(1,1)}. \qquad (5)$$

At $0(\varepsilon^2 e^0)$ order equations:

$$-vu_\xi^{(1,0)}+A_\xi^{(1,0)}=0,$$
$$-vA_\xi^{(1,0)}+p_\xi^{(1,0)}=0,$$
$$p^{(2,0)}=2u^{(2,0)}. \qquad (6)$$

At $0(\varepsilon^2 e^1)$ order equations:

$$-i\omega u^{(2,1)} - u_{\xi}^{(1,1)} + ikA^{(2,1)} + A_{\xi}^{(1,1)} = 0,$$
$$-i\omega A^{(2,1)} - vA_{\xi}^{(1,1)} + ikp^{(2,1)} + p_{\xi}^{(1,1)} = 0,$$
$$p^{(2,1)} = \left(1-\omega^2\right)u^{(2,1)} - 4i\omega v u_{\xi}^{(1,1)}. \qquad (7)$$

At $0(\varepsilon^2 e^2)$ order equations:

$$-\omega u^{(2,2)} + kA^{(2,2)} + ku^{(1,1)}A^{(1,1)} = 0,$$
$$-\omega A^{(2,2)} + kp^{(2,2)} + \frac{1}{2}\left[A^{(1,1)}\right]^2 = 0,$$
$$p^{(2,2)} = \left(1-4\omega^2\right)u^{(2,2)} + \left(2k^2 - 2 - \alpha\right)\left[u^{(1,1)}\right]^2. \qquad (8)$$

At $0(\varepsilon^3 e^0)$ order equations:

$$A_{\xi}^{(2,0)} - vu_{\xi}^{(2,0)} + \left(u^{(1,-1)}A^{(1,1)}\right)_{\xi} + \left(u^{(1,1)}A^{(1,-1)}\right)_{\xi} = 0,$$
$$vA_{\xi}^{(2,0)} - p_{\xi}^{(2,0)} + \left(A_{\xi}^{(1,-1)}A_{\xi}^{(1,1)}\right) + \left(A_{\xi}^{(1,1)}A^{(1,-1)}\right) = 0,$$
$$p^{(3,0)} = 2A^{(3,0)}. \qquad (9)$$

At $0(\varepsilon^3 e^1)$ order equations:

$$-i\omega u^{(3,1)} + u_{\tau}^{(1,1)} - vu_{\xi}^{(1,1)} + ikA^{(3,1)} + A_{\xi}^{(2,1)}$$
$$+ik\left(2u^{(1,-1)}A^{(2,2)} + 2u^{(2,2)}A^{(1,-1)}\right) = 0,$$
$$-i\omega A^{(3,1)} - vA_{\xi}^{(1,1)} + A_{\tau} + ikp^{(3,1)} + p_{\xi}^{(2,1)}$$
$$+ik\left(A^{(1,-1)}A^{(2,2)} + A^{(2,2)}A^{(1,-1)}\right) = 0,$$
$$p^{(3,1)} = \left(1-2\omega^2\right)u^{(3,1)} + 2v^2 u_{\xi\xi}^{(1,1)} - 4ikvu_{\xi}^{(2,1)}$$
$$+4iku_{\tau}^{(1,1)} + 2ik\left(u^{(1,-1)}u^{(2,2)} + u^{(2,2)}u^{(1,-1)}\right). \qquad (10)$$

We obtain from the Eqs. (8)

$$u^{(2,2)} = \frac{k^2(3-2\alpha) - 2 - \alpha}{2k^4}\left|u^{(1,1)}\right|^2,$$
$$A^{(2,2)} = \frac{\omega}{k}\frac{k^2(3-2\alpha) - 2 - \alpha}{2k^2}\left|u^{(1,1)}\right|^2. \qquad (11)$$

Similarly we can get from Eqs. (11)

$$iu_{\tau} + p_{\xi\xi} + q\left|u\right|^2 u = 0, \qquad (12)$$

where

$$p = \frac{\omega\left(\omega^3 - 2\omega^2 kv + k^2v^2\omega - 4v\omega^2k^3 + 2k^4v^2\omega\right)}{4k^5\omega^2},$$
$$q = \frac{\omega^2}{2k^3}\left[2\left(\frac{k^2(3-\alpha) - 2 - \alpha}{2k^2} - 1\right)\left(\frac{k^2(3-\alpha) - 2 - \alpha}{2k^4}\right)\right].$$

