INVENTORY MANAGEMENT FOR STOCHASTIC DEMAND AND LEAD TIME PRODUCTS

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Abstract- Inventory management is one of many revenue management techniques which focuses on balancing between the customer demand and supply. Inventory management plays a vital role in reducing the stocking cost while preparing enough products to guarantee the customers' satisfaction. This paper presents one of many inventory management strategies to deal with an uncertain behavior of various important factors, stochastic demand and lead time. Search algorithm and a simulation are used in order to determine the optimal reorder point (ROP*) according to important parameterse.g., the demand, lead time of products, and the minimum order quantity (MOQ).The search method in this paper is decrease the search space by half in every round. The result shows the company can reduce the reorder point for about 20-30%, but likely stay the same for the service level.

Keywords: Inventory Management, Reorder Point, Stochastic Demand, Stochastic Lead Time.

I. INTRODUCTION

Inventory is one of many ways to support fluctuations of demand and supply. An important thing to consider when manage inventory is the nature of a system. According to[1], the nature of an inventory system may be different based on: 1) repetitiveness of inventory decision, 2) source of supply, 3) knowledge of demand, 4) pattern of replenishment: continuous or periodic review, and 5) knowledge of lead time. There are various types of inventory models that are suitable for a variety of the inventory systems.

One of the easiest models for inventory is an economic order quantity model (EOQ model). Nahmias[2] argued that an EOQ model represents a concept in trading-off between several costs of inventory management. The objective of this inventory model is to minimize the expected inventory cost. Therefore, a quantity that have minimum inventory cost is called Economic order quantity (EOQ). An EOQ model is based on many assumptions, i.e., deterministic and constant demand, zero lead time, whole lot size received, and no shortage allowed. However, most of practical demands are indeterminable and stochastic.Thus, an inventory model with stochastic demand is called stochastic inventory model. Lieberman[3] pointed out that Two-bin System is a concept model to deal with a stochastic demand. Two-bin System is an inventory model that has two bins of storage. The capacity of the first bin is called Reorder Point (ROP). The system starts by using the second bin until it runs out. When the second bin is empty, there is a signal that calls an order for replenishment. The first bin is, then, used during the waiting for the replenishment period. This two-bin system is one of continuous review inventory systemmodels. As the computer systems and technology grow and become more famous, many inventory level tracking systems have more accuracy and precision. Therefore, an inventory level can be monitored in real-time, and managers can decide to place orders at a proper time. One of many indicators that determine when to place an order is called a Reorder Point (ROP). Managers should place an order at the point of time that the inventory level decreases to the ROP level. Moreover, another decision variable of this inventory model which should be taken into account is the lot size. Lot size is a quantity of the orders. This inventory model, which has ROP and lot size as decision variables, is called lot size-reorder point system or (s,Q) policy. There are three assumptions of (s,Q) policy: 1) continuous inventory levelreview, and 2) randomly and stationary demand: expected value of demand can be calculated, and 3) lead time is stationary and positive.

Lead time means a period of time between the order that is placed and the received products. That means the products do not receive immediately after placing the order. Therefore, an enough stock for the products that have to be used for serving the demand during the lead time period is the solution for this problem. The enough stock, which is used to serve the demand during the lead time period, is called the safety stock. Moreover, because most of the customer demands are stochastic, the safety stock should be increased to serve this uncertainty. Chopra[4] researched about the relationships between the lead time and the safety stock. The result in this paper concluded that the more number of lead time, even the lead time is deterministic; the greater the variation of the demand during the lead time period. This makes the safety stock and the ROP be higher to provide enough goods for the uncertainty of the demand.Furthermore, if the lead time is stochastic, the assumption that the lead time is stationary is no longer valid; therefore, the initial (s.Q) policy cannot be utilized. However, [2] and[5] stated that the inventory system, of which the lead time is stochastic, can still be managed. This can be achieved by taking the average and standard deviation of the lead time into account, and adapting the original calculation. Nevertheless, this system still

requires the lead time to be normally distributed, and most of the time, the distribution of the lead time is not normal. Therefore, this paper presents a way of determining the appropriate ROP that meets the service level requirement when the demand and lead time are stochastic with any distributions.

