A BLACK-SCHOLES APPROACH FOR THE PRICING OF ELECTRIC POWER OPTIONS IN TURKISH POWER MARKET

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Abstract—The electricity price is a stochastic decision variable which depends on the load, temperature, unit breakdowns, seasonal affects, and workdays etc. Wholesale electricity customers aim to minimize their cost through long term bilateral contracts. One way to deal with the problem is to get electricity options in Turkish derivative markets for future periods which need to be exercised before the expiration date. An option gives the right to consume 0.1 MWH of energy with the strike price for each hour of the month that is the option is exercised. We assume that a wholesale power costumer would like to have options from the market in an effort to get physical energy at the option expiration. The option is exercised only if the strike price is less than the average of the hourly day ahead power prices. This imposes a limit on the strike price that is then used in the Black-Scholes model. Using cyclic behavior of daily power prices and historical price data provided by the market authority, a simple model is developed to forecast the hourly power prices. Then the input is used in Black-Scholes model to estimate the value of the option. A numerical case study is developed and the results are presented.

Index Terms— Option Pricing, Black-Scholes Model, Electricity Price Forecasting, Power Trading

I. INTRODUCTION

In deregulated power markets the supply and demand of electricity determine the market prices. The supply and demand are two different and independent entities of the system. Each consumer with an oversized power consumption rate is free to select its own way of power supply. If the consumer has its own generation units, the power can be supplied by generating electricity from the units. The bilateral contract is common way that is used for power supply. Two companies can have a contract of which they define the price, capacity and delivery conditions. The risk is low and hence many large consumers prefer this option.

In deregulated markets it is also possible for a company to purchase low-cost power and sell it on the spot market in an effort to make profit. For such situations, the derivative markets are used to purchase and sell the options. An energy option is an agreement which is purchased from the derivative markets to exercise it according to predefined conditions. The option gives its owner the right of using the energy for a defined period of time and paying the corresponding strike price for the power. The European call options are common contracts that are used for such purposes. The option can be exercised on the maturity date if it is economic. If it is not exercised, the option expires at the end of the maturity period. The value of the option needs to be evaluated as it will be an important input for the decision making. The risk of loss and probability of profit should also have to be included to this process.

There is plenty of research in the literature that develops pricing methods on the stocks which mainly focusing on the well-known black and scholes model [1]. However, the pricing of energy derivatives require modified approaches as the electricity is a non-storable, special and real-time commodity. The researches on energy derivative pricing are limited. The author develops a monte carlo method to value a European call option in electricity purchase agreements in [2]. They first explain the process that large power consumers follow to purchase energy options and their motivation and then they develop a load consumption model to investigate the role of uncertainty in the option value. Authors explain the common methods used in pricing of energy derivatives in [3]. Then they use geometric Brownian motion and monte carlo simulation to estimate the forward prices and estimate the value of an option using black-scholes model.

In [4], authors explains basic concepts and formulations for pricing electricity derivatives in competitive markets. Authors investigate the mean reverting models for energy commodities especially for natural gas and crude oil in [5]. Numerical analyses are developed using binomial trees, finite difference method and monte carlo method. In [6], author presents the stochastic models for energy prices that also include the jumps and spikes. Author focuses on the pricing of swing options in energy markets in [7]. He presents pricing methods for call options, forwards, and options with payoff, hedging and risk. In [8], the author develop methods to estimate the value of the forward options. He use data from the Nord Pool and shows that black scholes method can be used for option pricing.

The price is volatile in developing markets like Turkey as the availability and price of resources show significant changes over time. The derivative market in Turkey has started its operations in 2005. The operations for electricity options started in November

2011 which is relatively new. On the other hand, the deregulation of Turkish power market started in 2003 and the restructuring process continues. The process of privatization of plants that belong to the state has not been completed yet. However, the independent system operator maintains an open market medium for supply side and demand side. The large power supplier, wholesale power consumer, power distribution companies and transmission organization are some of the natural member of the system.

The energy options that are provided in the derivative market help the market participants to price the electricity of future periods. The option seller (writer) sells an agreement to the option buyer in which a fixed amount of power is guaranteed for a so-called strike price that will be used during the defined time interval. An option seller and buyer have to pay a fixed amount of money as a deposit. The changes on asset prices are withdrawn from one account and transferred to other account. If the remaining balance drops below a sustainable level, the account holder is called for extra payment to continue the agreement. The option allows the buyer to use *O* MWh of energy for each hour of the month. Financial transactions are allowed and physical usage is not required.