By introducing the rescaling transformations

$$\xi = \sqrt{\frac{1}{k}}X, \quad \text{and} \quad \tau = \frac{k^6}{\omega^2}T.$$

In Eq. (12) we get the following perturbed nonlinear Schrödinger equation

$$iu_T + u_{XX} - 2\left|u\right|^2 u + \lambda\left[\delta_1 U_{XX} + \delta_2\left|U\right|^2 U\right] = 0, \qquad (13)$$

where

$$\delta_1 = \frac{1}{\alpha^2}\left(\frac{k^2v}{2\omega} - \frac{v^2k^3}{4\omega^2} + \frac{vk^4}{\omega} - \frac{v^2k^5}{2\omega^2}\right),$$
$$\delta_2 = -\frac{3k^2}{2\alpha^2} + \frac{k^4}{\alpha^2} - \frac{k^4}{4} - \frac{1}{2} - \frac{1}{\alpha^2k^2} - \frac{1}{\alpha k^2} - \frac{k^6}{16}.$$

The perturbed NLS equations describe the propagation of pressure pulse solitary waves in arterial elastic tube medium. We now carry out the multiple scale perturbation analysis by treating the terms proportional to λ in Eq. (13) as perturbation [23-25]. Considering u_0 as the exact solution corresponding to the unperturbed part ($\lambda = 0$), the envelope one soliton solution of Eq. (13) can be written as

$$u = \eta\,\mathrm{sech}\,\eta(\theta-\theta_0)\exp\left[i\xi(\theta-\theta_0) + i(\sigma-\sigma_0)\right], \qquad (15)$$

where, $\theta = 2\xi$, $\theta_x = 1$, $\sigma_t = \eta^2 + \xi^2$ and $\sigma_x = 0$ we write η, ξ, θ, θ_0 and σ_0 as the functions of the new time scale $T = \lambda t$. Hence the soliton solution of Eq. (13) can be written as

$$\hat{u} = \hat{u}(\theta, T : \lambda)\exp\left[i\xi(\theta-\theta_0) + i(\sigma-\sigma_0)\right].$$

The above assumption of quasi stationary seems to be conceptually simple generalization of the unperturbed form and is appropriate for explicit calculations. Upon substituting Eq. (15) in Eq. (13), we get

$$-\eta^2\hat{u}_1 + \hat{u}_{1\theta\theta} - 2\hat{u}^2\hat{u}^* = \lambda F\left(\hat{u}\right), \qquad (16)$$

where

$$F\left(\hat{u}\right) = i\hat{u}_t + \hat{u}\left(\xi_T(\theta-\theta_0) - \xi\theta_{oT} - \sigma\theta_{oT}\right)$$
$$-\delta_1\left(\hat{u}_{\theta\theta} + 2i\hat{u}_0\xi - \xi^2\hat{u}\right) + \delta_2\left|\hat{u}\right|^2\hat{u}. \qquad (17)$$

Expanding the $\hat{u}$ in terms of λ as

$$\hat{u}(\theta, T : \lambda) = \hat{u}(\theta, T) + \lambda\hat{u}_1(\theta, T) + \ldots \qquad (18)$$

Where $\hat{u}_0 = \eta\,\mathrm{sech}\,\eta(\theta-\theta_0)$ and substituting the Eq. (18) in Eq. (17) at *0* (λ) we get

$$-\eta^2\hat{u}_1 + \hat{u}_{1\theta\theta} + 4\hat{u}_0^2\hat{u}_1 - 2\hat{u}_0^2\hat{u}_1 = F_1\left(\hat{u}_0\right). \qquad (19)$$

The right hand side of Eq. (19) of the form,

$$F_1\left(\hat{u}_0\right) = i\hat{u}_{0T} + \hat{u}_0\left(\xi_T(\theta-\theta_0) - \xi\theta_{oT} - \sigma\theta_{oT}\right)$$
$$-\left\{\delta_1\left(\hat{u}_{0\theta\theta} + 2i\hat{u}_{0\theta}\xi - \xi^2\hat{u}\right) + \delta_2\hat{u}^3\right\} \qquad (20)$$

Substituting $\hat{u}_1 = \phi_1 + i\psi_1$ (ϕ_1 and ψ_1 are real) in Eq. (18), we obtain

$$L_1\phi_1 = -\eta^2\phi_1 + \phi_{1\theta\theta} - 6u_0^2\phi_1 = Re\,\hat{F}_1,$$
$$L_2\psi_1 = -\eta^2\psi_1 + \psi_{1\theta\theta} - 2u_0^2\psi_1 = Im\,\hat{F}_1. \qquad (21)$$

where $\mathrm{Re}\hat{F}_1(\hat{u}_0)$ and $\mathrm{Im}\hat{F}_1(\hat{u}_0)$ are real and imaginary parts of $\hat{F}_1(\hat{u}_0)$ given by Eq. (20) and L_1 and L_2 are self - adjoint operators. On solving Eq. (21), we obtain the perturbed soliton solution.