II. PROBLEM DESCRIPTION

As mentioned before, the inventory management based on natures of each inventory systems. Therefore, this paper studied and researched from raw data of the company in case studied. One of the important properties of the data from this company are stochastic demand and lead time. Both demand and lead time are expressed as each parameter of various distributions, i.e. normal, lognormal, exponential, and uniform distribution. Furthermore, the customers do not place an order every day; and more than one order per day for each product. Due to the uncertainty of demands and lead times, and the complex of orders, the inventory management in this paper is greater complicate than many simple system. The other simple systems trade-off between holding cost and ordering cost to manage an inventory. However, in this case the ordering cost is combined with a product price or assumed to be zero. Accordingly, in this paper uses the search algorithm to deal with the problems. The objective of this paper is to find appropriate ROP level that satisfies the target service level of each product. The appropriate ROP level is the minimum ROP level of which target service level; because the ordering cost in this paper is assumed to be zero, the order should be place more often, the number of ordering quantity is minimum. The target service level is an aim performance of the company in case studied. The service level of each product is calculated from the number of the orders that can deliver to the customer divided by the total order.

III. SOLUTION METHEDOLOGY

A. SEARCH METHOD

This section describes the method that is used to search for suitable ROP in any target service level. The simulation model is used to find service level at various values of ROP. Each service level is an expected service level resulted from five replicates simulation at each value of ROP. The result of this search is a minimum ROP level of which the service level meets target.There are many search algorithms that use to find solutions in inventory management. Some algorithm is simple, but some of them are complex. Nevertheless, Jans[6] explored that traditional methods beat meat-heuristics methods in standard inventory problems i.e. not a multi-echelons. Moreover, search algorithms have many objectives. The objective of this paper is the minimum ROP level that makes service level met the target service level. The searching algorithm that is presented in this paper is shown in the diagram in Fig.**1**.

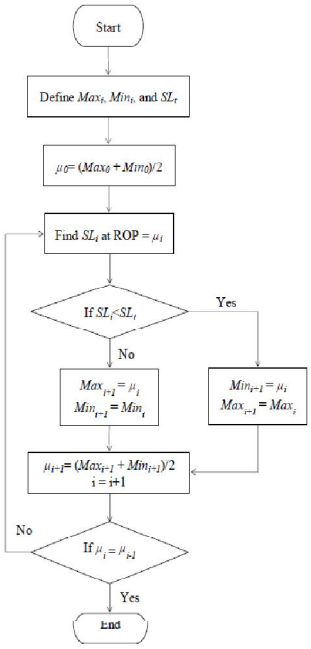


Fig.1. Search Recursive Procedure

Table I: The result example of a product

Distribution				ZAA072000242		%Customer			int. invent.	100		
Day	Expo.	0.06923				%count 1	0.8947368		MOQ	10		
Lead time	Expo.	-1.219	19.969			%count 2	0.1052632		Min ROP	1		
Demand	Lognormal	0.77062	3.4268			%count 3	0		Max ROP	500		
Service level	ROP	Average	rep1	rep2	rep3	rep4	rep5					
	73	0.7927495	0.79541109	0.7732057	0.800389	0.7928364	0.8019048		Simulate			
	ROP	Average	rep1	rep2	rep3	rep4	rep5		Actual Service level	0.429		
1	250	0.99017366	0.99424736	0.9893411	0.990375	0.98751201	0.9893925		Target Service level	0.8		
2	125	0.92994051	0.91503906	0.9258189	0.941571	0.92773438	0.9395393		ROP=	73		
3	63	0.74922899	0.76605944	0.7578947	0.757282	0.74857685	0.7163324					
4	94	0.86559213	0.85522531	0.8821811	0.863768	0.86923077	0.8575553		avg. inventory level	127.72675		
5	78	0.81085292	0.80813953	0.8059273	0.810395	0.81347656	0.8163265		rep1	rep2	rep3	rep4	rep5
6	70	0.78828675	0.78015564	0.7813102	0.77842	0.80250482	0.7990431		118.0255	131.95011	104.66985	113.3652	120.7813
7	74	0.80057259	0.80057526	0.7968902	0.795215	0.81341108	0.7967711						
8	72	0.78862309	0.81088825	0.7578051	0.803232	0.78181818	0.789372						
9	73	0.79549008	0.80365736	0.7891683	0.804971	0.78422053	0.7954329						
10	73	0.79507572	0.77347531	0.7913462	0.795299	0.79083095	0.8244275						
11	73	0.79124477	0.79107312	0.8027211	0.782359	0.78273521	0.7973359						

