AN APPLICATION OF ARTIFICIAL BEE COLONY ALGORITHM TO CARDINALITY CONSTRAINED PORTFOLIO OPTIMIZATION PROBLEM

CAN B. KALAYCI

Department of Industrial Engineering, Pamukkale University, Turkey
E-mail: cbkalayci@pau.edu.tr

Abstract— Portfolio selection problem is a real valued variables optimization problem of two conflicting objectives: profit maximization and risk minimization. The problem is to obtain an efficient frontier that corresponds to a set of pareto optimal solutions, since both objectives cannot be simultaneously achieved. In this paper, an artificial bee colony algorithm based solution approach is applied to solve cardinality constrained portfolio optimization problem. Computational results are promising where there is room for improvement.

Keywords— Portfolio Optimization, Cardinality Constraints, Metaheuristics, Artificial Bee Colony Algorithm.

I. INTRODUCTION

Realistic portfolio selection requires the satisfaction of some additional constraints such as cardinality considerations that impose lower/upper limits to the weights of assets and enforce the exact numbers of assets to be held in the portfolio. Chang, et al. [1] extended the mean-variance model [2, 3] introduced by Markowitzto a nonlinear programming model by adding a λ parameter that combines two objectives in order to track the efficient frontier by the help of a search algorithm. The problem under study is known to be computationally hard to handle. Therefore, a solution approach based on artificial bee colony algorithm [4] is applied to efficiently converge the efficient frontier that represents the best solutions possible for both objectives.

The mathematical formulation of cardinality constrained portfolio optimization problem is given as follows: Parameters: Nrepresents the number of assets available, $\mu_i$represents expected return of asset i, σ_{ij} representscovariance value between asset $i$and asset$j$, R^* represents expected return, $K$is the desired number of assets, $\varepsilon_i$is the minimum weight ofasset$i$.andδ_i is the maximum weight of asset i.Decision variables: w_i is the weight of asset i that is held in the portfolio while z_i is a binary variable which is equal to 1 if the asset i is held in the portfolio, 0 otherwise.

$$min \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \quad (1)$$

$$\sum_{i=1}^{N} w_i = 1 \quad (2)$$

$$\sum_{i=1}^{N} z_i = K \quad (3)$$

$$\varepsilon_i z_i \le w_i \le \delta_i z_i i = 1,\dots,N \quad (4)$$

$$z_i \in (0,1) i = 1,\dots,N \quad (5)$$

$$0 \le w_i \le 1, \quad i = 1,\dots,N \quad (6)$$

$$0 \le \varepsilon_i \le \delta_i \le 1, i = 1,\dots,N \quad (7)$$

Equation (1) minimizes risk while maximizing expected return for different values of λ parameter that is linearly increased from 0 to 1. Equation (2) ensures that the sum of asset weights is equal to 1 while Equation (3) guarantees that exactly K assets are held in the portfolio. Equation (4) imposes the lower and upper limit restrictions for weights of each asset. Equation (5), Equation (6) and Equation (7) define variable domains.Section 2 presents the methodology applied to the problem while computational results are given in Section 3. Finally, Section 4 concludes the paper.

II. METHOD

Artificial bee colony (ABC) algorithm, designed based on swarm intelligence concept that mimics the behavior of honey bees, is widely applied to tackle problems that fall into NP-Hard class of problems[5-7].In three phases, employed, onlooker and scout bees fly on the search space to exploit better solutions and explore new promising solutions. According to our knowledge, there are only six papers [8-13] that apply of ABC algorithm to portfolio optimization problem variants. This paper presents another application of ABC to cardinality constrained portfolio optimization problem. The pseudo code for artificial bee colony algorithm is given in Figure1.

