BONDGRAPH MODELING OF DYNAMIC INTERACTION BETWEEN THE CONCRETE SLAB AND SUBGRADE FOR HIGH SPEED TRACK

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Abstract - In the last few years there has been a major shift from petrol, diesel vehicles to electric vehicles and other alternate fuel vehicles, because of the limited availability of fossil fuels. High speed train is one such example of electric vehicles, that have witnessed an increasing demand in present times, especially in urban cities, due to its various advantages, namely, ease of use, economical, time saving, and comfortable. As a result of these factors, analysis and study of such systems, for their optimal performance and more economical operation, is of paramount concern. The present study is aimed to elucidate the effect of vehicle speed on the deflection between the concrete slab/subgrade contact, and track components. The interaction between the slab and the subgrade, affects the safety and comfort of the vehicle. The dynamic analysis of a three-dimensional, modal Bond graph model of the high-speed track components is carried out in this article. The concrete slab is modelled as a Euler-Bernaulli beam, and the beam is divided into 5 nodes. Two parallel slabs are joined with cement-asphalt mortar between them. The hence developed model is then simulated for train velocities ranging from 200kmph to 500kmph. The bond graph modelling and its simulation, is performed using SYMBOLS SHAKTI software.

Keywords - Bond Graph Modelling, Wheelset, Lateral Dynamics.

I. INTRODUCTION

The railway train running along a track comprises of various degrees of freedoms as it comprises of many parts. It is one of the most complicated dynamic systems in engineering, and the most important mode of public transport in a country like India where it runs more than 20,000 passenger trains daily.

All over the world, the railway industries are making considerable efforts to increase the speeds and load carrying capacity of trains. Karmakar et al, 1990 [1]modelled Electric Overhead Travelling (EOT) cranes subjected to severe dynamic loading, using bond graph technique.Zeng et. al, 1994 [3] described the bond graph modelling technique for vehicle-bridge coupling system.
The track was simplified as a lumped parameter system and the effective rail and sleeper masses were included in the vehicle-bridge coupling model.Nielsen et. al, 1995 [4] investigated the vertical dynamic behaviour for a railway bogie moving on a rail which was discretely supported\via rail pads\ by sleepers resting on an elastic foundation.

Application examples were given in which the influences of three types of practically important imperfections in the compound vehicle/track system were investigated. The third imperfection was a case where a single sleeper had lost its support due to erosion of the ballast.The dynamic of vertical interaction between a moving rigid wheel and a flexible railway track has been investigated by Bureikaet. al. (2002). [8].
Transient bending tensions in sleepers and rail were calculated. Kumar H.et. al. (2005) [10] Presented a numerical simulation of the lateral dynamic behaviour of a railway vehicle. Lateral dynamic natural frequency computed using 17dof model.Complete model of a railway vehicle, which has been assembled from wheel sets, car body and intermediate structure which have been flexible, and which have been connected by components such as springs and dampers as considered by Kumar et. al. (2009) [14].A combined vehicle-track bond graph model was developed to study the wheel/ track impact by Kumar et. al. (2018) [20].

It was noticed from simulation results that; a linear track model was found to be inappropriate for wheel/track interaction problems due to the extremely large value of impact load. The objective of the present study is to create a three-dimensional bond graph model of the concrete slab and sub grade system, simulation of vertical dynamics of high-speed railway track system, and study the effect of vertical dynamics by varying the operational speed of the high-speed train.

II. THE PROPOSED SYSTEM

To construct a modal model for the concrete slab – sub grade system, for high speed tracks, the beam (concrete slab) was divided into five modes. The contact between concrete slab and sub grade was considered as a spring-damper system, where the spring and damper, represented stiffness and damping properties, respectively.

The contact between the moving load and the concrete slab is modelled in a similar fashion. The two concrete slabs under the two parallel rails, connected with cement-asphalt mortar, are modelled in a way that, the same modes of both slabs are connected with the help of a spring damper system, representing the properties of cement-asphalt mortar.