1) Notations

i: the running number of each iterations
Max_i: the maximum value of the ROP range for loop i
Min_i: the minimum value of the ROP range for loop i
SL_i: expected service level of loop i
SL_t: target service level
μ_i: average of ROP range in loop i

2) Algorithm

The search method in this paper started by divided search space in to two equal sections. The concept of this search is selected a search section the efficient and cut out another half. As mentioned before,the objective of this paper is to find minimum ROP of which service level meets the target. Accordingly, the way to select a proper section is compared the mean ROP with the target service level. After selecting the proper section of a search space, the next iteration begins. The next iteration is finding a new ROP from the section that make you move forward to the solution. Next, repeat the divided search space until one ROP value left. This ROP is a minimum ROP which is the solution of the search method.

The process of the search method used in this paper presented in

Table **I**shows an ROP search example result for a product. It can be seen that each ROP is used five replicates to calculate the expected service level corresponding to that ROP. In this example, the ROP search range is 500. As expected, the number of the search iterations is approximately 9. The excess of the search iteration number is caused by the last condition that requires the current ROP equals to the previous ROP. Therefore, the simulation has to test the search iterations is calculated by counting the number of repetitions of dividing the search range by 2. In other words, it can also be obtained by using Eq. 1.

$$n = log\ (max\text{-}min)/log2 \qquad (1)$$

where$n$ is the approximate number of the search iterations. It should be noted that the calculated n should be rounded up.

For example, the minimum and maximum ROP are [1,500] that means the range of search is 500. Therefore, the number of search iterations is 9 (2^9=512). Moreover, the increase in the search range does not significantly increase the number of search iterations. For instance, by increasing the maximum value of the search range from 500 to 1000, double the maximum value, the number of iterations becomes 10 (2^{10}=1024), which is more than the original number of the search iterations by only 1

IV. SIMULATION

This section explains the simulation that is used Table II shows an example of simulation table of one

Fig.**1**. Firstly, defining initial parameters of searching which are searching range and target service level (SL_t). The searching ranges are defined with maximum value (Max_0), and minimum value (Min_0). The range of search can be any value, suggests start at 1 and finish at enough for demand value, the large value is better.Secondly, the mean value (μ_i) is found from the ROP range. Thirdly, the simulation program is used to find the expected value of service level at each ROP. The simulation in this paper uses five replicates for each ROP. Then, the resulted service level is compared with target service level. If the resulted service level does not reach target service level, the upper search space will be selected; else the lower search space will be selected. The upper search space selection means the value of the minimum parameter (Min_{i+1}) equals to the value of μ_i, and the value of the maximum parameter(Max_{i+1}) stays the same as previous, equals to the value ofMax_i.Then, the next ROP (μ_{i+1}) value is calculated from Max_{i+1} and Min_{i+1}. Finally, repeat to find the next ROP value, and continue searching until the ROP value equals to the previous ROP value. The ROP value which is the solution of the search method is a minimum ROP value (ROP*) of which the service level is more than or equal the target service level.

more iteration even though it has already found the solution. Another possibility, but unusual, is that the simulation contains uncertainty; that means, the same ROP can give different expected service level in a different test; thus the number of search may be fluctuated due to this inconsistency.

One of many advantages of the search algorithm presented in this paper is that the number of the search iterations can be predicted. The number of product which is used to monitor the service level at a specific set of parameters. There are many important variables and parameters in this simulation, they are:.