Algorithm: Artificial bee colony
Inputs: data (R_i, VC_{ij}) and parameters $(\varepsilon, K, E, ps, klimit, IT)$
Output: H
S: a set of assets in the index
N: number of assets
ps: population size
POP: population
E: number of λ
T: number of iteration
H: final population
EB: set of weights produced in employed bee phase
OB:set of weights produced in onlooker bee phase
f: fitness value
$limit_s$: limit values of the solutions in the population, $s = 1, \dots, ps$
r: a uniform random value, $r \in [-1,1]$
rk: a uniform random integer, $rk \in [1, ps]$
$klimit$: limit value that must be overcome for the introduction of the explorers bee
Start
$H = \emptyset$
$e = 1$
Repeat
$\lambda = (e-1)/(E-1)$
$W_i = \varepsilon + r(1-\varepsilon), \quad i = 1, \dots, ps \quad \forall W_i \in POP$
$W_i^{rep} \leftarrow Repair(N, K, S, W_i, z_i) \;\; i = 1, \dots, ps \quad \forall W_i \in POP$
$EB \leftarrow Evaluate\ fitness(EB, N, VC_{ij}, R_i, \lambda)$
$POP = \{W_1^{rep}, \dots, W_i^{rep} \dots, W_{ps}^{rep}\}$
*/*employed bee phase*/*
$s = 1$
Repeat
$k = rk, rk \in [1, ps], rk \neq s$
$EB_i \leftarrow POP_{si} + rPOP_{si} - POP_{ki} \qquad /* \; i = 1, \dots, N, s = 1, \dots, ps \; */$
$EB \leftarrow Repair(N, K, S, EB)$
$\{R^*, V^*, f_{EB}\} \leftarrow Evaluate\ fitness(EB, N, VC_{ij}, R_i, \lambda)$
If $f_{EB} < f_{POP_s}$ **then**
$POP_s \leftarrow EB$
$limit_s = 0$
Else $limit_s = limit_s + 1$
$s = s + 1$
Until $s = ps$
*/*onlooker bee phase*/*
$s = 1$
Repeat
$l \leftarrow roulette\ wheel(N, POP)$
$OB = POP_l$
$k = rk, rk \in [1, ps], rk \neq l$
$OB_i = POP_{li} + r * (POP_{li} - POP_{ki}), \quad i = 1, \dots, N$
$OB \leftarrow Repair(N, K, S, OB)$
$\{R^*, V^*, f_{OB}\} \leftarrow Evaluate\ fitness(OB, N, VC_{ij}, R_i, \lambda)$
If $f_{OB} < f_{POP_l}$ **then**
$POP_l \leftarrow OB$
$limit_l = 0$
Else $limit_l = limit_l + 1$
$s = s + 1$
Until $s = ps/2$
*/*Scout bee phase*/*
$s = 1$
Repeat
If $limit_s > klimit$ **then**
$W_i = \varepsilon + r(1-\varepsilon), \quad i = 1, \dots, ps \quad \forall W_i \in POP$
$SB \leftarrow Repair(N, K, S, W_i)$
$\{R^*, V^*, f_{SB}\} \leftarrow Evaluate\ fitness(SB, N, VC_{ij}, R_i, \lambda)$
$POP_s \leftarrow SB$
$limit_s = 0$
Until $s = ps$
*/*best solution in population add set of pareto optimal solutions */*
$H_e = POP_{best}$
Until $e = E$
Stop

Fig.1: Artificial bee colony algorithm

III. RESULTS AND DISCUSSION

Five different data sets for major capital market indices reported in the literature[1]; namely, Hang Seng (Hong Kong), DAX 100 (Germany), FTSE 100 (UK), S&P 100 (USA) and Nikkei 225 (Japan) are used for testing the proposed solution approach. In the literature, three performance measures are widely performed [14, 15] in order to calculate the errors between heuristic frontier and the standard efficient frontier: Mean Euclidian Distance (MED), Variance of Return Error (VRE) and Mean Return Error (MRE). Results obtained by ABC for all data sets with different K values is reported in Table 1. Efficient frontiers obtained by ABC on NIKKEI 225,Hang Seng, DAX 100, FTSE 100 and S&P 100 and are demonstrated in Figures 1-5, respectively.

Table 1: Results obtained by ABC for all data sets with different K values

	MED	VRE	MRE
K	*Hang Seng (Hong Kong)*		
5	0.0000	0.6388	0.3698
10	0.0002	4.0354	1.1899
20	0.0005	24.5434	2.7012
	DAX 100 (Germany)		
5	0.0001	3.2006	0.9817
10	0.0002	15.9678	1.6927
20	0.0002	32.7496	0.4768
	FTSE 100 (UK)		
5	0.0000	2.0435	0.0758
10	0.0001	4.8364	0.3925
20	0.0001	16.5120	0.3951
	S&P 100 (USA)		
5	0.0000	4.2491	0.7168
10	0.0001	5.4077	1.2344
20	0.0002	17.7206	0.7221
	Nikkei 225 (Japan)		
5	0.0000	0.7508	0.0927
10	0.0000	1.9813	0.8514
20	0.0002	20.5082	1.2086

MED: Mean Euclidian Distance,VRE: Variance of Return Error, MRE: Mean Return Error

