DYNAMIC BEHAVIOUR OF FOUR-WHEEL-STEERING GROUND VEHICLE USING 9DOF MODEL UNDER STEERING INPUT CONSIDERING ROLL, YAW AND PITCH

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Abstract - Deriving the equations of motions for a ground vehicle needs to be based on assumptions which has been provided in the first part of this article and then it is followed by deriving 9 equations of motions including Roll, Pitch, Yaw, Lateral Equation of motion, Vertical Displacement of 4 un-sprung masses and one for the sprung mass for a passive suspension system considering a linear tire model. This model is then used to make steering effect comparison between 4WS and 2WS.Results show that a 4WS can avoid the obstacle without changing its heading. Besides, lateral displacement under same steering input is higher than in 2WS.The model can also consider the signal coming from the road.

Keywords - Four Wheel Steering Vehicle, Roll, Yaw, Pitch, 9 DOF Vehicle Dynamics

I. INTRODUCTION

In conventional 2WS, vehicle goes where the front wheels are pointed while the back of the car gets dragged along. A two-wheel-steering vehicle can only avoid an obstacle by changing its heading (yaw angle). Unlike a 2WS, a 4WS vehicle by steering the four wheels in the same direction, can avoid obstacle without changing its heading [1]. In 4WS, at high speeds the back wheels turn in sympathy with the front wheels which allows the car to almost shift and not being dragged as it is the case for 2WS. In lower speeds the back wheels of a 4WS can go in the opposite direction of the front wheels which gives a much tighter turning radius and higher maneuverability. Another interesting maneuver can be the ability to toe-in during a hard stop as a skier does. For ground vehicle, the most contributing factor is the tire which varies with friction and normal load [2]. Moreover, Longitudinal and lateral forces are coupled and highly nonlinear. Needless to say, roll motion can affect the yaw motion and the dynamics at low speed differ from the one in high speed [3].
However, two different set of angular equations of motion has been presented in this paper. One with respect to the body center of gravity. In this case, body needs to be isolated and the suspension forces play direct role and its set of results can be used to evaluate the suspension system effectiveness. However, the driver is mostly familiar with the second set of angular equation of motion which is with respect to the roll center. In this set, normal forces on tires come to the equation and suspension forces become internal forces.

II. ASSUMPTIONS

- Planar motion, Roll about x axis, Pitch about y axis and Yaw about z axis.
- Constant tire contact.

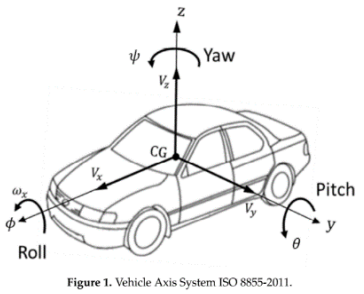


Figure 1. Vehicle Axis System ISO 8855-2011.

Figure 1.Coordinate System. Adapted from [8]

- The vehicle will be lumped into two set of un-deformable masses: the sprung mass (Includes the vehicle body, engine, passengers and so on.) and the un-sprung mass (Includes the wheels, suspension, brakes and so on.). In addition, due to the difference between front and rear axle, the un-sprung mass is decomposed into two different masses(M_{uf} and M_{ur}).[4]
- Since the roll center is considered to be fixed, it has been chosen to be the origin of chassis coordinate as well as moving-vehicle coordinate. The un-sprung mass is not subject to neither the roll nor the pitch motions. Three different coordinate systems need to be defined.
- Earth fixed Inertial reference frame
- Chassis coordinate or vehicle fixed
- Vehicle moving with the same origin of chassis coordinate.

Vehicle moving coordinate can pitch and roll around the fixed vehicle coordinate while chassis origin can yaw around the earth fixed coordinate. The driver is most familiar with chassis coordinate reference.

- For some passenger sedans, such as the Ford Taurus, the roll axis can be assumed horizontal (Demerly, 2000).
- Longitudinal acceleration is equal to zero. Since the only forces which are acting in the

longitudinal direction are longitudinal tire forces then according to Newton Second law, it can be assumed to have no longitudinal tire force.

- It has been assumed to have the symmetry in the geometry of the vehicle in all three planes which allows to consider

$I_{xy} = I_{xz} = 0.$

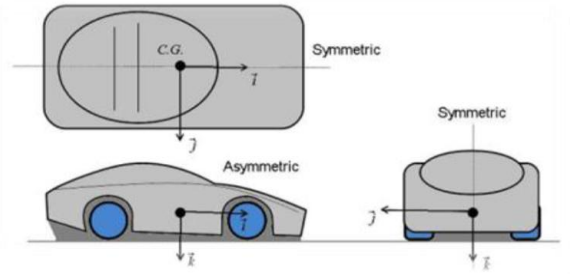


Figure 2.Symmetric Planes. adapted from [8]

- This model is only valid for normal driving condition which there is a linear relationship between slip angle and the lateral force of the tire. It is assumed that normal forces depend only on the weight distribution of the car and that these forces are constant during the vehicle movement. In other words, neither longitudinal nor lateral load transfer is considered within this paper [5].For small sideslip angle α the lateral tire characteristics is linear and lateral force F_y is equal to sideslip angle multiplied by the nominal cornering stiffness [6]. For small slip angles the cornering stiffness is independent of the road surface and remain almost at the same value until a certain magnitude of the slip angle is reached. Beyond this value the cornering stiffness's decrease rapidly and may become negative at large side slip angle [7].