As $\hat{u}_{\theta\theta}$ and $\hat{u}_0$ are solutions of the homogenous parts of Eq. (21) respectively. We have the secularity conditions,

$$\int_{-\alpha}^{+\alpha} \hat{u}_{0\theta}\, Re\,\hat{F}_1\, d\theta = 0,\ \int_{-\alpha}^{+\alpha} \hat{u}_0\, Im\,\hat{F}_1\, d\theta = 0. \qquad (22)$$

Upon substituting the values of $\hat{u}_0$, $\hat{u}_{0\theta}$, $Re\hat{F}_1$ and $Im\hat{F}_1$ into the above integrals and after evolution, we finally obtain

$$\xi_T = \frac{\eta^2 \delta_1}{2}, \text{ and } \eta_T = 0. \qquad (23)$$

Integrating once again the Eq. (36), we obtain the amplitude and velocity of the soliton as

$$\xi = \xi_0 + \frac{\delta_1}{8}\eta_0^4 T \text{ and } \eta = \eta_0 \qquad (24)$$

Here ξ_0 and η_0 are the initial velocity and amplitude of the soliton. From the above equation it is evident that the amplitude of the soliton varies linearly with time t and travels with constant amplitude η_0. The perturbed solution of Eq. (24) can be constructed by solving Eq. (13) for ϕ_1 and ψ_1.

$$\phi_1 = c_1\phi_{11} + c_2\phi_{12} - \phi_{11}\int_{-\alpha}^{\theta} \phi_{12}\, Re\,\hat{F}_1 d\theta + \phi_{12}\int_{-\alpha}^{\theta} \phi_{11}\, Re\,\hat{F}_1 d\theta,$$

$$\psi_1 = c_3\psi_{11} + c_4\psi_{12} - \psi_{11}\int_{-\alpha}^{\theta} \psi_{12}\, Im\,\hat{F}_1 d\theta + \psi_{12}\int_{-\alpha}^{\theta} \psi_{11}\, Im\,\hat{F}_1 d\theta. (25)$$

Where c_1, c_2, c_3, and c_4 are arbitrary constants. The construction in explicit forms of general solutions of ϕ_{11}, and ϕ_{12} require evolution of the integrals in the last two terms in Eq. (25). In order to find the general solution of ϕ_1, we remove the secular terms using the boundary condition $\phi_1(0)|_{\theta_0=0} = 0$ and $\phi_{10}(0)|_{\theta_0=0} = 0$ and choosing

$$c_1 = -\left\{\frac{\eta}{4}\left(\frac{1}{2}\delta_1 + \delta_2\right) + \frac{3}{4\eta}\left(\xi\theta_{0T} + \sigma_{0T} - \xi^2\delta_1\right)\right\},$$

$$c_2 = \left\{\frac{\eta^2}{4}(\delta_1 + \delta_2) + \frac{1}{2}\left(\xi\theta_{0T} + \sigma_{0T} - \xi^2\delta_1\right)\right\}. \qquad (26)$$

The final form of the general solution ϕ_1 can written as

$$\phi_1(\theta) = \begin{bmatrix} \left\{\frac{\eta}{2}\left(\eta(\theta-\theta_0) - \frac{1}{4}\eta(\theta-\theta_0)^2 - \frac{3}{4}\right)\delta_1 - \right. \\ \left. \frac{1}{4}(\theta-\theta_0)\left(\xi\theta_{0T} + \sigma_{0T} - \xi^2\delta_1\right) + \frac{3\eta^2}{2}\delta_2\right\} \\ sech\,\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0) - \left[\frac{\eta^2}{8}(\theta-\theta_0)\delta_1\right] \\ sech\,\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0) + \frac{\eta^2}{4}(\theta-\theta_0)\delta_1 \\ sech\,\eta(\theta-\theta_0) \end{bmatrix}, (27)$$

and secular terms can be removed choosing c_3, c_4 to be zero and applying the boundary condition gives

$$c_3 = -\frac{1}{2}\left(\frac{1}{2}(\theta-\theta_0)_T + \xi\delta_1\right), \text{ and}$$

$$c_4 = 2\eta\left(\frac{1}{2}(\theta-\theta_0)_T + \xi\delta_1\right). \qquad (28)$$

