CONJUGATE FREE CONVECTION IN A SQUARE CAVITYFILLED WITH NANOFLUID AND HEATED FROM BELOW BYSPATIAL WALL TEMPERATURE

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Abstract- The problem of conjugate free convective in a square cavity filled with nanofluid and heated from below by spatial wall temperature is studied numerically using the finite difference method. Water-based nanofluid with copper nanoparticle are chosen for the investigation. Governing equations are solved over a wide range of nanoparticle volume fraction ($0 \le \phi \le 0.2$), wave number ($0 \le \lambda \le 4$) and thermal conductivity ratio ($0.44 \le K_r \le 6$).Theresults presented for values of the governing parameters in terms of streamlines, isotherms and average Nusselt number. It is found that the flow behavior and the heat distribution are clearly enhanced with the increment of the non-uniform heating.

Keywords- Conjugate, free convection, Nanofluid, Spatial temperature.

I. INTRODUCTION

Laminar free convective heat transfer is a significant phenomenon in engineering systems due to its wide applications in operations of solar collectors, heat exchangers, storage tanks, double pane windows, etc. Ostrach [1] introduced number of these applications. Many researchers have considered nanofluids as working mediums that enhance thermal conductivity with the presence of nanoparticles, thus making nanofluids appear as a decent candidate for heat removal devices in workable, fluid-based, thermal applications. A very first comprehensive workon natural convection in partially occupying nanofluids in cavities was done by Khanafer et al. [2]. The work of Jou and Tzeng [3] considered the natural convective heat transfer in nanofluids occupying a rectangular cavity. Nasrin and Parvin [4] used the finite element method to study the buoyancy-driven flow and heat transfer in a trapezoidal cavity filled with water-Cu nanofluid.

Conjugate natural convection heat transfer in cavities has received much attention because of its importance to many engineering systems, such as solar energy collectors, material processing, heat preservation of thermal transport circuits, building energy components, and the cooling of electrical units. House et al. [5] investigated the effect of a centred heat-conducting body on the natural convection heat transfer in a square cavity. Zhao et al. [6] studied the effect of a centred heat-conducting body on the conjugate natural convection heat transfer in a square cavity.

The problem of natural convection in closed cavities with boundary walls, including spatial wall temperatures, has been considered in a few studies. Saeid and Yaacob [7] considered numerically the natural convection in a square cavity filled with pure fluid in addition to a non-uniform hot-wall temperature and a uniform cold-wall temperature. However, conjugate free convective in a square cavity filled with nanofluid and heated from below by spatial wall temperature has not yet been undertaken. The aim of this study is to investigate the effect of spatial wall temperature variation on conjugate free convective in a square cavity filled with nanofluid.

II. MATHEMATICAL FORMULATION

Consider two-dimensional steady free convective in a square cavity with length L, as illustrated in Fig 1. The bottom horizontal wall of the cavity is heated to spatial temperature $\bar{T}_h$ and the top horizontal wall is maintained at a constant cold temperature T_c while the vertical walls are kept adiabatic. The boundaries of the cavity are assumed to be impermeable, the fluid within the cavity is a water–based nanofluids having Cu nanoparticles. The Boussinesq approximation is applicable, the nanofluid physical properties are constant except for the density. By considering these assumptions, the continuity, momentum and energy equations for the laminar free convective can be written as follows:

$$\frac{\partial u_{nf}}{\partial x}+\frac{\partial v_{nf}}{\partial y}=0, \tag{1}$$

$$u_{nf}\frac{\partial u_{nf}}{\partial x}+v_{nf}\frac{\partial u_{nf}}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p_{nf}}{\partial x}+\nu_{nf}\left(\frac{\partial^2 u_{nf}}{\partial x^2}+\frac{\partial^2 u_{nf}}{\partial y^2}\right), \tag{2}$$

