ARTIFICIAL BEE COLONY BASED OPTIMUM DESIGN OF RETICULATED DOMES TO LRFD-AISC

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Abstract— This study focuses on developing a structural optimization algorithm based on artificial bee colony method for solving structural design problems with discrete variables under external loading. Artificial Bee Colony algorithm is one of the swarm intelligent optimization algorithms which is inspired by honey bee foraging. The structural optimization algorithm developed minimizes the weight of a steel structure under the provisions of LRFD-AISC. A geometrically nonlinear reticulated lamella dome is designed by the algorithm developed as an example. It is shown that the performance of the proposed algorithm is quite efficient and the artificial bee colony algorithm provides another alternative for finding the optimal design of structural optimization problems.

Index Terms— Metaheuristic Search Techniques, Artificial Bee Colony, Harmony Search Optimization, Reticulated Lamella Domes, Steel Structures

I. INTRODUCTION

The methods developed for solving optimization problems vary greatly. The methods which deals with linear objective function and constrains belong to linear programming. If the objective function is nonlinear and/or constraints are nonlinear then one has to use one of the methods of nonlinear programming. These techniques require the gradient calculations of the objective function and design constraints. However, there are other methods they use direct search techniques. In recent years there is a new trend called metaheuristic algorithms which mimics natural phenomena into a numerical optimization techniques [1]. These algorithms do not require computation of gradients of objective function and constraints and they handle them as they are regardless of their mathematical complexity. Furthermore they equally work efficiently for optimization problems with continuous and/or discrete variables. It is this last feature of these techniques which made them quite popular in engineering optimization. This is due to the fact that most engineering optimization problems require consideration of discrete design variables. Among these evolutionary algorithms have found wide application in structural optimization and large number of structural optimization techniques developed such as genetic algorithms that are based on the concept of the survival of fittest in the last decades. These are followed by several other metaheuristic techniques that make use of quite different concepts. For example while the simulated annealing algorithm is based on the principle of annealing of metals and cuckoo search technique makes use of the reproduction strategy of cuckoo birds. What makes these techniques powerful are that they work equally well for continuous and discrete optimization problems as well as in optimization problems with large number of design variables and design constraints [2]. They found large area of application in structural optimization problems where the main goal is to minimize the weight of structures while satisfying all design requirements imposed by design codes [3-6]. Metaheuristic search methods perform better compare to the conventional mathematical programming and have better global search abilities than the classical optimization algorithms [7, 8]. The random selection and the information obtained in each cycle are used to choose the new points in the subsequent cycles. These algorithms do not require for a given function to be derivable, and explicit relationship between the objective function and constraints is not needed. Artificial Bee Colony (ABC) is an innovative computational metaheuristic method to solve optimization problems. At its core, ABC optimization technique takes the advantage of a relatively simple concept [9].

In this paper, the ABC algorithm is used to obtain optimum design of steel structures such as domes. These steel structures are designed according to AISC-LRFD specifications [10] and the optimal designs are discussed.

II. OPTIMUM DESIGN FORMULATION

Find a vector of integer values $\boldsymbol{I}$ representing the sequence numbers of $\boldsymbol{N_d}$ design variables within the design pools prepared for the design variables each of which contains certain number of discrete values,

$$I^T = \left[I_1, I_2, \ldots, I_{N_d} \right] \tag{1}$$

where, I_1 is the sequence number in the discrete list for number of rings, I_2 is the sequence number in the discrete list for height of crown and the remaining I_3, $I_4,\ldots,I_{Nd}$ are the sequence numbers in the steel tubular section list for member groups to minimize the weight (W) of a dome,

$$\text{Min } W = \sum_{i=1}^{ng} m_i \sum_{j=1}^{s_i} l_j \qquad (2)$$

Subject to;

$$\delta_{kl} < \delta_k < \delta_{ku}, \qquad k = 1, 2, \ldots, p \qquad (3)$$

For $\frac{P_{ur}}{\phi P_{nr}} \geq 0.2$

$$\frac{P_{ur}}{\phi P_{nr}} + \frac{8}{9}\left(\frac{M_{uxr}}{\phi_b M_{nxr}} + \frac{M_{uyr}}{\phi_b M_{nyr}}\right) \leq 1 \qquad (4)$$