III. METHOD

3.1 Deriving the position, velocity and acceleration of the origin of the chassis coordinate system w.r.t. inertial reference frame. [Appendix A].

$$\vec{R}_O = (X\cos(\psi) + Y\sin(\psi))\vec{\imath} + (Y\cos(\psi) - X\sin(\psi))\vec{\jmath}$$

$$\vec{V}_O = \left(\dot{X}\cos(\psi) + \dot{Y}\sin(\psi)\right)\vec{\imath} + \left(\dot{Y}\cos(\psi) - \dot{X}\sin(\psi)\right)\vec{\jmath}$$

$$\vec{a}_O = (\dot{\vec{V}}_{Ox} - \dot{\psi}\vec{V}_{Oy})\vec{\imath} + (\dot{\vec{V}}_{Oy}$$

3.2 Deriving the position, velocity and acceleration of the front and rear un-sprung mass and sprung mass

[Appendix B]

$$\vec{V}_{uf}$$

$$\vec{V}_{ur}$$

$$\vec{V}_{sprung} = \begin{bmatrix} \left(\vec{V}_{Ox} - l_s sin\theta\dot{\theta} + h_s\, sin\phi sin\theta\,\dot{\phi} - h_s cos\phi cos\theta\,\dot{\theta} - h_s sin\phi\dot{\psi}\right) \\ \left(\vec{V}_{Oy} + h_s cos\phi\dot{\phi} + l_s\dot{\psi}cos\theta - h_s\dot{\psi}cos\phi sin\theta\right) \\ \left(l_s cos\theta\,\dot{\theta} + h_s sin\phi cos\theta\,\dot{\phi} + h_s sin\theta cos\phi\,\dot{\theta}\right) \end{bmatrix}$$

$$\vec{a}_{uf} = \left(\vec{V}_{Ox} - \dot{\psi}\vec{V}_{Oy} - l_f\dot{\psi}^2\right)\vec{\imath} + \left(\vec{V}_{Oy} + \dot{\psi}\vec{V}_{Ox} + l_f\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{a}_{ur} = \left(\vec{V}_{Ox} - \dot{\psi}\vec{V}_{Oy} + l_r\dot{\psi}^2\right)\vec{\imath} + \left(\vec{V}_{Oy} + \dot{\psi}\vec{V}_{Ox} - l_r\ddot{\psi}\right)\vec{\jmath}$$

$$\begin{aligned}\vec{a}_{sprung\ -x} = \vec{V}_{Ox} &- l_s cos\theta\dot{\theta}^2 - l_s sin\theta\,\ddot{\theta} + h_s\, sin\phi sin\theta\,\ddot{\phi} \\ &+ h_s\, sin\phi cos\theta\,\dot{\phi}\dot{\theta} + h_s\, sin\theta cos\phi\,\dot{\phi}^2 \\ &- h_s cos\phi cos\theta\,\ddot{\theta} + h_s cos\phi sin\theta\,\dot{\theta}^2 + h_s\, sin\phi cos\theta\,\dot{\phi}\dot{\theta} \\ &- \dot{\psi}(\vec{V}_{Oy} + h_s cos\phi\dot{\phi} + l_s\dot{\psi}cos\theta \\ &- h_s\dot{\psi}cos\phi sin\theta)\end{aligned}$$

$$\begin{aligned}\vec{a}_{sprung\ -y} = \dot{V}_{Oy} &+ h_s cos\phi\ddot{\phi} - l_s\dot{\psi}\dot{\theta}sin\theta + l_s\ddot{\psi}co - h_s\dot{\psi}\dot{\theta}cos\phi cos\theta \\ &+ h_s\dot{\psi}\dot{\phi}sin\phi sin\theta \\ &- h_s\ddot{\psi}cos\phi sin\theta + \dot{\psi}(V_{0x} - l_s sin\theta\dot{\theta} + h_s\, sin\phi sin\theta\dot{\phi} \\ &- h_s cos\phi cos\theta\dot{\theta} - h_s sin\phi\dot{\psi})\end{aligned}$$

$$\begin{aligned}\vec{a}_{sprung\ -z} = (&l_s cos\theta\ddot{\theta} - l_s sin\theta\dot{\theta}^2 + h_s sin\phi cos\theta\ddot{\phi} \\ &- h_s sin\phi sin\theta\dot{\phi}\dot{\theta} + h_s cos\phi cos\theta\dot{\phi}^2 \\ &+ h_s sin\theta cos\phi\ddot{\theta} \\ -h_s sin\theta sin\phi\dot{\theta}\dot{\phi} &+ h_s cos\theta cos\phi\dot{\theta}^2 + h_s sin\theta cos\phi\ddot{\theta} \\ &- h_s sin\theta sin\phi\dot{\theta}\dot{\phi} + h_s cos\theta cos\phi\dot{\theta}^2)\end{aligned}$$