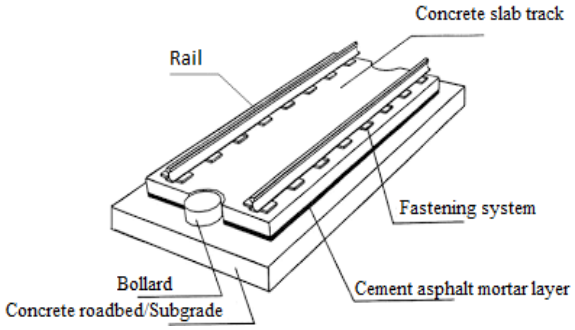


Fig.1. Physical model of the concrete slab and sub grade system.

Equation 3.4, 3.5, 3.7, and 3.8, are used to write expressions for the bond graph elements. After that, the bond graph was checked for errors and warning, and then the equations were generated successfully. The bond graph system was then compiled and simulated.

III. NUMERICAL MODELLING

In this paper the track, i.e., the concrete slab is modelled as Euler-Bernoulli beam.

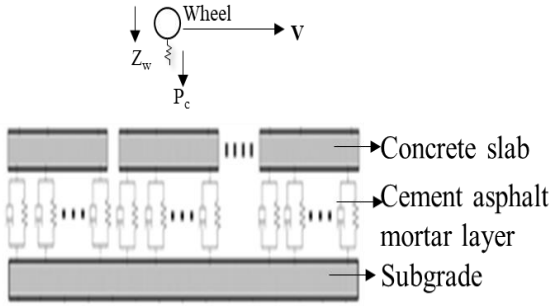


Fig.2. Computational model of concrete slab and subgrade system.

If the transverse deflection of the beam is taken as $z_s(x,t)$, where x denotes the longitudinal position of the beam. Then, the partial differential equation of the beam can be derived and given as –

$$EI_s \frac{\partial^4 z_{s(x,t)}}{\partial x^4} + \rho_r A_r \frac{\partial^2 z_{s(x,t)}}{\partial t^2} = P_c(t)\delta(x - x_w) \quad (3.1)$$

Here,

EI_s - Vertical bending stiffness of concrete slab
ρ_s – Density of concrete slab
A_s - Area of cross section of the concrete slab
$P_c(t)$ - Contact force at the wheel/rail/slab interface.
x - The longitudinal position of the concrete slab with respect to the left end support of the beam.
x_w - The position of the wheel
$\delta(.)$ - Dirac's delta function.

The partial differential equation is converted into normal derivatives by variable-separation method. Solution by this method comes out to be an infinite sum of product of shape function $Y_i(x)$ and modal amplitude $q_i(t)$, which represents the vertical motion of the slab.In the present analysis total 5 modes are considered.The displacement of the rail beam at x from the beam end at time t is given by

$$z_r(x,t) = \sum_{i=1}^{NM} Y_i(x) q_i(t) \quad (3.2)$$

where NM is the number of modes
Both the ends of the concrete slab beam are pivoted, therefore, at both the ends, the deflections and the bending moments are equal to zero.Applying these boundary conditions, shape function and natural frequencies for i^{th} mode, are obtained, as given by Eq (3.3) and Eq. (3.4) respectively. [19]

$$Y_i(x) = \sin\left(\frac{i\pi x}{L_s}\right) \quad (3.4)$$

$$\omega_i = \left(\frac{i\pi x}{L_s}\right)^2 \sqrt{\frac{EI_s}{\rho_s A_s}} \quad (3.5)$$

where,
L_s is the length of the concrete slab
Substituting Eq. (3.4) to Eq. (3.5)and integrating with respect to x along the length of beam, considering the orthogonality property of shape functions, one obtains Eq. (3.6).