For $\frac{P_{ur}}{\phi P_{nr}} < 0.2$

$$\frac{P_{ur}}{2\phi P_{nr}} + \left(\frac{M_{uxr}}{\phi_b M_{nxr}} + \frac{M_{uyr}}{\phi_b M_{nyr}}\right) \leq 1 \qquad (5)$$

$$\frac{\phi_v V_{nr}}{V_{ur}} \geq 1.0, \; r=1,2,\ldots,nm \qquad (6)$$

$$\det(\mathbf{K}) > 0 \qquad (7)$$

where, m_i in (2) is the unit weight of the tubular section selected from steel tubular section list of LRFD-AISC for the dome member belonging to group i, s_i is the total number of members in group i, and ng is the total number of groups in the dome system. l_j is the length of member j . δ_k in (3) is the displacement of joint k and δ_{ku} and δ_{kl} are its upper and lower bounds, respectively. p is the total number of restricted displacements.

Equations (4) and (5) represent the strength requirements for a member subjected to both bending and axial force according to LRFD. In this inequalities ϕ_b is the resistance factor for flexure given as 0.9, ϕ is the resistance factor for compression or tension given respectively as 0.85 and 0.90, M_{uxr} is the required flexural strength relating to strong axis (x) bending, M_{uyr} is the required flexural strength relating to weak axis (y) bending, M_{nxr} and M_{nyr} are the nominal flexural strength relating to strong axis (x) bending and weak axis (y) bending respectively. P_{ur} is the required compressive or tensile strength, and P_{nr} is the nominal compressive or tensile strength. For compressive strength, P_{nr} is computed from;

$$P_{nr} = A_g F_{cr} \qquad (8)$$

where, F_{cr} is calculated as in the following;

For $\lambda_c \leq 1.5 \quad F_{cr} = (0.658^{\lambda_c^2}) F_y \qquad (9)$

For $\lambda_c > 1.5 \quad F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right] F_y \qquad (10)$

where, in (8) A_g is the gross area of a lamella, network or geodesic domes' member, and F_{cr} is found from (9) or (10) in which F_y is the specified yield stress taken as 250MPa and λ_c is obtained from;

$$\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}} \qquad (11)$$

where, K is the effective length factor taken as 1, *l* is the length of a dome member, r is governing radius of gyration about the axis of buckling, and E is the modulus of elasticity. For tensile strength, P_{nr} is computed from;

$$P_{nr} = A_g F_y \qquad (12)$$

where, F_y is the specified minimum yield stress and A_g is the gross area of a lamella, network or geodesic domes' member.

Equation (6) represents the shear strength requirement in load and resistance factor design according to LRFD. In this inequality ϕ_V represents the resistance factor for shear given as 0.9, V_{nr} is the nominal strength in shear and V_{ur} is the factored service load shear for member r.

The algorithm developed in this study randomly selects set of pipe sections for the member groups from the pipe sections list [11]. With these sections geometrically nonlinear analysis is carried out under the given loading as explained in detail in Section 4. If during this analysis or at the end of the analysis if determinant of the stiffness matrix (det(**K**) in (7)) of the latticed dome turns negative, this set of sections are rejected and a new set is selected. However if at the end of the nonlinear analysis the determinant remains positive then program takes the member end forces from the nonlinear analysis and goes and checks whether the strength and displacement constraints are satisfied. If any of these constraints is not satisfied this set once more is rejected and a new set is randomly generated. The algorithm continues generating set of sections until all the constraints are satisfied. This feasible solution is kept in the harmony memory matrix. The strength constraints themselves do not have any ability to detect the overall instability.

III. TOPOLOGY ANATOMY OF LAMELLA DOMES

The common form of single layer lamella is shown in Fig.1. In a lamella dome, it is possible to generate the structural data for the geometry if three parameters are known. These parameters are the diameter D of the dome, the total number of rings n_r, and the height of the crown h. Once the values of these parameters are decided, the total number of members, member incidences, total number of joints, and joint

coordinates of the dome can be obtained automatically.