3.3 Adding up the acceleration of the three masses in y direction to get the lateral equation of motion.

$$\begin{Bmatrix} \sin\phi \approx \phi & \cos\phi \approx 1 & \sin\theta \approx \theta & \cos\theta \approx 1 \\ \phi\theta \approx 0 & \dot{\phi}\dot{\theta} \approx 0 & \dot{\phi}^2 \approx 0 & \dot{\theta}^2 \approx 0 \end{Bmatrix}$$

It is assumed to have relatively small values of roll and pitch angles and angular velocities with respect to yaw dynamics

$$\vec{a}_{ufy} = \left(\dot{\vec{V}}_{0y} + \dot{\psi}\vec{V}_{0x} + l_f\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{a}_{ury} = \left(\dot{\vec{V}}_{0y} + \dot{\psi}\vec{V}_{0x} - l_r\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{a}_{sprung\ -y} = (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})\vec{\jmath}$$

$$\begin{aligned}\Sigma F_y = M\left(\dot{V}_{0y} + \dot{\psi}V_{0x}\right) + M_s\left(h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta\right) \\ + \ddot{\psi}(M_{uf}l_f - M_{ur}l_r)\end{aligned}$$

3.4 Deriving the side-slip angle and lateral tire force [Appendix C]

$$\alpha_f = \delta_f - \gamma_{front} = \delta_f - \frac{V_{0y} + l_f\dot{\psi}}{V_{0x}} = \delta_f - \beta - \frac{l_f\dot{\psi}}{V}$$

$$\alpha_r = \delta_r - \gamma_{rear} = \delta_r - \frac{V_{0y} - l_r\dot{\psi}}{V_{0x}} = \delta_f - \beta + \frac{l_r\dot{\psi}}{V}$$

$$F_{yfl} = F_{yfr} = c_f\alpha_f$$

$$F_{yrl} = F_{yrr} = c_r\alpha_r$$

c_r&c_f : Tire cornering stifness

Obtaining the lateral tire forces, Lateral equation of motion can be written as:

$$\begin{aligned}M\left(\dot{V}_{0y} + \dot{\psi}V_{0x}\right) + M_s\left(h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta\right) \\ + \ddot{\psi}(M_{uf}l_f - M_{ur}l_r) = \\ \left(F_{yfl} + F_{yfr}\right)cos\delta_f + \left(F_{yrl} + F_{yrr}\right)cos\delta_r \\ + (F_{xfl} + F_{xfr})sin\delta_f \\ + (F_{xrl} + F_{xrr})sin\delta_r\end{aligned}$$

Since it has been assumed to have no longitudinal acceleration, Longitudinal tire forces become zero

and it can be written in its final form:

$$M(\dot{V}_{0y} + \dot{\psi}V_{0x}) + M_s(h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta) + \ddot{\psi}(M_{uf}l_f - M_{ur}l_r) = (F_{yfl} + F_{yfr})\cos\delta_f + (F_{yrl} + F_{yrr})\cos\delta_r$$

3.5 Deriving the Angular Equation of motion with respect to the body centre of gravity (BCOG). [Appendix D]

$$M_x = (I_{xx} - 2I_{xz}\theta) * \ddot{\phi} + \ddot{\psi}[I_{xz} + \theta * (I_{xx} - I_{zz})] + \ddot{\theta}\phi * (I_{xz} - I_{zz}\theta) = (F_{s3} - F_{s4}) * d + (F_{s1} - F_{s2}) * c - (h_o + h_s) * (F_{ylf} + F_{yfr} + F_{yrl} + F_{yrr})$$

$$M_y = I_{yy} * \ddot{\theta} + \ddot{\psi}\phi * (I_{zz}) + \ddot{\phi}\phi * (I_{zx}) = (F_{s3} + F_{s4}) * b - (F_{s1} + F_{s2}) * a$$

$$M_z = [I_{zz} + \theta * (2I_{xz})] * \ddot{\psi} + \ddot{\theta}\phi * (I_{zz} - I_{yy}) + \ddot{\phi}\theta * (I_{xx} - I_{zz}) + I_{zx} * \ddot{\phi} = l_f[F_{yfl} + F_{yfr}] - l_r[F_{yrl} + F_{yrr}]$$

$F_{s1}, F_{s2}, F_{s3}, F_{s4}$: Suspension Forces . Appendix[F]
$F_{yrl}, F_{yrr}, F_{yrl}, F_{yrr}$: Lateral Tire Force. Appendix[C]