$$m_i \ddot{q}_i + k_i q_i = P_c(t) Y_i(x_w) \quad (3.6)$$

where, m_i is the modal mass and k_i is the i^{th} modal stiffness

$$m_i = \frac{\rho_s A_s L_s}{2} \quad i = 1,2 \ldots . NM \quad (3.7)$$

$$k_i = m_i \omega_i^2 \quad i = 1,2 \ldots . NM \quad (3.8)$$

The modal momentum is given by the expression –

$$p_i = m_i \dot{q}_i \quad (3.9)$$

Therefore, the equation (3.6) can be written as –

$$\frac{dp_i}{dt} = -k_i q_i + P_c(t) Y_i(x_w) \quad (3.10)$$

and

$$\frac{dq_i}{dt} = \frac{p_i}{q_i} \quad (3.11)$$

This reduces the equation, which is an i^{th} modal equation, in the form of q_i and p_i, into 2, first order state equations.

The equation 3.4 is used in the expression of transformer element in the bond graph. Similarly, equations (3.7) and (3.8) are used in the expression of stiffness properties of the concrete slab

IV. BOND GRAPH MODELLING

The resulting modal bond graph model for the concrete slab/sub grade system is as shown below –

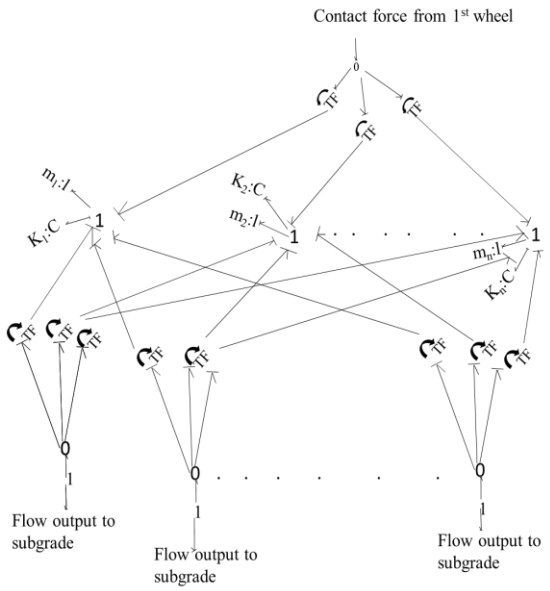


Fig.3.A representativeBond Graph model of the concrete slab and sub grade system.

Here, C172, C164, C28, C35, and C31, represent the stiffness properties of the sub grade. R173, R1655, R29, R36, and R32, represent the damping properties of the sub grade. The 1-elements, joined to the 0-elements, by transfer functions represent the nodes of the beam (concrete slab).

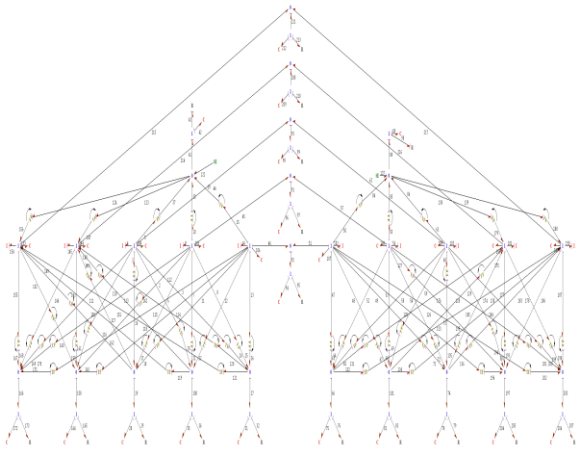


Fig.4. Three-dimensional Bond graph modal model of the concrete slab and sub-grade system, subjected to a constant moving load.

When the bond graph shown above, for a single rail, is to be converted in 3-D, a system of springs and dampers denoting the Cement-asphalt mortar is used to join the two rails to model the whole track. The bond graph model for the same is shown below –
It is a three-dimensional modal model of the concrete slab – sub grade system, subjected to moving load of the train. The concrete slab is divided into five modes.