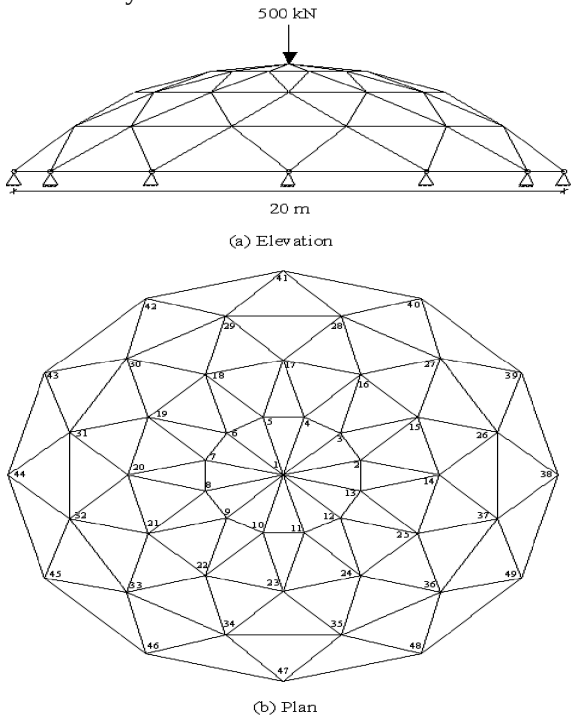


Fig.1. Lamella Dome.

The distances between the rings in the dome on the meridian line are generally made to be equal. It can be easily seen from the figure that all the joints are located with equal distance between each other on same the ring in both domes. The top joint at the crown is numbered as first joint (joint number 1). There are 12 joints on each ring in lamella dome. The first joint on the first ring is numbered as joint 2. The joint numbers of all other first joints of on other rings are computed from $[J_{r1}+ (r-1)*12]$ where *r* is the ring number, and J_{r1} is the first joint number of the first ring namely 2 for lamella dome. It is worthwhile to mention that all of the first joints of the odd numbered rings (ring 1, ring 3, ring 5,...) are located on the radius that makes angle of 15^o with the x-axis and similarly, the first joints of the even numbered rings (ring 2, ring 4,…) are located on the intersection points of that ring and the x-axis in lamella dome. For example in lamella dome, the first joint number of the third ring is numbered as 2 + (3-1)*12 = 26 and it makes 15^o angle with x-axis. On the other hand, the first joint number of the fourth ring is numbered as 2 + (4-1)*12 = 38 and it is on x-axis. Every other joint on rings is numbered in a regular sequence. Member incidences are arranged in similar manner. First member is taken as one and connects joint 1 to joint 2 which makes angle of 15^o with x-axis in lamella dome. The other 11 members connect joint 1 to joints 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 in lamella dome. This is followed by the members that connect joints on the ring in lamella domes 2-3, 3-4, 4-5, 5-6, 6-7, 7-8, 8-9, 9-10,10-11, 11-12, 12-13, 13-2. This process is repeated for each ring and member incidences for all the members in the domes are determined and stored in an array.

The members grouping is decided such that members between each ring are to be made one group and the members on each ring are another group. The diagonal members between the crown and the first ring are group 1, the members on the first ring are group 2, the members between ring 1 and 2 are group 3 and the group number of members on the ring 2 is 4 and so forth, then the total number of groups in the dome becomes twice the total number of rings in the design problem. For example if the total number of rings in the dome is selected as three, then the remaining design variables becomes 7, six of which are the sectional designations to be selected for each group and the last one is the height of the dome.

IV. GEOMETRIC NONLINEARITY

The behavior of latticed domes is nonlinear due to change of geometry under external loads. This is due to the imperfections arising either from the manufacturing process and/or from the construction of the structure. Furthermore they are sometimes subjected to equipment loading concentrated at the crown in addition to uniform gravity loading. This also makes it necessary to check the overall stability during the analysis to ensure that the structure does not lose its load carrying capacity due to instability. The elastic instability analysis of space frames involves repeated stiffness method analysis of the structure at progressively increasing load factor. At each increment of the load factor, nonlinear analysis of the structure is carried out. For this, the stiffness matrix for a three-dimensional space member that includes the effect of flexure on axial stiffness and the stiffness against translation is derived. The details of this derivation and related terms of a nonlinear stiffness matrix of a space member are given in [12]. The stiffness matrix of a stable structure is positive-definite. During the nonlinear analysis iteration, the determinant of the overall stiffness matrix is checked to determine whether at any load increment it becomes negative. This is an indication of a loss of stability of the structure and the load factor which causes this is identified as the critical load factor. The detailed steps of the elastic instability analysis of space frames are given in [13] and are not repeated in this paper.