3.6 Deriving the Angular Equation of motion with respect to the roll centre (RC). [Appendix E]

$$M_x = \ddot{\phi}(I_{xx} + M_s * h_s * h_s) + \ddot{\psi}\theta(I_{xx} - I_{zz} + M_s * h_s * h_s - M_S * l_s * l_s) + M_{uf}\left(h_{uf}\dot{\vec{V}}_{0y} + h_{uf}\dot{\psi}\vec{V}_{0x} + h_{uf}l_f\ddot{\psi}\right) + M_{ur}\left(-h_{ur}\dot{\vec{V}}_{0y} - h_{ur}\dot{\psi}\vec{V}_{0x} + h_{ur}l_r\ddot{\psi}\right) + M_s\left(l_s\ddot{\theta}(h_s\phi) + \left(\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x}\right)(l_s\theta + h_s)\right) = c * (F_{zfl} - F_{zrf}) + d * (F_{zlr} - F_{zrr}) + h_o * (F_{ylf} + F_{yfr} + F_{yrl} + F_{yrr})$$

$$M_y = \ddot{\theta}(I_{yy} + M_s * h_s * h_s + M_s * l_s * l_s) + \ddot{\psi}\phi(I_{zz} - I_{yy} - M_s * h_s * h_s) + M_{uf}h_{uf}\dot{\psi}\vec{V}_{0y} + M_{ur}h_{ur}\dot{\psi}\vec{V}_{0y} - M_s l_s\ddot{\theta}(l_s - h_s\theta) = a * (F_{zfl} + F_{zfr}) - b * (F_{zrl} + F_{zrr})$$

$$M_z = \left(I_{zzuf} + I_{zzur} + m_{uf} * l_f^2 + m_{ur} * l_r^2 + I_{zz} + M_s * l_s * l_s\right) * \ddot{\psi} + \phi\ddot{\theta}(I_{zz} - I_{yy} - M_s * h_s * h_s) + \ddot{\phi}\theta(I_{xx} - I_{zz} - M_s * h_s * h_s) + M_{uf}\left(l_f\dot{\vec{V}}_{0y} + l_f\dot{\psi}\vec{V}_{0x} + l_f^2\ddot{\psi}\right) + M_{ur}\left(-l_r\dot{\vec{V}}_{0y} - l_r\dot{\psi}\vec{V}_{0x} + l_r^2\ddot{\psi}\right) + M_s(l_s - h_s\theta)(\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x}) = l_f[F_{ylf} + F_{yfr}] - l_r[F_{yrl} + F_{yrr}]$$

$F_{zfl}, F_{zfr}, F_{zrl}, F_{zrr}$: Vertical Force on each tire. [section 3.7]

3.7 Deriving the vertical motion of un-sprung masses and the sprung mass.

For Un-Sprung Rear Right

$$M_{urr}\ddot{z}_{urr} = -K_{srr}(z_{urr} - z_s - b\theta + d\phi) - c_{srr}(\dot{z}_{urr} - \dot{z}_s - b\dot{\theta} + d\dot{\phi}) - K_{trr}(z_{urr} - z_{rrr})$$

For Un-Sprung Rear Left

$$M_{ulr}\ddot{z}_{ulr} = -K_{slr}(z_{ulr} - z_s - b\theta - d\phi) - c_{slr}(\dot{z}_{ulr} - \dot{z}_s - b\dot{\theta} - d\dot{\phi}) - K_{tlr}(z_{ulr} - z_{rlr})$$

For Un-Sprung Front Left

$$M_{ulf}\ddot{z}_{ulf} = -K_{slf}(z_{ulf} - z_s + a\theta - c\phi) - c_{slf}(\dot{z}_{ulf} - \dot{z}_s + a\dot{\theta} - c\dot{\phi}) - K_{tlf}(z_{ulf} - z_{rlf})$$

For Un-Sprung Front Right

$$M_{urf}\ddot{z}_{urf} = -K_{srf}(z_{urf} - z_s + a\theta + c\phi) - c_{rsf}(\dot{z}_{urf} - \dot{z}_s + a\dot{\theta} + c\dot{\phi}) - K_{trf}(z_{urf} - z_{rrf})$$

Sprung mass

$$M_s\ddot{Z}_s = K_{srr}(z_{urr} - z_s - b\theta + d\phi) + c_{srr}(\dot{z}_{urr} - \dot{z}_s - b\dot{\theta} + d\dot{\phi}) + K_{slr}(z_{ulr} - z_s - b\theta - d\phi) + c_{slr}(\dot{z}_{ulr} - \dot{z}_s - b\dot{\theta} - d\dot{\phi}) + K_{slf}(z_{ulf} - z_s + a\theta - c\phi) + c_{slf}(\dot{z}_{ulf} - \dot{z}_s + a\dot{\theta} - c\dot{\phi}) + K_{srf}(z_{urf} - z_s + a\theta + c\phi) + c_{srf}(\dot{z}_{urf} - \dot{z}_s + a\dot{\theta} + c\dot{\phi})$$

Suspension Front Left

F_{zrr}

Suspension Front Left

F_{zrl}

Suspension Front Left

$F_{zfl} = k_{tfl} * (z_{ufl} - z_{rfl})$

Suspension Front Left

F_{zfr}

Suspension Rear Right

$$K_{srr}(z_{urr} - z_s - b\theta + d\phi) + c_{srr}(\dot{z}_{urr} - \dot{z}_s - b\dot{\theta} + d\dot{\phi})$$

