THE DEPENDENCE OF THE TRANSVERSE RELAXATION TIME ON THE ORBITAL AND SPIN QUANTUM NUMBERS

ASMA MOHAMMED ELBASHIR SAAD

Department of Physics college of Science and Humanities in AL- Kharj, Prince Sattam Bin Abdulaziz University, AL- Kharj 11942 Saudi Arabia,
Alzaiem Alazhri University Faculty of Education Physics Department, Sudan
E mail, Asmamohammed72@yahoo.com

Abstract- Internal quantum and local magnetic field relaxation time was derived using classical electromagnetism laws. This expression relates the magnetic flux density to the magnetic moment. The quantum expression for magnetic moment and flux density were used to find the excitation energy due to the internal magnetic field. This excitation energy expression together with the uncertainty principle were used to derive the transverse relaxation time. The expression obtained shows that the transverse relaxation time depends on the spin, orbital quantum numbers as well as the total quantum number and Landau split factor.

Keywords - Magnetic Moment, Magnetic Flux Density, Orbital Quantum Numbers, Spin Quantum Number, Total Quantum Number, Transverse Relaxation Time, Uncertainty Principle.

I. INTRODUCTION

Magnetic resonance imaging technique is now a day one of the most desirable techniques in soft tissues medical diagnosis. The control of image quality depends on relaxation times as well as on spin density [1, 2]. To find good image quality needs knowing mare parameters that affect the relaxation times and spin density. Two approaches were used to describe this phenomenon. The classical electromagnetic was used to describe and find a useful expressions for longitudinal and transverse relaxation times [3]. This approach measures the relaxation times on the basis of the time elapsed for the s x and y components of the magnetic moments to decay from maximum to minimum value. The second approach is based on quantum laws including spin concept and Zeeman effect beside uncertainty These laws are used to find longitudinal and trans verse relaxation times [4, 5]. The magnetic resonance imaging technique is now a day one of the most popular known techniques used for soft tissues since it gives good image quality. This is since the image quality depends on three parameters. These parameters are longitudinal, transverse, and spin density [6,7].Different attempts were made to improve the quality of the image. Most of these attempts concentrates on the protocols used and experimental techniques. In the work done by Lawrence [8] the study is concerned with improving image quality by having prior knowledge about the patient and relaxation by changing MRI working conditions and patient orientation. This improvement should rest on recent revolution in. optical Microscopic including diffraction phenomenon. This new trend is promising but it needs a long time for readjustment and orientation of the patient. The work done by Odey [9] studied the improve of image quality using Block equation beside some classical and quantum laws. These equations have been used for determining the factors that affect relaxation times, to see how to control these factors so as to get good image quality. This approach is promising, but needs more support by determining more factors that affect image quality. The work done by Asma [10] used also quantum laws to
find more new factors that affect the transverse relaxat ion time. This approach is based on the Schrodinger e quation in the energy space .Where she assume sine, c osine and exponential solution. This approach is base d on the Schrodinger equation in the energy space .Wh ere she assume sine, cosine and exponential solution. This solution shows that the transverse relaxation time on electric dipole moment, internal magnetic field as well as the spin quantum number.

II. PROCEDURE

The research methodology is based on deriving mathematically the transverse relaxation time using classical electromagnetism laws that relates the magnetic flux density to the current beside using quantum expression for the angular momentum that is related to the magnetic moment, beside the uncertainty principle

III. RESULTS AND DISCUSSION

Internal quantum magnetic moment
The magnetic moment is defined in terms of the current i and the area A, enclosed by it to be in the form:

$$P_m = iA \qquad (1)$$

But the current i generated by the electron of charge e, moving around a nucleus in circular orbit of radius r, frequency f, angular frequency ω, is given by

$$i = -ef = \frac{e\omega}{2\pi} \quad (2)$$

Where the area is given by

$$A = \pi r^2 \quad (3)$$

On the other hand the orbital angular momentum L is given by

$$L = mvr = m\omega r^2 \quad (4)$$

Inserting (2), (3) and (4) in (1) yields the orbital magnetic momentum in the form:

$$P_\circ = iA = \frac{-e\omega}{2\pi}(\pi r^2) = \frac{-e\omega r^2}{2}$$

$$P_\circ = \frac{-em\omega r^2}{2m} = \frac{-e}{2m}\underline{L} \quad (5)$$

L= orbital angular momentum
J= quantum angular momentum
S= spin

Similarly the spin magnetic moment P_s is related to the spin angular momentum according to the relation.

$$\underline{P}_S = \frac{-e}{m}\underline{s} \quad (6)$$

The total magnetic moment P_m resulting from spin and orbital motion is given by

$$\underline{P_m} = \frac{e\hbar}{2m} g_J J, \qquad \underline{P_m} = -\mu_B g_J \underline{J} \quad (7)$$