A power consumer can purchase the power from the spot market in which he accepts all high price risks. On the other hand, it is possible to forecast the forward electricity prices considering the supply and demand of the participants. It is natural that the forecasted prices have deviations from the actual prices but the forecasted prices provide a confidence interval and base ground for decision maker. In this research, we develop a price forecasting method and then using the forecasted prices we show that it is possible to find a value for the energy option.

II. PROBLEM FORMULATION

As mentioned earlier, the energy purchase agreement established at time *t* is an option that requires buyer and seller to commit a required price. The option gives the right to use a *O* MWh of energy for each hour of the corresponding month with the cost of the strike price *X*. The appendix provides a list of notations. Assuming an *M*-days month and h-hours days, the total cost of exercising the option becomes

$$C_{option} = OMhX \quad (1)$$

As an example let's consider a month with 30 days and 24 hours in each day. The cost of option for the month $C_{option} = 720OX$. Assume that the energy is purchased from the spot market if the option is not exercised. Then total cost of the energy supply is

$$C_{spot} = O\sum_{t=1}^{Mh} E(P_t) \quad (2)$$

The option is a European call option and the option is exercised if it is in the money. Hence the payout of the option is:

$$\pi = \max(O\sum_{t=1}^{Mh} E(P_t) - OMhX, 0) \quad (3)$$

The payout equation imposes a limit on the exercise price of the option in such a way that the exercise price is limited with the average of market prices. It can be expressed as follows:

$$O\sum_{t=1}^{Mh} E(P_t) - OMhX \geq 0 \quad (4)$$

and hence using basic calculus we get:

$$X \leq \frac{\sum_{t=1}^{Mh} E(P_t)}{Mh} = E(\bar{P}_t) = \bar{P} \quad (5)$$

If the option is purchased at the current time then the value of the option according to Black-Scholes equation:

$$V = S_0 N(d_1) - e^{-rT} XN(d_2) \quad (6)$$

If the value of the option is attractive, then the option can be used to get power for the month it is exercised. Often times the option buyer waits until t_1 to purchase the option. Then the value of the option at time *t* is:

$$V_t = S_t e^{-r(T-t_1)} N(d_1) - e^{-r(T-t_1)} XN(d_2) \quad (7)$$

To find an acceptable option value, one can have an upper bound for an option value by plugging in (5) to (7) and hence get:

$$V_t' = S_t e^{-r(T-t_1)} N(d_1) - e^{-r(T-t_1)} \bar{P} N(d_2) \leq V_t \quad (8)$$

and subject to:

$$d_1 = \frac{\ln(\frac{S_{t_1}}{\bar{P}}) + 0.5\sigma^2 (T - t_1)}{\sigma\sqrt{T - t_1}} \quad \text{and}$$

$$d_2 = d_1 - \sigma\sqrt{T - t_1} \quad (9)$$

An option buyer evaluates the value of the offer and if the value is larger than V_t', there is a risk that loss can occur. Note that a proper electricity price forecasting method should be employed to estimate the average of day-ahead hourly prices.

The power price is a stochastic decision variable which depends on the load, temperature, unit breakdowns, workdays etc. The hourly price in a day has a cyclic behavior with random deviations which need to be estimated. Time series methods, stochastic price generation models, arima models are some methods used in the literature to forecast the power prices[9]. In these models, historical data is used as a reference to estimate the future prices. Historical load data, temperature and hourly prices are some inputs that are commonly used.

Turkish power market is dominated by bilateral contracts as the deregulation and privatization are still

incomplete. Government based resources still have the important share. Besides the load and price data along with the generation information are not available for market participants. The competition is still not widely effective on the prices as the resources are limited. Therefore, we use a historical price based regression method to estimate the hourly prices.

Suppose that HP_t is the historical power price at time t for the past year. Then we assume that it is possible to have an equation:

$$P_t = \alpha + \beta HP_t + \varepsilon \quad (10)$$

with further assuming ε as the noise with $N(0,\sigma_p)$. The accurate calculation of α *and* β leads to an approximated estimate of real power price for hour t. Note that α *and* β can be estimated only if there is real data and it is in usable format.