Suspension Rear Left

$$K_{slr}(z_{ulr} - z_s - b\theta - d\phi) + c_{slr}(\dot{z}_{ulr} - \dot{z}_s - b\dot{\theta} - d\dot{\phi})$$

Suspension Front Left

$$K_{slf}(z_{ulf} - z_s + a\theta - c\phi) + c_{slf}(\dot{z}_{ulf} - \dot{z}_s + a\dot{\theta} - c\dot{\phi})$$

Suspension Front Right

$$K_{srf}(z_{urf} - z_s + a\theta + c\phi) + c_{srf}(\dot{z}_{urf} - \dot{z}_s + a\dot{\theta} + c\dot{\phi})$$

Appendix A: Deriving the position of the origin of the chassis coordinate system w.r.t. inertial reference frame

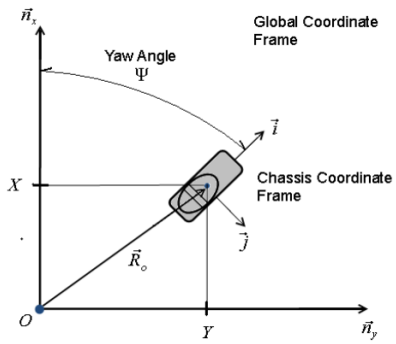


Figure 2.Adapted from [8]

$\vec{R}_O = X\vec{n}_x + Y\vec{n}_y$ Position of the origin of Chassis frame in inertial reference frame

$R_z = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \end{bmatrix}$ The chassis coordinate origin rotate w.r.t. inertial reference frame about z direction

$$\begin{bmatrix} \vec{n}_x \\ \vec{n}_y \\ \vec{n}_z \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Change of coordinate from Inertial reference frame to chassis coordinate frame

$\vec{n}_x$ $\vec{n}_y$

$$\vec{R}_O = (X\cos(\psi) + Y\sin(\psi))\vec{\imath} + (Y\cos(\psi) - X\sin(\psi))\vec{\jmath}$$

$$\vec{V}_O = \frac{d(\vec{R}_O)}{dt} = (\dot{X}\cos(\psi) + \dot{Y}\sin(\psi))\vec{\imath} + (\dot{Y}\cos(\psi) - \dot{X}\sin(\psi))\vec{\jmath} + (-X\dot{\psi}\sin(\psi) + Y\dot{\psi}\cos(\psi))\vec{\imath} + (-Y\dot{\psi}\sin(\psi) - X\dot{\psi}\cos(\psi))\vec{\jmath} + (X\cos(\psi) + Y\sin\psi)\dot{\imath} + (Y\cos\psi - X\sin\psi)\dot{\jmath}$$

$\vec{\jmath} = \dot{\psi}\vec{k} * \mathrm{J}$ $\vec{\imath} = \dot{\psi}\vec{k} * \mathrm{I}$

The origin of the chassis coordinate system can rotate along z direction w.r.t. Inertial reference frame

$$\vec{a}_{ur} = \left(\dot{\vec{V}}_{Ox} - \dot{\psi}\vec{V}_{Oy} + l_r\dot{\psi}^2\right)\vec{\imath} + \left(\dot{\vec{V}}_{Oy} + \dot{\psi}\vec{V}_{Ox} - l_r\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{V}_O = (\dot{X}\cos(\psi) + \dot{Y}\sin(\psi))\vec{\imath} + (\dot{Y}\cos(\psi) - \dot{X}\sin(\psi))\vec{\jmath}$$

$\vec{a}_O = \frac{d^2(\vec{R}_O)}{dt^2} = \frac{d(\vec{V}_O)}{dt}$

$$\vec{a}_O = \vec{V}_{Ox}\vec{\imath} + \vec{V}_{Oy}\vec{\jmath} + \vec{V}_{Ox}\vec{\imath} + \vec{V}_{Oy}\vec{\jmath}$$

using the expression for the time derivative of unit vectors

$$\vec{a}_O = (\dot{\vec{V}}_{Ox} - \dot{\psi}\vec{V}_{Oy})\vec{\imath} + (\dot{\vec{V}}_{Oy} + \dot{\psi}\vec{V}_{Ox})\vec{\jmath}$$

Appendix B: Deriving the position, velocity and acceleration of the front un-sprung mass.