μ_B =Bohr magnetron

$\hbar$ = Planck Constant/2π

$$\underline{P}_m = -\mu_B \langle \underline{L} + 2\underline{S} \rangle \quad (8)$$

$$\mu_B = \frac{e\hbar}{2m}$$

The parameter g_J can simply be given by

$$g_J = \frac{\frac{3}{2}J^2 + (\frac{1}{2})\underline{S}^2 \frac{1}{2}\underline{L^2}}{\underline{J^2}} = \frac{3}{2} + \frac{s(s+1) - L(L+1)}{2J(J+1)}$$

$$\underline{J} = \hbar\sqrt{J(J+1)}$$

This relation can be found by setting

$$J = \underline{L}.\underline{S} \quad (9)$$

$$< \underline{L} + 2s >= \underline{L} + 2\underline{S} \quad (10)$$

to get

$$g_J \underline{J} \cdot \underline{J} = (\underline{L} + 2\underline{S}) \cdot (\underline{L} + \underline{S}) = L^2 + S^2 + 2\underline{L} \cdot \underline{S}$$

$$\therefore g_J J^2 = L^2 + 2S^2 + 3L \cdot S \text{ , } g_{J=\text{parameter}}$$

But $\underline{J}.\underline{J} = (\underline{L} + \underline{S}).(\underline{L} + \underline{S}) = L^2 + S^2 + 2\underline{L}.\underline{S}$

$$\therefore \underline{L}.\underline{S} = \frac{1}{2}J^2 - \frac{1}{2}L^2 - \frac{1}{2}S^2,$$

Hence

$$g_J = \frac{\frac{3}{2}\underline{J}^2 + \frac{1}{2}\underline{S}^2 - \frac{1}{2}\underline{L}^2}{J^2}$$

$$g_{Jj} J.J = (\underline{L} + 2\underline{s}).(\underline{L} + s)$$

$$\therefore g_J J^2 = L^2 + 2s^2 + 3L - s$$

$$\underline{j}.j = (\underline{L} + s).(\underline{L} + \underline{s}) = L^2 + s^2 + 2\underline{L}.\underline{S}$$

$$\therefore \underline{L}.\underline{s} = \frac{1}{2}J^2 - \frac{1}{2}L^2 - \frac{1}{2}s^2$$

But

Hence

$$g_J J = \frac{\frac{3}{2}J_J^2 + \frac{1}{2}\underline{s}^2 - \frac{1}{2}\underline{L}^2}{J^2} \quad (11)$$

If n atoms per unit volume align themselves along the x- axis thus the component of magnetic moment in x along the x- axis is given by [1, 2, 3, 4]

$$M_x = nP_m = -n\mu_B g_J \underline{J} \quad (12)$$

Where magnetic moment M_x change from 0 to max value during a time interval T_2

A. Internal local magnetic field relaxation time:

The electron revolving around a nucleus can produce a magnetic field of flux intensity. If the electron revalues with frequency f in circulator orbit of radius r, then according to Biot- Savart Low it produces the magnetic field of flux intensity [5]:

$$B_e = \frac{\mu_\circ fe}{2r} \quad (13)$$

$$i = fe \quad (14)$$

The magnetic moment produced by such an electron is given by

$$P_m = iA = i(\pi r^2) = \pi i r^2 \quad (15)$$

To get

$$g_J \underline{J} \cdot \underline{J} = (\underline{L} + 2\underline{S}) \cdot (\underline{L} + \underline{S}) = L^2 + S^2 + 2\underline{L} \cdot \underline{S}$$

$$\therefore g_J J^2 = L^2 + 2S^2 + 3L \cdot S$$

But $\underline{J}.\underline{J} = (\underline{L} + \underline{S}).(\underline{L} + \underline{S}) = L^2 + S^2 + 2\underline{L}.\underline{S}$

$$\therefore \underline{L}.\underline{S} = \frac{1}{2} J^2 - \frac{1}{2} L^2 - \frac{1}{2} S^2,$$

To get

$$g_J \underline{J} \cdot \underline{J} = (\underline{L} + 2\underline{S}) \cdot (\underline{L} + \underline{S}) = L^2 + S^2 + 2\underline{L} \cdot \underline{S}$$

$$A = \pi r^2 \qquad (16)$$

From z atoms with mean radius r permeability $\mu_\circ$, the internal atomic magnetic field equals

$$B_{as} = \frac{\mu_\circ z f e}{2r} \qquad (17)$$

Thus the internal atomic magnetic field generated by one electron is related to the magnetic moment through the relation [17]

$$B_e = \frac{\mu_\circ p_m}{2\pi r^3} \qquad (18)$$

Since the current for the whole atom is

$i = zfe.$

Hence the field of the atom is related to the magnetic moment also as follows:

$$B_a = \frac{\mu_\circ p_m}{2\pi r^3}$$

But the magnetic moment M is defined in terms of the number of dipoles N divided by the volume V to be

$$M = \frac{N P_m}{V} \qquad (19)$$

If the atomic radius is r, thus one atom exists in a volume v^a is given by:

$$\frac{N}{V} = \frac{1}{V_a} = \frac{1}{\frac{4}{3}\pi r^3} = \frac{3}{4\pi r^3}$$