A. Solution approach

We consider a company that purchases energy options from the Turkish derivative market in an effort to consume the low-cost power or sale this power to market for profit. The main objective of the problem is to find a decision framework to value the option. Black-scholes model is selected as a tool to evaluate the value of the option.

In order to have an upper bound on the option value, we developed a price forecasting method based on the historical price data. We first use the latest available data that is closer to the option exercise date to calculate the parameters α *and* β . Then use the parameters to calculate the hourly prices of the month of interest. The pseudo code of the solution approach is given in Fig. 1.

1. START
2. Set $t=t_1$, maturity time =T, interest = r
3. Get historical energy option data for last 4 months
4. Estimate σ
5. Get historical power price data for each hour of last
6. year
7. Estimate α *and* β parameters for regression equation
8. Estimate σ_p for power prices
9. Generate hourly price scenarios for month of interest
10. based on α *and* β
11. Calculate $\bar{P}$ based on the prices
12. Calculate *d1*, *d2*, *N(d1)* and *N(d2)*
13. Estimate $V_t^{'}$ that will be an upper bound for an option value
14. End

Fig.1. Pseudo code of the multi-period optimization and simulation

Note the parameters that affect the electricity price have the same effect on the both option month and the previous months. Hence, we assume that it is not a strong assumption to use the same regression formulae to calculate hourly electricity prices of exercise month.

Another important parameter to calculate is the volatility of the option price through time. To estimate the volatility of the option price, we use the method employed by Blanco and Soronow, (2001). They use logarithmic price changes to calculate the volatility of the option. The natural log of price changes for each period are assumed as continuous returns and their standard deviation is computed. Then the volatility is estimated based on the number of periods that the price data is available.

Once the average power price is found and volatility of option price is estimated, *d1* and *d2* which will be used to in cumulative normal distribution are computed. The black and scholes model provides an option value for the option buyer. This value then can be used to make a decision for power supply.

III. NUMERICAL EXAMPLE

We assume that a wholesale power supplier needs to have 10 MWh of power for the April, 2012. The value of O, amount of power per option is determined as 0.1 MWH in Turkish Power market. They can procure the power from the market with the cost of spot market price which is open to risk of higher prices. They would like to purchase option contracts from the derivative market at the beginning of March 2012. A feasibility analysis is needed before the actual decision is made.

Since each option provides 0.1 MWh of power for each hour of the April (which is 30x24 =720 hours), they need to purchase 100 options to satisfy the demand. Each option requires a payment of 1200 TL to initiate the contract and in total a cost of 120000 TL to the company. The analysis presented below is only for one option.

It is assumed that t=0, T =1 month (1/12), r = 10% compounded continuously. The derivate market in Turkey started to do energy sales operations on November 2011. Therefore, no annual data is available for the option prices. However, the option prices of past 5 weeks are collected as shown in Table I.

Table I. Option prices in Turkish derivative market

Option Price(TL)	Return
110	
144	0.2693
147.3	0.022
134.8	-0.0887
129.5	-0.0401
Standard deviation	0.159039
Volatility (σ)	1.146655

The returns of the options are found using logarithmic scale of the change between two consecutive periods. Then the standard deviation of the returns is used to find the annual volatility of the option.

The next step is to estimate the parameters that will be used to estimate the hourly electricity prices. The latest available data that can be used is the prices of February 2011 and 2012. Using the regression analysis, the parameters are estimated as $\alpha = 0.63827$ and $\beta = 97.12$. To show the relationship between actual and forecasted prices, we plot the forecasted and actual prices of February 2012 in Figure 2.

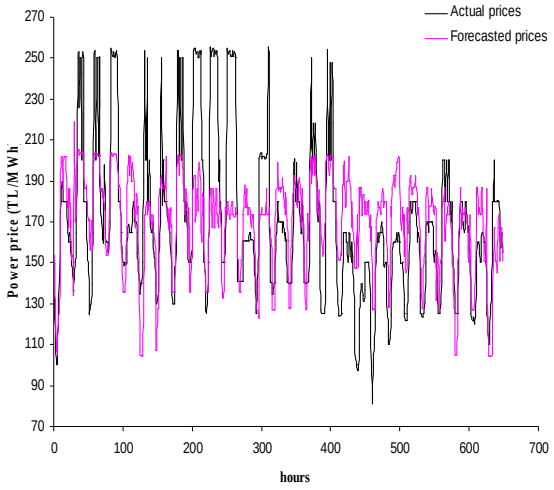


Fig. 2. Actual and forecasted power prices of February 2012

Using the relationship Pt=97.12+0.63827HP_t, we estimate the power prices for April 2012. Fig. 3 shows the estimated prices for April. The average of hourly prices plays an important role in Black-scholes model. $\bar{P}$ is estimated as 152.11 TL/MWh and used as an input for the model.