$\vec{R}_{uf}$

$\vec{r}_{uf}$

$\vec{V}_{uf} = \frac{d(\vec{R}_{uf})}{dt} = \frac{d(\vec{R}_O)}{dt}$

$\frac{d(\vec{r}_{uf})}{dt}$

$\vec{k} = \dot{\psi}\vec{k} * k{=}0$ $\vec{\imath}$ $\vec{\jmath}$

$\vec{V}_{uf}$

$$\vec{a}_{uf} = \frac{d(\vec{V}_{uf})}{dt} = \vec{V}_{Ox}\vec{\imath} + \left(\vec{V}_{Oy} + l_f\dot{\psi}\right)\vec{\jmath} + \vec{V}_{Ox}\vec{\imath} + (\vec{V}_{Oy} + l_f\dot{\psi})\vec{\jmath}$$

$$\vec{a}_{uf} = \left(\vec{V}_{Ox} - \dot{\psi}\vec{V}_{Oy} - l_f\dot{\psi}^2\right)\vec{\imath} + \left(\dot{\vec{V}}_{Oy} + \dot{\psi}\vec{V}_{Ox} + l_f\ddot{\psi}\right)\vec{\jmath}$$

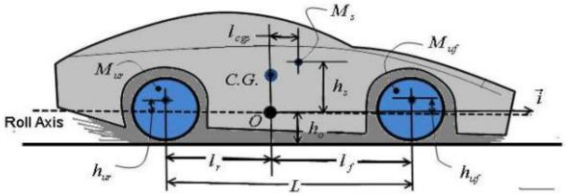


Figure 3.Adapted from[8]

Appendix B: Deriving the position, velocity and acceleration of the rear un-sprung mass

Due to the relative motion of the sprung mass with respect to chassis coordinate frame, it is necessary to define the transformation matrix with respect to its roll and pitch.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \end{bmatrix} \quad R_y = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \end{bmatrix}$$

$$R = R_x * R_y = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ \sin(\theta)\sin(\phi) & \cos(\phi) & \cos(\theta)\sin(\phi) \\ \cos(\phi)\sin(\theta) & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix}$$

$$\begin{pmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix}$$

$\vec{i}_s$

$\vec{j}_s = sin\phi sin\theta\vec{\imath} + cos\phi\vec{\jmath}$

$\vec{k}_s = cos\phi sin\theta\vec{\imath} - sin\phi\vec{\jmath} + sin\phi cos\theta\vec{k}$

$\vec{r}_s$

$$\vec{r}_s = (l_s cos\theta - h_s cos\phi sin\theta)\vec{\imath} + (h_s sin\phi)\vec{\jmath} + (-l_s sin\theta - h_s cos\phi cos\theta)\vec{k}$$

$\vec{R}_s$

$$\vec{V}_s = \frac{d(\vec{R}_s)}{dt} = \frac{d(\vec{R}_O)}{dt}$$

$$\frac{d(\vec{r}_s)}{dt} = (-l_s sin\theta\dot{\theta} + h_s sin\phi sin\theta\dot{\phi} - h_s cos\phi cos\theta\dot{\theta} h_s sin\phi\dot{\psi})\vec{\imath} + (h_s cos\phi\dot{\phi} - h_g sin\phi\dot{\phi} + l_s\dot{\psi}cos\theta - h_s\dot{\psi}cos\phi sin\theta)\vec{\jmath} + (l_s cos\theta\dot{\theta} + h_s sin\phi cos\theta\dot{\phi} + h_s sin\theta cos\phi\dot{\theta})\vec{k}$$

$$\vec{V}_s = \begin{bmatrix} (\vec{V}_{Ox} - l_s sin\theta\dot{\theta} + h_s sin\phi sin\theta\dot{\phi} - h_s cos\phi cos\theta\dot{\theta} - h_s sin\phi\dot{\psi}) \\ (\vec{V}_{Oy} + h_s cos\phi\dot{\phi} + l_s\dot{\psi}cos\theta - h_s\dot{\psi}cos\phi sin\theta) \\ (l_s cos\theta\dot{\theta} + h_s sin\phi cos\theta\dot{\phi} + h_s sin\theta cos\phi\dot{\theta}) \end{bmatrix}$$

$\vec{a}_s$

$d(\vec{V}_s)$

$$\vec{a}_{sx} = \vec{V}_{Ox} - l_s cos\theta\dot{\theta}^2 - l_s sin\theta\ddot{\theta} + h_s sin\phi sin\theta\ddot{\phi} + h_s sin\phi cos\theta\dot{\phi}\dot{\theta} + h_s sin\theta cos\phi\dot{\phi}^2 - h_s cos\phi cos\theta\ddot{\theta} + h_s cos\phi sin\theta\dot{\theta}^2 + h_s sin\phi cos\theta\dot{\phi}\dot{\theta} - \dot{\psi}(\vec{V}_{Oy} + h_s cos\phi\dot{\phi} + l_s\dot{\psi}cos\theta - h_s\dot{\psi}cos\phi sin\theta)$$

$$\vec{a}_{sy} = \dot{V}_{Oy} + h_s cos\phi\ddot{\phi} - l_s\dot{\psi}\dot{\theta}sin\theta + l_s\ddot{\psi}cos\theta - h_s\dot{\psi}\dot{\theta}cos\phi cos\theta + h_s\dot{\psi}\dot{\phi}sin\phi sin\theta - h_s\ddot{\psi}cos\phi sin\theta + \dot{\psi}(V_{Ox} - l_s sin\theta\dot{\theta} + h_s sin\phi sin\theta\dot{\phi} - h_s cos\phi cos\theta\dot{\theta} - h_s sin\phi\dot{\psi})$$

$$\vec{a}_{sz} = (l_s cos\theta\ddot{\theta} - l_s sin\theta\dot{\theta}^2 + h_s sin\phi cos\theta\ddot{\phi} - h_s sin\phi sin\theta\dot{\phi}\dot{\theta} + h_s cos\phi cos\theta\dot{\phi}^2 + h_s sin\theta cos\phi\ddot{\theta} - h_s sin\theta sin\phi\dot{\theta}\dot{\phi} + h_s cos\theta cos\phi\dot{\theta}^2 + h_s sin\theta cos\phi\ddot{\theta} - h_s sin\theta sin\phi\dot{\theta}\dot{\phi} + h_s cos\theta cos\phi\dot{\theta}^2)$$