V. ARTIFICIAL BEE COLONY (ABC)

The artificial bee colony algorithm, one of the recent metaheuristic search algorithms, was developed by Karaboga and Basturk in 2007 [14]. This algorithm is based on characteristic behavior of honey bee swarms. In the artificial bee colony algorithm, all bees are categorized in three main groups. These are employed bees, onlooker bees and scout bees. The first group of bees is the *employed bees* that locate

food source, evaluate its amount of nectar and keep the location of better sources in their memory. These bees when fly back to hive they share this information to other bees in the dancing area by dancing. The dancing time represents the amount of nectar in the food source. The second group is the *onlooker bees* who observe the dance and may decide to fly to the food source if they find it is worthwhile to visit the food source. Therefore foods sources reach in the amount of nectar attract more onlooker bees. The third group is *scout bees* that explore new food sources in the vicinity of the hive randomly. The employed bee whose food source has been abandoned by the bees becomes a scout bee. Each employed bee in the colony goes to one food source and this food source is selected only by one employed bee. Therefore, number of employed bees in the artificial bee colony algorithm is equal to number of food sources. By the time the food source is exhausted, onlooker and employed bees of this food source are replaced by scout bees. Then, these bees start finding new food sources by making random search. The main steps of the algorithm, described as pseudo code, are given below [14]:

```
Initialize
Find initial food positions randomly
For i = 1 to Maximum iteration number
 For i=1 to Number of food Source
  Place the employed bees on the food sources in the memory;
  Calculate nectar amount of food source i
  If(nectar amount of new food source is more than the old one
)
   Replace
  End if
 End
 Calculate probability of food sources
 For i=1 to Number of food Source
  Place the onlooker bees on the food sources depending on the
probability
 End
 If(any food is exhausted)
  Send the scouts to the search area to discover new food
sources;
 End if
End
```

For structural design problems, all available frame designs are represented as food sources which are used by bee swarms in the artificial bee colony algorithm. Amount of each food source represents weight of steel space frame. Steps of artificial bee colony algorithms for optimum design of space frame problems are defined as:

Step1: Search parameters of artificial bee colony algorithm are defined in this step. These are number of employed bees (NEB), number of onlooker bees (NOB), number of cycles and control parameter adjusting the food source (limit). In the algorithm, number of onlooker bees is equal to number of employed bees.

Step2: After defining search parameters, all foragers in the colony search food source randomly. This means NEB+NOB frame designs are generated randomly. Generated frame designs are evaluated and penalized in accordance with their weights and constraints violations. Penalized weight of each frame design is calculated by following function.

$$W_p = W.(1+C)^2 \qquad (13)$$

where, W is the value of objective function values given in (2), W_p is the penalized weight of dome, C is the total constraint violation value calculated from the sum of the values of constraints violation functions shown in (14), ε is penalty coefficient taken as 2.

$$C = \sum C_s + \sum C_d + \sum C_v \qquad (14)$$

where, C_s, C_d and C_v, are the constraints violation functions for strength, deflection and shear constraints functions given in (3) to (6) respectively. In general form, constraints violation functions can be expressed as:

$$C_i = \begin{cases} 0 & \text{if } g_i(x_j) \le 0 \quad i = 1,\dots NC \\ g_i(x_j) & \text{if } g_i(x_j) \le 0 \quad j = 1,\dots NG \end{cases} \qquad (15)$$

where, $g_i(x)$ is i[th] constraints function, x is the vector of design variables, NC is the number of constraint functions and NG is the total number of member groups in the optimization problem.