Then found the final form of the general solution ψ_1

$$\psi_1(\theta) = -\frac{\eta}{2}(\theta-\theta_0)\left(\frac{1}{2}(\theta-\theta_0)_T + \xi\delta_1\right) sech\,\eta(\theta-\theta_0). (29)$$

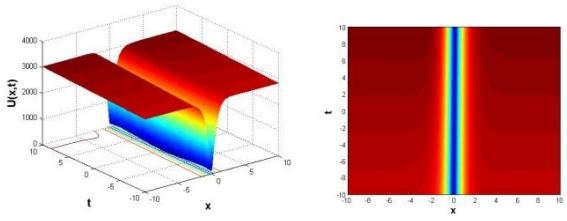
Thus, the general perturbed soliton solution u_1 is obtained as

$$u_1 = \begin{bmatrix} \left\{\frac{\eta}{2}\left(\eta(\theta-\theta_0) - \frac{1}{4}\eta(\theta-\theta_0)^2 - \frac{3}{4}\right)\delta_1 - \right. \\ \left. \frac{1}{4}(\theta-\theta_0)\left(\xi\theta_{0T} + \sigma_{0T} - \xi^2\delta_1\right) + \frac{3\eta^2}{2}\delta_2\right\} \\ sech\,\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0) - \left[\frac{\eta^2}{8}(\theta-\theta_0)\delta_1\right] \\ sech\,\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0) + \frac{\eta^2}{4}(\theta-\theta_0)\delta_1 \\ sech\,\eta(\theta-\theta_0) + i\begin{bmatrix} -\frac{\eta}{2}(\theta-\theta_0)\left(\frac{1}{2}(\theta-\theta_0)_T + \xi\delta_1\right) \\ sech\,\eta(\theta-\theta_0) \end{bmatrix} \end{bmatrix}. (30)$$

III. RESULTS AND DISCUSSION

In this section, the results obtained from this theory are compared with the experimental measurements given by McDonald for a dog. These experimental results are summarized as follows [12] -for the femoral artery
r_0=1.5x10^{-3}m, h_0/r_0=0.12, ρ=1.05x10^{-9}kg/m^3,
E=14.1x10^{5}pa, v=1m/s, p_m-p_e=110mHg=110x10^{-9}pa.

The effect of the nonlinear wall parameter on the perturbed soliton is well presented by plotting the real ϕ_1 and imaginary ψ_1 part of Eqs. (27 and 29) as shown in Figs. (2, 3). Fig. (2) shows the real part of the perturbed soliton solution of Eq. (29) with the choices of parameters for k=0.1, ω=0.09, □□□=0.08, □=0.001, θ_0=0.00001, a=0.8 □□=0.8 and varying the values of nonlinear elastic wall parameter. From the results it is found that for increasing the nonlinear parameter λ the amplitude of the pressure pulse waves is increased. Similarly, we also constructed imaginary part of the perturbed soliton solution of Eq. (28) with the various choices of λ. It explores the small amplitude fluctuation in the localized region of the soliton. Fig. (4) displays the perturbed soliton solution of Eq. (30) by varying nonlinear wall parameter while keeping the other parameter constant. This figure shows that the nonlinear parameter λ increases the wave profile of the soliton a perturbed and also the amplitude of the soliton is increased. This explores that the nonlinearity has severe impact on the propagation of blood flow in arteries depending on the strength of the pressure pulse wave velocity.



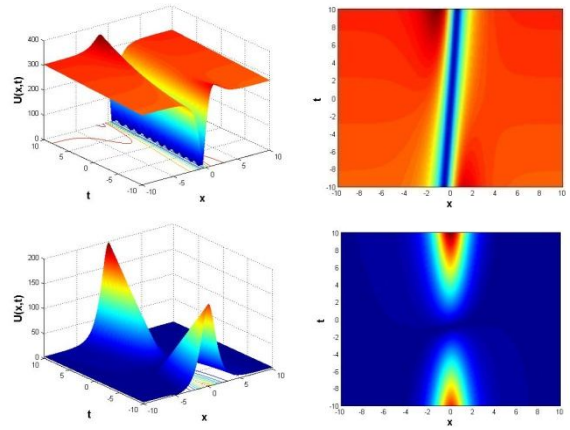


FIG 2: Real part of the perturbed soliton solution for λ=0.1x10^{-5} to λ=0.1x10^{-7}, k=0.1, ω=0.09,□□□□=0.08, □=0.001, θ_0=0.00001, a=0.8, □$_0$=0.8.