Appendix C: Deriving the side-slip angle and lateral tire force

$$slip\ angle = \begin{pmatrix} angle\ between\ the\ center\ line \\ of\ the\ tire\ and\ longitudinal\ axis(\delta) \end{pmatrix} - \begin{pmatrix} angle\ between\ the\ tire\ velocity \\ and\ longitudinal\ axis(\gamma) \end{pmatrix}$$

$\vec{R}_{tire\ center}$ $\vec{r}_{tc,front}$ $\vec{r}_{tc,rear}$

$$\vec{V}_{tc} = \frac{d(\vec{R}_{tc})}{dt} = \frac{d(\vec{R}_O)}{dt}$$

$\frac{d(\vec{r}_{tc,front})}{dt}$ $\frac{d(\vec{r}_{tc,front})}{dt}$

$\vec{\imath}$

$$\vec{V}_{tc,fronr} = \vec{V}_{Ox}\vec{\imath} + (\vec{V}_{Oy}$$

$$\vec{V}_{tc,rear} = \vec{V}_{Ox}\vec{\imath} + (\vec{V}_{Oy}$$

$$\tan(\gamma_{front}) = \gamma_{front}$$

$$V_{Oy} + l_f\dot{\psi}$$

$$\tan(\gamma_{rear}) = \gamma_{rear}$$

$$-\frac{V_{Oy} - l_r\dot{\psi}}{}$$

$$\alpha_f = \delta_f - \gamma_{front} = \delta_f - \frac{V_{Oy} + l_f\dot{\psi}}{V_{Ox}} = \delta_f - \beta - \frac{l_f\dot{\psi}}{V}$$

$$\alpha_r = \delta_r - \gamma_{rear} = \delta_r - \frac{V_{Oy} - l_r\dot{\psi}}{V_{Ox}} = \delta_f - \beta + \frac{l_r\dot{\psi}}{V}$$

$$F_{yfl} = F_{yfr} \qquad F_{yrl} = F_{yrr}$$

Appendix D:Deriving the Angular Equation of motion with respect to the body centre of gravity (BCOG).

For the sprung mass the Inertia tensor is given as the following which is with respects to the centre of gravity of the body:

$$I_{sb} = \begin{bmatrix} I_{xxs} & 0 & I_{xzs} \\ 0 & I_{yys} & 0 \\ I_{zxs} & 0 & I_{zzs} \end{bmatrix}$$

The sprung mass can roll about x direction and pitch around y direction.

$$R = R_x * R_y = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ \sin(\theta)\sin(\phi) & \cos(\phi) & \cos(\theta)\sin(\phi) \\ \cos(\phi)\sin(\theta) & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix}$$

$$I_{sc} = \mathbb{R}\, I_{sb}\, \mathbb{R}^T$$

$$I_{sc11} = Ixx * cos(\theta)^2 + Izz * sin(\theta)^2$$

$$I_{sc12} = Ixx * cos(\theta) * sin(\phi) * sin(\theta) - Izz * cos(\theta) * sin(\phi) * sin(\theta)$$

$$I_{sc13} = Ixx * cos(\phi) * cos(\theta) * sin(\theta) - Izz * cos(\phi) * cos(\theta) * sin(\theta)$$

$$I_{sc21} = Ixx * cos(\theta) * sin(\phi) * sin(\theta) - Izz * cos(\theta) * sin(\phi) * sin(\theta)$$

$$I_{sc22} = Iyy * cos(\phi)^2 + Izz * cos(\theta)^2 * sin(\phi)^2 + Ixx * sin(\phi)^2 * sin(\theta)^2$$

$$I_{sc23} = Izz * cos(\phi) * cos(\theta)^2 * sin(\phi) - Iyy * cos(\phi) * sin(\phi) + Ixx * cos(\phi) * sin(\phi) * sin(\theta)^2$$

$$I_{sc31} = Ixx * cos(\phi) * cos(\theta) * sin(\theta) - Izz * cos(\phi) * cos(\theta) * sin(\theta)$$

$$I_{sc32} = Izz * cos(\phi) * cos(\theta)^2 * sin(\phi) - Iyy * cos(\phi) * sin(\phi) + Ixx * cos(\phi) * sin(\phi) * sin(\theta)^2$$

$$I_{sc33} = Izz * cos(\phi)^2 * cos(\theta)^2 + Ixx * cos(\phi)^2 * sin(\theta)^2 + Iyy * sin(\phi)^2$$