V. SIMULATION STUDY

The results obtained, in the form of graphs are shown in this section, for the simulation of the bond graph shown in figure 3.3. The interpretation of the results is discussed in the next chapter. After defining all the expressions in the bond graph model and after the generation of all the equations successfully, the bond graph is then simulated. All the values of parameters are to be entered for the generation of graphs in the simulation window, and an appropriate time duration is chosen for the same.

The values of all the properties of the elements, used in the expressions and in the final calculations of the model, during the simulation are shown in the table below –

Properties	Values
Stiffness properties of Sub grade (C_sub)	60×10^6 N/m
Damping properties of Sub grade (R_sub)	90×10^3 Ns/m
Density of Concrete Slab (ρ)	2500 kg/m^3
Length of Concrete Slab (L)	15 m
Area of cross section of a single Concrete Slab (A)	0.435 m^2
Stiffness properties of Concrete slab (C_contact)	87×10^9 N/m
Stiffness properties of Cement-Asphalt Mortar (C_lat)	90×10^7 N/m
Damping properties of Cement-Asphalt Mortar (R_lat)	83×10^3 Ns/m
Flexural Rigidity of Concrete Slab (EI)	13.25×10^6 Nm^2
Static load acting on the beam (Concrete slab)	80×10^3 N

Table 5.1: Material properties of the components of high-speed track

The parameters mentioned above are used during the simulation of the final bond graph. The graphs generated henceforth can be altered by varying the values of parameters in the "parameters" section in the simulation window. The trend and the time of operation for the system to become stable, or tend towards becoming stable, is primary concern.

At various operational speeds, ranging from 200 kmph to 500kmph, the simulation is run, for 10 seconds, to obtain the deflection curves at different modes of the beam (concrete slab), the sub grade, and also at the Cement-asphalt mortar joint, for lateral deflection.

The Fast Fourier Transform (FFT) is generated for the same, at different modes and at different speeds, for a time duration of 40 seconds, which in turn gives the magnitude versus frequency graphs.

VI. RESULTS AND DISCUSSION

Fast Fourier Transform (FFT) for concrete slab –

6.1 Mode – 1

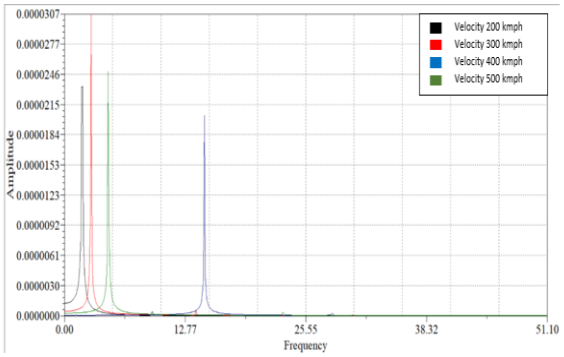


Fig.5. Deflection (m) vs Frequency (Hz) Fast Fourier Transform curve of Mode 1 of Concrete Slab at different velocities.

In the above graph, figure 4.61, it can be observed that the maximum value of deflection shoots up at the velocity of 300 kmph, at a frequency of about 2.5 Hz

6.2 Mode– 2

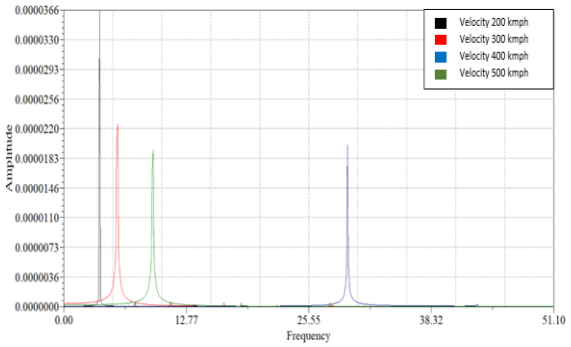


Fig.6. Deflection (m) vs Frequency (Hz) Fast Fourier Transform curve of Mode 2 of Concrete Slab at different velocities.