$$u_{nf}\frac{\partial v_{nf}}{\partial x}+v_{f}\frac{\partial v_{nf}}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p_{nf}}{\partial x}+\nu_{nf}\left(\frac{\partial^2 v_{nf}}{\partial x^2}+\frac{\partial^2 v_{nf}}{\partial y^2}\right)+\beta_{nf}\,g\left(T_{nf}-T_c\right), \tag{3}$$

$$u_{nf}\frac{\partial T_{nf}}{\partial x}+v_{nf}\frac{\partial T_{nf}}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T_{nf}}{\partial x^2}+\frac{\partial^2 T_{nf}}{\partial y^2}\right), \qquad (4)$$

and the energy equation for the impermeable wall is:

$$\frac{\partial^2 T_w}{\partial x^2}+\frac{\partial^2 T_w}{\partial y^2}=0. \qquad (5)$$

where x and y are the Cartesian coordinates measured in the horizontal and vertical directions respectively, g is the acceleration due to gravity. We assumed that the temperature of the hot vertical wall has a sinusoidal variation almost a minimum value of $\bar{T}_h$ in the form

$$T_h(y)=\bar{T}_h+\varepsilon\left(\bar{T}_h-T_c\right)\left[1-\cos\left(\frac{2\pi\lambda x}{L}\right)\right]. \qquad (6)$$

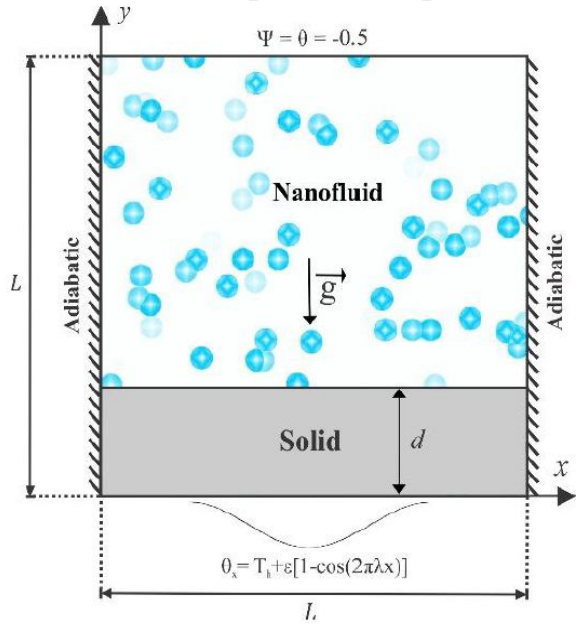


Fig.1. Physical model of convection in a square cavity together the coordinate

The effective thermal diffusivity α_{nf} of the nanofluid, the effective density ρ_{nf} of the nanofluid and the effective dynamic viscosity μ_{nf} of the nanofluid are defined as

$$\alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}},\ \rho_{nf}=(1-\phi)\rho_{bf}+\phi\rho_{sp},\quad \frac{\mu_{nf}}{\mu_{bf}}=\frac{1}{(1-\phi)^{2.5}}, \qquad (7)$$

Where, $\phi$is the solid volume fraction of nanoparticles, theheat capacitance of the nanofluids given is

$$(\rho Cp)_{nf}=(1-\phi)(\rho Cp)_{bf}+\phi(\rho Cp)_{sp}. \qquad (8)$$

The thermal expansion coefficient of the nanofluids can bedetermined by

$$\beta_{nf}=(1-\phi)(\beta)_{bf}+\phi\beta_{sp}, \qquad (9)$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_{bf}+\phi(\rho\beta)_{sp}. \qquad (10)$$

The thermal conductivity based on Maxwell-Garnett's (MG)model is given below:

$$\frac{k_{nf}}{k_{bf}}=\frac{k_{sp}+2k_{bf}-2\phi(k_{bf}-k_{sp})}{k_{sp}+2k_{bf}+\phi(k_{bf}-k_{sp})}. \qquad (11)$$