Step3: After evaluation process, bees having the best dome designs become employed bees. Then, employed bees start to generate a new dome designs by using the old one as follows:

$$v_{ij} = x_{ij} + \emptyset_{ij} \cdot (x_{ij} - x_{kj}) \quad i,k = 1,2,\dots,NEB \quad j = 1,2,\dots,NG$$

where, i represents employed bee number index, k and j are randomly chosen indexes. Although k is generated randomly, it is not equal to k. $\emptyset_{ij}$ is a uniformly distributed random number between [-1, 1]. This parameter adjusts size of neighborhood dome design region. Then, new dome designs generated from employed bees are evaluated and their penalized weights are calculated by using aforementioned process. After evaluation process, penalized weights of new dome designs and old dome designs are compared. If penalized weight of the new dome design is better than the old one, the old dome design is replaced with the new one. This process is called greedy selection.

Step 4: After finding new dome designs and replacements, all employed bees return their hive and start their waggle dance. Waggle dance of employed

bees are related to penalized weight of dome designs. The remainders of the bees (onlooker bees) watch the waggle dance and make a decision. This decision process of each onlooker bee depends on its probability value associated with dome design. Probability of i^{th} dome design is calculated according to i^{th} onlooker bee by using following function,

$$P_i = \frac{(W_p)_i}{\sum_{i=1}^{NOB}(W_p)_i} \quad (17)$$

Then onlooker bees generate new dome designs by using (16) and make greedy selection the same as the case of employed bees.

Step 5: If dome design cannot be replaced with the old dome design, this dome design is abandoned and the employed bee associated with that fame design becomes a scout bee. Scout bees generate new dome designs by using random selection process the same as step 2.

The steps 3 and 5 are repeated until a pre-assigned maximum iteration number is reached.

VI. DESIGN EXAMPLE

The optimum topological design algorithm presented is used to determine the optimum topology of single layer lamella dome shown in Fig.1. The design pool for the total number of rings for the domes contains 3 values that are 3, 4, and 5. For the crown height, a list is prepared starting from 1m to 8.75 m with the increment of 0.25 m. There are 32 values altogether for the harmony search algorithm to choose from. Among the steel tubular sections given in LRFD-AISC, 37 steel tubular sections are selected as the standard list. The sectional designations selected vary from PIPST13 to PIPDEST203 where abbreviations ST, EST, and DEST stands for standard weight, extra strong, and double-extra strong respectively. The yield strength is taken as 250 MPa. The modulus of elasticity for the steel is taken as 205 kN/mm^2. The diameters of all domes are taken as 20 m. The limitations imposed on the joint displacements are given in Table 1.

It is apparent from the table that both upper and lower bound limitations on the restricted joint displacements are assumed as the same value. However it should be noticed that while the upper bound values are applied to those joint displacements which are on the negative direction of global Y-axis, the lower bound values are applied to the joint displacements that may be on the positive direction of the same global axis. Lamella dome is designed to be subjected to equipment loading of 500 kN at its crown. The maximum number of searches is taken as 20000 in each design case. This number is determined after carrying out several designs with a larger number of iterations and it is noticed that the result obtained within the 20000 iterations remains the same even if the iterations continued further. It is noticed in all these cases that the strength limitations are dominant in the design problem. In the optimum dome while the strength ratios were equal to 1 or very close to 1, the values of the restricted displacements were much less than their upper and lower bounds. This is because; consideration of geometric nonlinearity affects the behavior of the dome drastically. During optimization process, it is observed that while the displacement constraints dominates the design problem when linear-elastic behavior is considered, it becomes the strength (stability of members) constraints in the case when geometric nonlinearity is taken into account. Hence consideration of geometric nonlinearity makes it difficult to predict whether the displacement constraints will be dominant in the design problem or not as oppose to linear elastic behavior. Certainly in the optimum results it is the stability constraints that govern the design. This is why optimum designs have small displacements as shown in Table 2. Also, the optimum design of lamella dome obtained using artificial bee colony algorithm is compared with those obtained with classical harmony search optimization (CHSO) algorithm taken from literature [11] in Table 2.