This

$$M = \frac{3P_m}{4\pi r^3} \qquad (20)$$

Using (17) in (19) yields

$$M = \frac{3}{4\pi r^3} \frac{B}{\mu_\circ} (2\pi r^3) = \frac{3}{2} \frac{B}{\mu_\circ} \qquad (21)$$

The energy gained by the proton due to the effect of internal magnetic flux density B_i is given by:

$$V_m = m_L \left(\frac{eh}{2m}\right) B_i \qquad (22)$$

Depends on the orbital and spin as well as the total quantum number beside Landau split factor.

According to equation (21) the local magnetic flux density is given by

$$B_i = \frac{2}{3} \mu_0 M \qquad (23)$$

But according to equation (12) in section (A)

$$V_m = \frac{2}{3} m_L \left(\frac{eh}{2m}\right) \mu_0 \mu_B g n J \qquad (24)$$

Thus the excitation energy is given by

$$\Delta_E = V_m \qquad (25)$$

Hence the transverse relaxation time is given according to the uncertainty principle by

$$T_2 = \Delta t = \frac{h}{\Delta E} = \frac{h}{V_m} = \frac{4m}{3e\mu_0 \mu_B g n J m_L} \qquad (26)$$

Discussion

The classical electromagnetic theory has been used to relate the magnetic moment to the internal current of atoms in equation (1).The magnetic moment is then related to the angular momentum and then to the orbital quantum number in equation (5).The spin magnetic moment has been also related to the spin quantum number according to equation (6).The quantum total magnetic moment has been also related to the magnetic moment of one atom in equation (12).The magnetic flux density is also related to the atomic current in equation (13).This relation is then used to relate it to the magnetic moment of the atom in equation (18).These relations are used to find the magnetic moment in terms of the magnetic flux density in equation (21).The expression of the excitation energy gained by the proton due to the effect of the local magnetic field was found in equation (22).This expression together with the uncertainty principle has been used to find transverse relaxation time in equation (26).This relationship shows that the trans verse relaxation time is quantized and depends on the orbital, spin and total quantum number as well as Landau split factor .

IV. CONCLUSION

The classical electromagnetic laws together with the quantum uncertainty principle were used to find the excitation energy due to the effect of the internal magnetic field and the transverse relaxation time. This expression shows that the transverse relaxation time. The analysis shows that the transverse relaxation time

ACKNOWLEDGMENT

The authors would like to thank Prof. Dr.Mubark Dirar Abdallah Sudan University for Science and

Technology for his generous assistance and support during this work

REFERENCES

[1] R. P. Ferman, A. R. Hipps, Quantum mechanics and Path Integral, Mc Grows Hill, New York, 1965..
[2] P. T. Mathews, Introduction to Quantum Mechanics, 3rd Ed, London, 1974.
[3] W. V. Houston, Principle of Mathematical physics, London, 1948.
[4] A.V. Kavokin, M.R. Vladimirova, M.A. Kaliteevski, O. Lyngnes, J.D. Berger, H.M. Gibbs, G. Khitrova, Phys. Rev. B 56, 1087, 1997.
[5] A.V. Kavokin, M.R. Vladimirova, M.A. Kaliteevski, O. Lyngnes, J.D. Berger, H.M. Gibbs, G. Khitrova, Phys. Rev. B 56, 1087, 1997
[6] A. Beiser, concept of modern physics, 207, 1948.
[7] Kamal. Ali. Hamad, PhD, Longitudinal Relaxation time τ_1 for time Dependent Perturbation, 2002.
[8] Bydder GM, Steiner RE, Young IR et al, AJNR. Clinical NMR imaging of the brain, 139: 215-36, 1982.
[9] Lawrence L. Wald (2019), Ultimate MRI, J. Of Magnetic Resonance 306.
[10] Odey Samuel Onwu, Oluwaseu Michael Dada, Osmotayo, Bamidele, Awojoyobe (2014), Physics and mathematics of magnetic resonance imaging for nano medicine :an overview, World J. of translation medicine, 2014 April 12;12;3(1).
[11] Asma M. Elbashir Saad, M. Dirar, Sawsan Ahmed Elhouri (2019), Time depndent Schrodinger equation for two level systems to find transverse relaxation time, Global J. of Engineering Science and Researchs [Saad, 6(3):March 2019.
[12] Asma M. Elbashir Saad, M. Dirar, Sawsan Ahmed Elhouri (2019), Time depndent Schrodinger equation for two level systems to find transverse relaxation time, Global J. of Engineering Science and Researchs [Saad, 6(3):March 2019.

★★★