In terms of the stream function $\psi$and the vorticity ω, whichare defined in the usual way as:

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}, \qquad (12)$$

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \qquad (13)$$

Now we introduce the following non-dimensional variables:

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ \Omega=\frac{\omega L^2}{\alpha_{bf}},\Psi=\frac{\psi}{\alpha_{bf}},\quad \theta=\frac{T-T_c}{\bar{T}_h-T_c},P=\frac{L^2 p}{p\alpha^2}. \qquad (14)$$

This then yields the dimensionless governing equations are:

$$\frac{\partial^2\Psi_{nf}}{\partial X^2}+\frac{\partial^2\Psi_{nf}}{\partial Y^2}=-\Omega_{nf}, \qquad (15)$$

$$\frac{\partial\Psi_{nf}}{\partial Y}\frac{\partial\Omega_{nf}}{\partial X}-\frac{\partial\Psi_{nf}}{\partial X}\frac{\partial\Omega_{nf}}{\partial Y}=\frac{\mu_{nf}}{\mu_{bf}}\left[\frac{Pr}{(1-\phi)+\phi\frac{\rho_{sp}}{\rho_{bf}}}\right]\times\left(\frac{\partial^2\Omega_{nf}}{\partial X^2}+\frac{\partial^2\Omega_{nf}}{\partial Y^2}\right)+\frac{\beta_{nf}}{\beta_{bf}}RaPr\left(\frac{\partial\theta_{nf}}{\partial X}\right), \qquad (16)$$

$$\frac{\partial\Psi_{nf}}{\partial Y}\frac{\partial\theta_{nf}}{\partial X}-\frac{\partial\Psi_{nf}}{\partial X}\frac{\partial\theta_{nf}}{\partial Y}=\frac{(\rho Cp)_{bf}}{(\rho Cp)_{nf}}\frac{(k)_{nf}}{(k)_{bf}}\times\left(\frac{\partial^2\theta_{nf}}{\partial X^2}+\frac{\partial^2\theta_{nf}}{\partial Y^2}\right), \qquad (17)$$

$$\frac{\partial^2\theta_w}{\partial X^2}+\frac{\partial^2\theta_w}{\partial Y^2}=0. \qquad (18)$$

Where $Ra_{bf}=g\rho bf\beta_{bf}(\bar{T}_h-T_c)L^3/(\mu_{bf}\alpha_{bf})$ is the Rayleighnumber, and $Pr=\nu_{bf}/\alpha_{bf}$is the Prandtl number for the basefluid.

The dimensionless boundary conditions of (15)–(18) are:

$$\theta_w(X,0)=0.5+\varepsilon\left[1-\cos(2\pi\lambda X)\right];\quad \theta_{nf}(X,1)=-0.5 \qquad (19)$$

$$\partial\theta_w(0,Y)/\partial X=0\ ;\ \partial\theta_{nf}(0,Y)/\partial X=0 \qquad (20)$$

$$\partial\theta_w(1,Y)/\partial X=0\ ;\ \partial\theta_{nf}(1,Y)/\partial X=0 \qquad (21)$$

$$\theta_{nf}(X,D)=\theta_{nf}(X,D)\ ;\quad \partial\theta_{nf}(X,D)/\partial Y=K_r\partial\theta_w(X,D)/\partial Y \qquad (22)$$

Where $K_r=K_w/K_{bf}$is the thermal conductivity ratio. Thelocal Nusselt numbers along the hot and the cold horizontalwalls can be defined as:

$$Nu_h=hL\frac{k_w}{k_{bf}}=\left(-\frac{\partial\theta_w}{\partial Y}\right)_{Y=0}, \qquad (23)$$

$$Nu_c = hL\frac{k_{nf}}{k_{bf}} = \left(-\frac{\partial\theta_{nf}}{\partial Y}\right)_{Y=1}. \qquad (24)$$