Table 1. Displacement restrictions of single layer lamella dome

Algorithm Used		Classical Harmony Search Algorithm	Artificial Bee Colony Algorithm
Optimum number of rings		3	3
Optimum height of crown (m)		2.5	5.5
Optimum tubular section designations	Group1	PIPEST 102	PIPST 127
	Group2	PIPST 25	PIPST 89
	Group3	PIPST 76	PIPST 64
	Group4	PIPEST 51	PIPST 38
	Group5	PIPST 76	PIPST 64
	Group6	PIPST 13	PIPEST 13
Maximum displacement (mm)		-14.83	-3.88
Maximum strength ratio		0.93	1.00
Weight (kg)		3490.4	3456.9

Table 2. Optimum designs for single layer lamella dome

	Displacement limitations (mm)					
	X-direction		Y-direction		Z-direction	
Joint #	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound
1	-	-	-	-	28 28	-
2	33 33	-	33 33	-	28 28	-
3	33 33	-	33 33	-	28 28	-

The minimum weights obtained by each technique for lamella dome is given in Table 2. For this structure, ABC technique gives the least weight, which is 3456.9 kg. This design is tabulated in Table 2 with pipe section designations attained for each member group, and is considered to be the optimum solution of the problem reached in the present study. The CHSO method achieved heavier design than ABC. The optimum number of rings is obtained as 3 by both algorithms. The design history graph is shown in Fig. 2. The CHSO has appeared the fastest approach reaching the final design within the vicinity of 2000 cycles. On the contrary, ABC method displayed a slow and steady progress towards the optimum design of lamella dome, but finally yields the optimum weight indicating the relative effectiveness of this algorithm. This indicates the consistent behavior of this method in obtaining the solution of this kind of particular structural design problem. Moreover, optimum design derived by using ABC algorithm is shown in Fig.3.

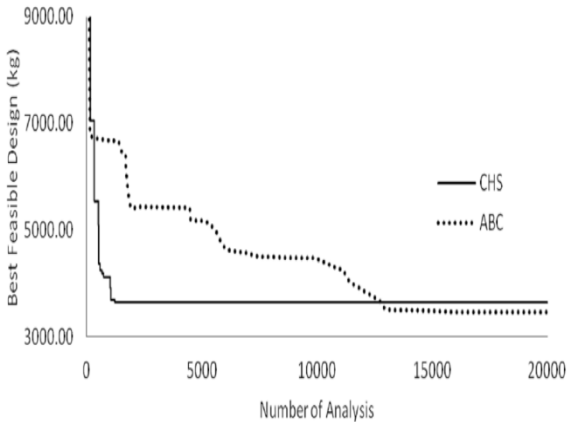


Fig.2. Design history graph for single layer lamella dome.

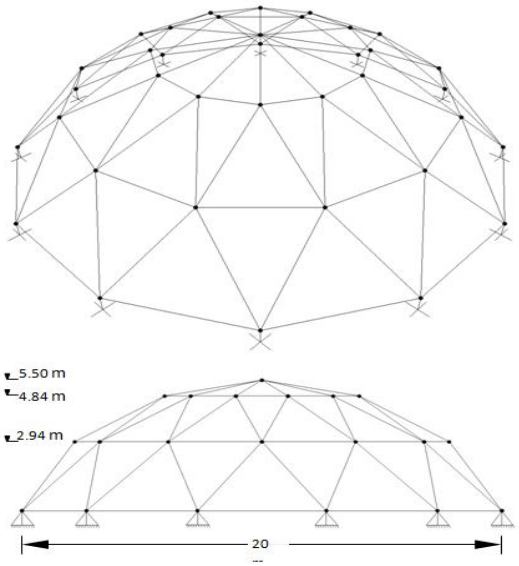


Fig.3. 3D and side views of optimum single layer lamella dome obtained by ABC algorithm

CONCLUSIONS

ABC algorithm is used to determine the optimum geometry and member sizes for the reticulated lamella dome subject to concentrated load at its crown point. The diameter D of the dome, the total number of rings n_r, and the height of the crown h are allowed to be design variables. The structural response of the dome structure is considered to be geometrically nonlinear in order to attain more realistic solution. It is observed that the strength ratio response of the dome is affected considerably by the geometric nonlinearity. In comparison with other heuristic algorithm, it seems that the ABC method has better performance than CHSO technique for optimum design of single layer lamella dome structures.

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