1. ROP

This is the decision variable which is the solution of this simulation. It can be expected that the service level strongly depends on the ROP. With higher ROP, the service level tends to be higher too.

2. MOQ

The minimum order quantity (MOQ) is the minimum quantity that cannot be changed and is determined by the supplier. This factor also affects the inventory level, and the service level.

3) Lead time distribution

As stated previously, the original inventory

Table II. The value can be 1, 2, and 3 according to describe the behavior of the inventory system presented in this paper. Simulation is a method that deals with uncertainties of many situations, and is used to study behavior of many systems. Simulation can help a company to find an appropriate answer for a situation. One of many

advantages of the simulation method is that it does not affect the real system. The system can be monitored, examined its problem, altered its system, and improved without having to disturb the original system. As a result, the cost of implementing the simulation method is low. In this case, the demand and lead time are stochastic; therefore, the simulation
Table II
system requires the lead time to be normally distributed. In order to simulate the inventory system of which the lead time is not normally distributed, the lead time distribution and its parameters have to be taken into account. The difference in lead time affects the result.

4) Day distribution

Since the customers do not order every day, the day in which the order takes place must be identified. The method is suitable for exploring and improving the inventory system. The
inventory level and service level according to the ROP, and behavior of demand and lead time can be monitored through this simulation. It should be noted that the simulation in this paper is done by Microsoft Excel.
interarrival between days data are collected, tested their distributions, and used in this simulation.

5) Orders per day

This factor represents the number of orders per day. This factor is defined by probability percentage. For example, the probability that the product has one customer order per day, the value of the parameter is 1, is 86% in the to the studied products.

Table II: Simulation table example

ROP	9		Initial	Inventory			is in?			Cust. Order			Inventory level			Shortage		
Day	order?	LT	invent	level	Cust. No.		1	2	3	1	2	3	1	2	3	1	2	3
	MOQ	0.04959			%count 1	0.86				1.34								
0.10487	55	0			%count 2	0.14				2.61						Shortage	230	
				avg=	%count 3	0										all	1079	
Expo.		Expo.		152.38						Log						serv. level	0.79	
					sum	1128												
1	0	x	100	100		2	1	1	0	9	84	0	91	7	7	91	7	0
22	55	29	7	7		2	1	1	0	13	20	0	-6	-26	-26	-6	-26	0
28	0	x	-26	29		1	1	0	0	6	0	0	23	23	23	23	0	0
28	0	x	23	78		1	1	0	0	14	0	0	64	64	64	64	0	0
32	0	x	64	119		1	1	0	0	7	0	0	112	112	112	112	0	0
42	0	x	112	112		1	1	0	0	7	0	0	105	105	105	105	0	0
43	0	x	105	105		2	1	1	0	2	20	0	103	83	83	103	83	0

6) Order quantity distribution

After the proportion of the customer number has been identified, the quantity of each order must be generated. This factor determines the quantity of each order according to its distribution.

7) Service level

The service level is the result from the simulation. It is affected by all the factors listed above. The service level is calculated by dividing the number of the orders that can deliver to the customer by the total order. It is the performance indicator for this inventory management system.

The simulation concept in this paper can be described in two sections.

1) Notation

i: the number of day, $i = 1, 2, 3, \ldots, n$
j: the number of order, $j = 1, 2, 3$
DR: the day on which orders arrive
R_i: the amount of products received in day i
O_i: the amount of products ordered in day i
I_i: the inventory level of day i
D_{ij}: the amount of customer demand of order j in day i
Y_{ij}: 1; if the there are enough product to serve customer demand j in day i
0; otherwise
ROP: reorder point
MOQ: minimum order quantity
LT: Lead time
SL: service level
Total: total orders