Finally, the average Nusselt number can be defined as:

$$\overline{Nu} = \int_0^1 \left[-\left(\frac{k_{nf}}{k_{bf}}\right)\frac{\partial\theta}{\partial Y}\right] dX. \qquad (25)$$

III. RESULTS AND DISCUSSION

In this section, we present numerical results for thestreamlines and isotherms with various values of nanoparticlevolume fraction ($0 \le \phi \le 0.2$), wave number ($0 \le \lambda \le 4$), thermal conductivity ratio ($0.44 \le K_r \le 6$),and other parameters fix at Rayleigh number ($Ra = 10^5$),nondimensional temperature ($\varepsilon = 0.4$), ratio of wall thickness($D = 0.3$) and Prandtl number$Pr = 6.2$. The values ofthe average Nusselt number have been calculated for variousvalues of $\lambda$and ϕ.

Fig. 2 shows the effects of the wave number on thestreamlines (left) and isotherms (right) of waterCu at $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. Fig. 2 (a) presents theeffect of low wave number ($\lambda = 0.2$). On the streamlines andthe isotherm patterns. The pure fluid flow structure appearswith singular streamlines cell in the clockwise direction,while two streamlines cells occur for the nanofluid withinthe cavity. When the streamlines circulated as vortices inthe clockwise direction (negative sign of Ψ), the strength ofthe flow circulation is denoted as $\Psi_{\min}$.By the nanofluidaddition ($\phi = 0.05$), the thermal conductivity increases which leads to increase the strength of the flow circulation (see$\Psi_{\min}$values).The convection heat transfer enhanced by thenon-uniform heating; as such the isotherm patterns of the purefluid tend to take almost a horizontal lines, while the nanofluidisotherm patterns appear with almost a curved lines.

Increasing wave number up to 0.9 leads to increase the strength of theflow circulation. Significant change appears on the flow motionespecially for the pure fluid, the pure fluid streamline cell tendsto break into two cells, one in the clockwise direction closeto the right wall and one in anti-clockwise direction near tothe left wall. Clearly, the isotherm patterns are influenced bythe increase of λ value, next to the horizontal, the isothermpatterns arise with irregular lines. At a higher λ value ($\lambda = 3$),the streamlines are significantly affected, the streamlines forthe pure fluid are almost similar to that for the nanofluid. Thestrength of the flow circulation decreases due to the strongeffect of the non-uniform heating.The isotherm patterns nearto the bottom wall are transferred from irregular-shape to thespots-shape due to the high non-uniform heating, as displayedin Fig. 2 (c).