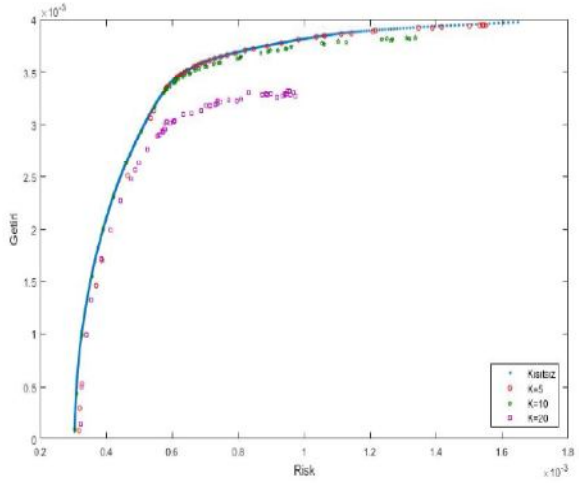


Fig.1: Efficient frontier obtained by ABC on NIKKEI 225 data set for different *K* values

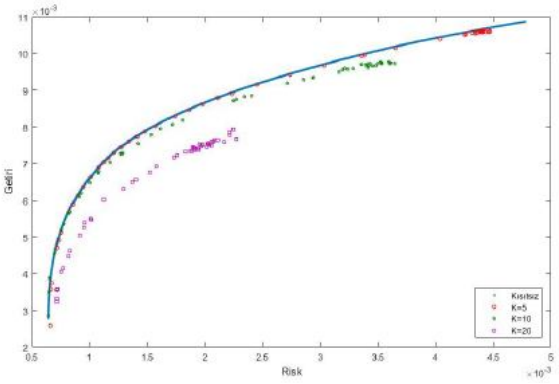


Fig.2: Efficient frontier obtained by ABC on Hang Seng data set for different *K* values

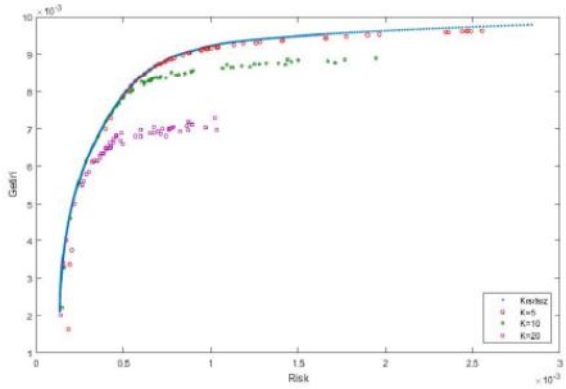


Fig.3: Efficient frontier obtained by ABC on DAX 100 data set for different *K* values

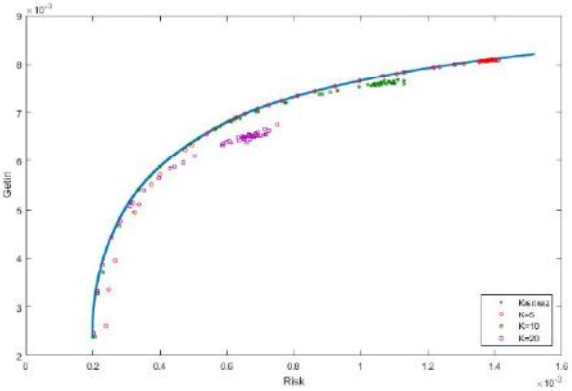


Fig.4: Efficient frontier obtained by ABC on FTSE 100 data set for different *K* values

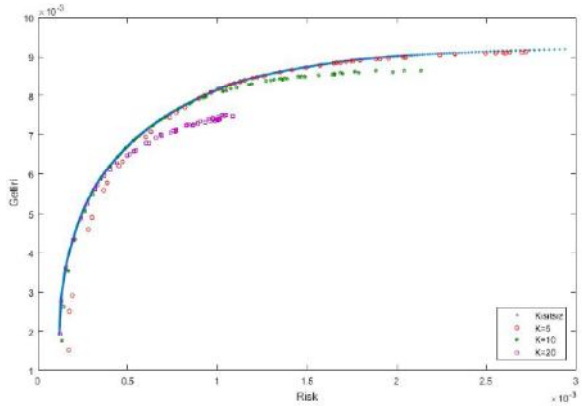


Fig.5: Efficient frontier obtained by ABC on S&P 100 data set for different *K* values

CONCLUSIONS

In this study, an artificial bee colony algorithm based solution approach is applied to solve cardinality constrained portfolio optimization problem. Computational results are promising where there is room for improvement.

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