In the above graph, figure 4.62, it can be observed that the maximum value of deflection shoots up at the velocity of 200 kmph, at a frequency of about 3.5 Hz

6.3 Mode - 3

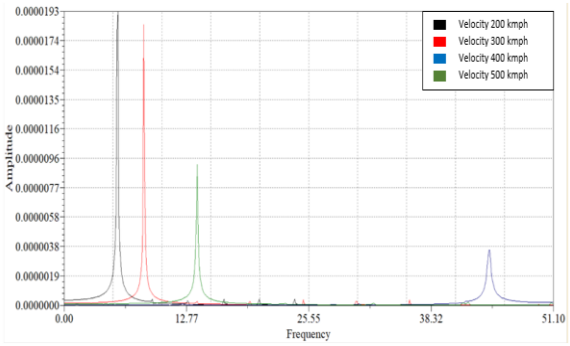


Fig.7. Deflection (m) vs Frequency (Hz) Fast Fourier Transform curve of Mode 3 of Concrete Slab at different velocities.

In the above graph, figure 4.63, it can be observed that the maximum value of deflection shoots up at the velocity of 200 kmph, at a frequency of about 5.5 Hz

6.4 Mode - 4

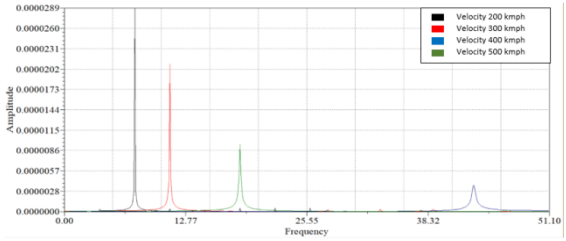


Fig.8. Deflection (m) vs Frequency (Hz) Fast Fourier Transform curve of Mode 4 of Concrete Slab at different velocities.

In the above graph, figure 4.64, it can be observed that the maximum value of deflection shoots up at the velocity of 200 kmph, at a frequency of about 7 Hz

6..5 Mode – 5

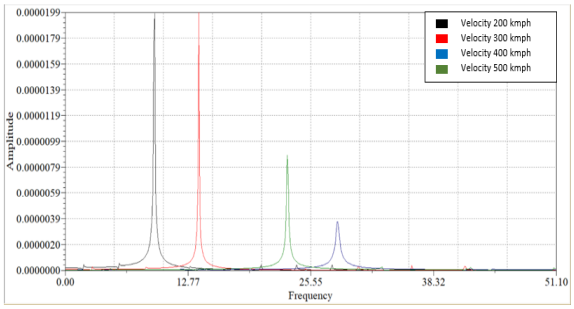


Fig.9. Deflection (m) vs Frequency (Hz) Fast Fourier Transform curve of Mode 5 of Concrete Slab at different velocities.

In the above graph, figure 4.65, it can be observed that the maximum value of deflection shoots up at the velocity of 200 kmph and 300 kmph, at a frequency of about 9 Hz and 13 Hz, respectively.

VII. CONCLUSIONS

The Fast Fourier Transform (FFT) of the concrete slab, at different modes, for different velocities, depicts that the excitation of Mode 1 is highest at the speed of 300 kmph, at a value of 0.0307 mm. The excitation of Mode 2 is highest at the speed of 200 kmph, at a value of approximately 0.0366 mm. The excitation of Mode 3 is highest at the speed of 200 kmph, at a value of 0.0193 mm, closely followed by 300 kmph speed, at a value of 0.0183 mm. The excitation of Mode 4 is also highest at the speed of 200 kmph, at a value of 0.0289 mm. And the excitation of Mode 5 is highest at the speed of 200 and 300 kmph, both, at a value of 0.0199 mm. This observation hints that the maximum deflection occurs around the velocity of 200 kmph and in mode 2.

The present study deals with a two-layer problem of beam modelling, in three-dimensions, for a concrete slab – sub grade system. In the future the same methods can be extended for multi-layer bond graph modelling in three-dimensions. A more complex system, involving the vehicle as well, on the multi-

layered track, can be performed, using bond graph capsules and ports.

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