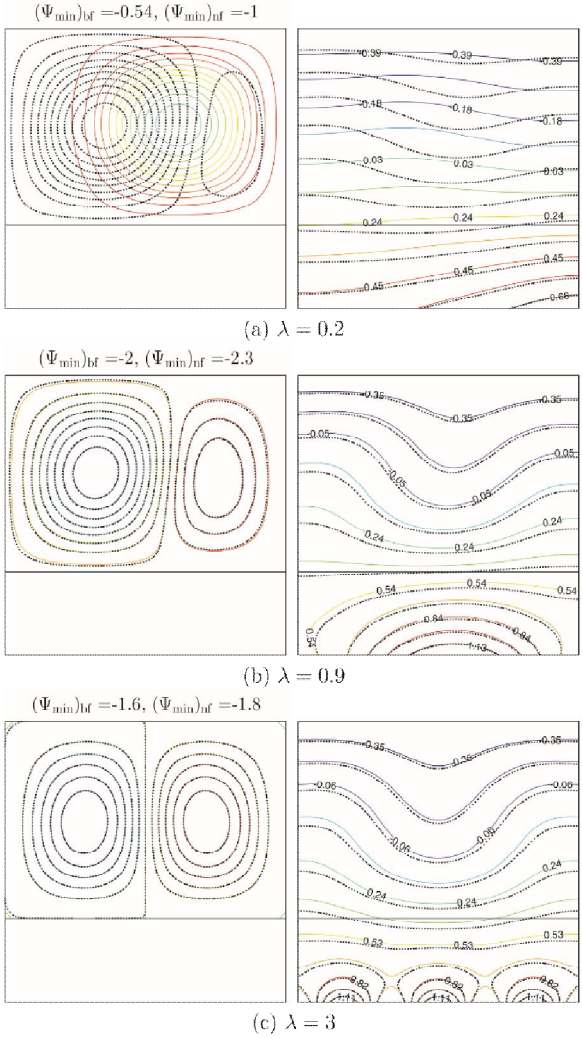
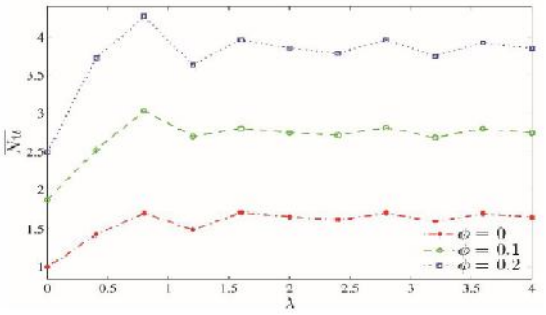
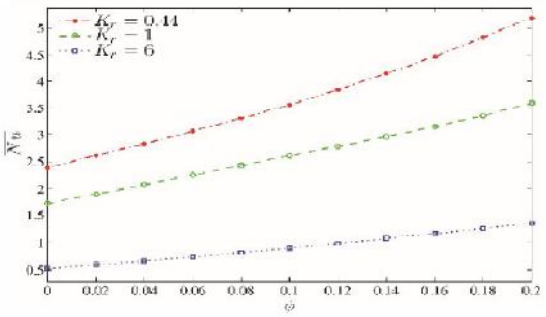


Fig.2. Streamlines (left) and isotherms (right) evolution by wave numberfor $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$.

Fig. 3 (a) illustrates the effect of nanoparticle volumefractions on the average Nusselt number with wave numberfor waterCu at $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. Weobserve that the convection heat transfer is enhanced by theraise of wave number, the significant increasing appears forlower λvalues ($\lambda \ge 2$). Increasing λvalues leads to decreaseand then increase the average Nusselt number which depictsthe different enhancement of the effects of the spatial heatingon the convection heat transfer from the uniform heating.Maximum values of average Nusselt number appear togetherwith higher value of nanoparticle volume fractions due tothe higher thermal conductivity of the nanoparticles. Fig. 3(b) presents the effect of thermal conductivity ratio on theaverage Nusselt numbers with nanoparticle volume fractionsat $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. Due to theincrease in convection by the addition of nanoparticles, theaverage Nusselt number increases. A significant enhancement in the convection heat transfer is obtained by lower thermalconductivity ratio$K_r = 0.44$, which leads to the maximumvalue of the average Nusselt number.



(a)



(b)

Fig.3. (a) variation of average Nusselt numbers interfaces with wave numberfor different nanoparticle volume fraction; (b) variation of average Nusselt numbers interfaces with nanoparticle volume fraction for different thermal conductivity ratio.

CONCLUSIONS

The present numerical study consider the effect of spatialwall temperature variation on conjugate free convection in asquare cavity filled with nanofluid and heated from below. TheFinite Difference Method (FDM) has been used numericallyto solve the dimensionless governing equations (15)-(18)subject to the boundary conditions (19)-(22). The streamlines areaffected by applying the non-uniform heating, the pure fluidflow structure appears with singular streamlines cell in theclockwise direction, while two streamlines cells occur forthe nanofluid within the cavity. A significant enhancementin the convection heat transfer is obtained by lower thermalconductivity ratio, which leads to the maximum value of theaverage Nusselt number.

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