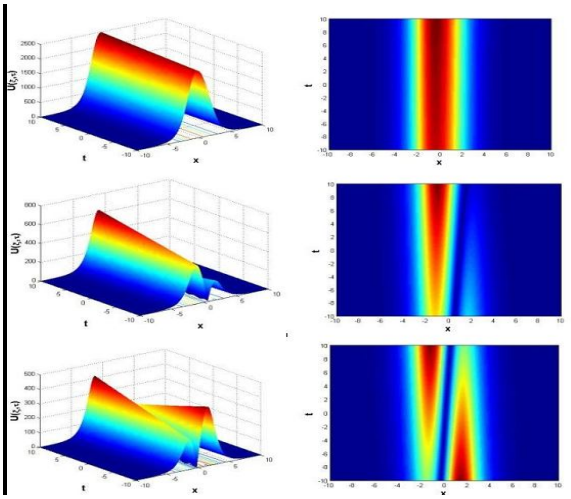


FIG 3: Imaginary part of the perturbed soliton solution for λ=0.1x10^{-3} to λ=0.1x10^{-5}, k=0.8, ω=0.1, □□□=0.007, □=0.09, θ_0=0.1, a=1.6.

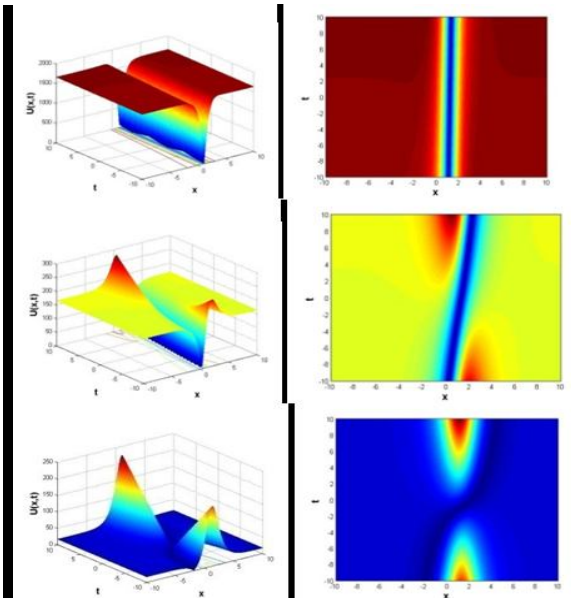


FIG 4: Perturbed soliton solution for λ=0.1x10^{-6} to λ=0.1x10^{-8}, k=0.1, ω=0.1, □□□=0.1, □=0.001, θ_0=1.2, a=0.9, □$_0$=0.1.

CONCLUSIONS

In this paper, we investigated the perturbed soliton excitation in nonlinear blood pressure pulse waves in thin elastic tubes of arteries. We have chosen a model of blood vessals which have incompressible inviscid fluid with nonlinear elastic walls. The propagation of nonlinear waves in such a medium is studied using the long wave approximation method. The model equation of the ideal fluid for slowly varying pressure waves in the arterial parameters is governed by the perturbed nonlinear Schrödinger (NLS) equation are obtained interms of amplitude and velocity. The NLS type equations solved for multiple scale perturbation method through which the blood vessel wall motion under the blood pressure pulse waves is established. Moreover, the investigation clearly shows that theoretical possibility for the well behavior of the pressure pulse waves and parameter influence of the vascular diseases. This theoretical model is calculated in more particularly valid in the small range of the pressure pulse wave's excitations in the elastic tube of arteries. Therefore, the nonlinear parameter plays an important role in many biological functions of circulatory process in different parts of human and mammalian bodies. We analyzed the wave behaviour in elastic tube of arteries with perturbed soliton excitations by using mathematical model. These mathematical model can be used in the analysis and understanding of the effects of vascular diseases and thrombosis on blood pressure pulse waves. Finally, we concluded that our results may be potentially useful for future experiments in various cardiovascular diseases.

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