V. SIMULATION PROCESS

The simulation model used in this paper begins with receiving important parameters. The parameters are ROP level, MOQ level, lead time distribution, day distribution, number of orders per day, and order quantity per order distribution. It should be noticed that ROP level is a decision variable of this simulation. After receiving all important data, each row is simulated for one day. The inventory level of the present day is the inventory on the previous day plus the amount of products received; and decreases by the demand. The inventory level equation becomes:

$$I_{i-1} + R_i = I_i + \sum D_{ij} \qquad (2)$$

When the inventory level reduces to ROP level, the company should place an order. The quantity of the

ordering product is a multiple of minimum order quantity (MOQ). We have:

$$O_i - kMOQ \quad (3)$$

where k is an integer.
However, the company is not receives the products suddenly after ordering. They have to wait for the lead time period. The day that the company obtains the products can be expressed as

$$DR = i + LT \quad (4)$$

$$R_{i+LT} = O_i \quad (5)$$

VI. RESULTS

As stated before, the objective of this paper is to find the minimum ROP value that meets the target service level. The result of this paper is an ROP value that can be applied to a real situation. The data which are used in this paper are raw data from a studied company. This study focused on many products of the company, but in this paper, a result example of five products was presented. The example of the result is presented in Table III.

Table III: ROP based on parameters result example

Product	Demand (unit)	*LT* (day)	MOQ (unit)	SL_t	ROP* (unit)
1	1.9	9.7	10	0.80	31
2	3.1	6.9	12	0.80	32
3	7.1	4.6	12	0.80	44
4	4.7	6.7	10	0.80	46
5	4.0	6.8	12	0.80	45

The minimum ROP values (ROP*) for each set of parameters of the five products are shown in Table III. The minimum ROP (ROP*) depends heavily on the factors presented in the table. The parameters included in the table are the demand per day, average lead time, MOQ, and the target service level. Demand and lead time have their own distributions; both are stochastic. MOQ is determined by the supplier and cannot be altered. The target service level is decided by the company and it can be changed accordingly to appropriate situations.
It can be observed that the demand and lead time have a positive impact on the optimal ROP. That means as these two factors increase, the optimal ROP increase. In the result shown above, one target service level is used as this is the requirement of the company. However, the target service level should also have a positive influence on the optimal ROP. If the target service level were higher, the optimal ROP value would be higher too. From the results above, it can also be compared further with the actual parameters and performance as shown in Table IV.

From the result of the searching, the result and the actual values are compared as revealed in Table IVTable IV: Result comparison with actual values. The comparison in this table are the difference between the minimum ROP (ROP*) and the actual reorder point (ROP); and the difference between the target service level (SL_t) and the actual service level (*SL*) of each product.
Table IV: Result comparison with actual values

Product	ROP* (unit)	ROP (unit)	% diff.	SL_t	*SL*	diff.
1	31	40	0.23	0.8	0.83	0.03
2	32	48	0.33	0.8	0.83	0.03
3	44	84	0.48	0.8	0.82	0.02
4	46	100	0.54	0.8	0.82	0.02
5	45	60	0.25	0.8	0.80	0

It can be instantly noticed that the optimal ROP values for all the products that are included in the results are improved. If the company implemented this simulation method, the ROP values would be reduced; as a result, the company would not need to stock as much as before. Moreover, the difference between the target service level (SL_t) and the service level (*SL*) for each product is negligible. That means the stock can be decreased while maintaining the overall satisfaction of the customers.

CONCLUSION

An inventory management is a method for efficiently control inventory. It helps a company to decide when to order and how much to order. The proper inventory managing makes the company have appropriate stock level, reduce the total cost as a result, and keep high service level.
The objective of the research in this paper is to find the suitable ROP level that satisfies the target service level (SL_t) for each product. There were previous studies on this topic stated that the demand and lead time distributions had to be normal in order to use the original formula to determine the appropriate ROP. In this paper, with the aim of deciding the suitable ROP level (ROP*) of each product, the search method was used. The search algorithm concept presented was to reduce the search space by half in every loop until the search space becomes a value.According to the variability of products, the simulation method was used in order to examine the overall behavior of the demand and stock level according to important parameters. One of the benefits of this method is less number of search iterations is required for obtaining the answer. The results showed that the demand per day, average lead time, minimum order quantity (MOQ), and the target service level affected the suitable ROP level (ROP*). Moreover, it can be investigated that the suitable ROP level (ROP*) reduced from actual reorder point (ROP), but the service level remained approximately the same.

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