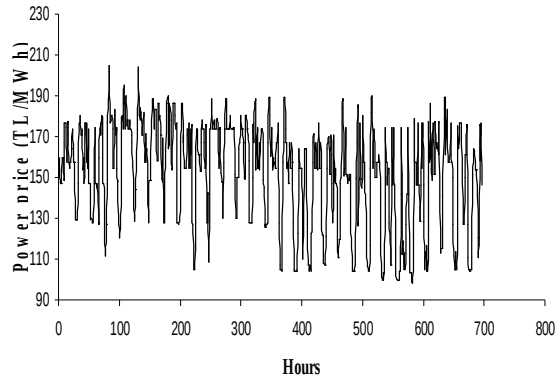


Fig. 3. Estimated hourly day-ahead power prices for April 2012

The parameters for the cumulative normal distribution are estimated as *N(0.11)= 0.54* and *N(-0.22)= 0.41*. The Black scholes model is solved and the upper bound on the option value $V_t^{'}$ is estimated as 1,004 TL. This value is an upper bound for the value of the option. In order words, if the calculated option value of is less than 1,004 TL, then the option can be purchased. We perform a sensitivity analysis to show the effect of the average power prices on the option value. Fig. 4 shows the results.

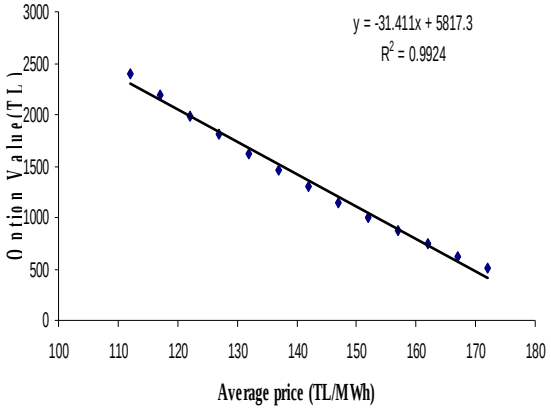


Fig. 4. Actual and forecasted power prices of February 2012

An interval of 5 TL/MWh is selected to determine the average prices that will be used in the sensitivity analysis. The relationship between average power price and the option value is close to linear. Note that as the estimated average power price decreases, the value of option increases. It is worth mentioning that the figure is only valid for the exercise month for given parameters and should be estimated for any new parameter set.

CONCLUSION

The value of option plays an important role in derivative markets no matter the option is purchased for physical power supply or for financial transaction purposes. The estimated forward prices are commonly used to make a decision in the derivative markets. In this research, we have developed a model that combines estimated power prices with the black-scholes model to provide an option value for the option buyers. A sensitivity analysis is provided that shows the value of the option for different average prices. The model can be extended by including stochastic features of the option and power prices. The price forecasting method can further be improved if the load data is employed. However, the model with its current form can be used in Turkish derivative market if the price forecasting method is tested and verified with real data.

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Appendix

Notation

S_t = Option price at time t
X = Strike price of the option
V = Value of the option
T = Maturity time of the option
r = Risk free interest rate discounted continuously
σ = volatility of the future prices
σ_p= Volatility of the option prices
d_1, d_2= Black-scholes model parameters
$N(.)$ = Cumulative standard normal distribution
$N(d_1)$=Probability of change in the option
$N(d_2)$ = The probability that the asset price will be above the exercise price
P_t = The power price at hour t (TL/MWH)
HP_t = Historical power price at hour t (TL/MWH)
h = number of hours in a day
M = Number of days in a month
t = hour index
$E(P_t)$ = Expected power price at hour t
π = Payout of the option
$\bar{P}$ = Average of hourly day-ahead power prices
α and β = Regression parameters to estimate the forward power prices
ε = Deviations from the actual power prices
$V_t^{'}$ = Bound on the value of the option
C_{option}= The cost of energy with option
C_{spot} = The cost of energy in spot market
O = Amount of power energy in each option (MWh)

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