Mathematically, the time derivative of arbitrary vector q can be written as follows:

$$\frac{d\vec{q}}{dt} = \frac{d}{dt}(q\hat{e}) = \frac{dq}{dt}\hat{e} + q\frac{d\hat{e}}{dt}$$

$$\frac{d\vec{q}}{dt} = \dot{q}\hat{e} + \vec{\omega} \times \vec{q}$$

Then:

$$Angular\ Momentum: H = I_{sc}\omega = \begin{bmatrix} I_{sc11}\dot{\phi} + I_{sc12}\dot{\theta} + I_{sc13}\dot{\psi} \\ I_{sc21}\dot{\phi} + I_{sc22}\dot{\theta} + I_{sc23}\dot{\psi} \end{bmatrix}$$

$$\sum M = \frac{dH}{dt} = \frac{\partial H}{\partial t} + \overrightarrow{\omega_c} \qquad \overrightarrow{\omega_c} \qquad \omega = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix}$$

$$\frac{\partial H}{\partial t} = I_{sc}\dot{\omega}$$

$$\begin{bmatrix} I_{sc11}\ddot{\phi} + I_{sc12}\ddot{\theta} + I_{sc13}\ddot{\psi} \end{bmatrix}$$

$$\overrightarrow{\omega_c} \times \vec{H} = \begin{bmatrix} -\dot{\psi}(I_{sc21}\dot{\phi} + I_{sc22}\dot{\theta} + I_{sc23}\dot{\psi}) \\ \dot{\psi}(I_{sc11}\dot{\phi} + I_{sc12}\dot{\theta} + I_{sc13}\dot{\psi}) \end{bmatrix}$$

$$\sum M = \frac{d\hat{H}}{dt} = \frac{\partial H}{\partial t} + \overrightarrow{\omega_c} \times \vec{H} = \begin{bmatrix} I_{sc11}\ddot{\phi} + I_{sc12}\ddot{\theta} + I_{sc13}\ddot{\psi} - \dot{\psi}(I_{sc21}\dot{\phi} + I_{sc22}\dot{\theta} + I_{sc23}\dot{\psi}) \\ I_{sc21}\ddot{\phi} + I_{sc22}\ddot{\theta} + I_{sc23}\ddot{\psi} + \dot{\psi}(I_{sc11}\dot{\phi} + I_{sc12}\dot{\theta} + I_{sc13}\dot{\psi}) \\ I_{sc31}\ddot{\phi} + I_{sc32}\ddot{\theta} + I_{sc33}\ddot{\psi} \end{bmatrix}$$

After simplifications:

$$\begin{Bmatrix} \sin\phi \approx \phi & \cos\phi \approx 1 & \sin\theta \approx \theta & \cos\theta \approx 1 \\ \phi\theta \approx 0 & \dot{\phi}\dot{\theta} \approx 0 & \dot{\phi}^2 \approx 0 & \dot{\theta}^2 \approx 0 \end{Bmatrix}$$

$$M_x = (I_{xx} - 2I_{xz}\theta) * \ddot{\phi} + \ddot{\psi}[I_{xz} + \theta * (I_{xx} - I_{zz})] + \ddot{\theta}\phi * (I_{xz} - I_{zz}\theta)$$

$$M_y = I_{yy} * \ddot{\theta} + \ddot{\psi}\phi * (I_{zz}) + \dot{\phi}\phi * (I_{zx})$$

$$M_z = [I_{zz} + \theta * (2I_{xz})] * \ddot{\psi} + \ddot{\theta}\phi * (I_{zz} - I_{yy}) + \ddot{\phi}\theta * (I_{xx} - I_{zz}) + I_{zx} * \ddot{\phi}$$

Appendix E: Deriving the Angular Equation of motion with respect to the roll centre (RC).

For the sprung mass the Inertia tensor is given as the following which is with respects to the Roll Centre:

$$\vec{a}_s = l_s\ddot{\theta}\vec{k} + (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})\vec{j}$$

$$\vec{r}_s \times \vec{a}_s = -l_s\ddot{\theta}(l_s - h_s\theta)\vec{j} + (l_s - h_s\theta)(\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})\vec{k} + (l_s\ddot{\theta}(h_s\phi) + (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})(l_s\theta + h_s)\vec{i}$$

$$I_{sb} = \begin{bmatrix} I_{xxs} & 0 & I_{xzs} \\ 0 & I_{yys} & 0 \\ I_{zxs} & 0 & I_{zzs} \end{bmatrix} + \begin{bmatrix} M_s h_s^2 & 0 & M_s h_s l_{cgs} \\ 0 & M_s\left(h_s^2 + l_{cgs}^2\right) & 0 \\ M_s h_s l_{cgs} & 0 & M_s l_{cgs}^2 \end{bmatrix}$$

For the un-sprung mass, the Inertia tensor is given as the following which is with respects to the Roll Centre:

$$I_{usc}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & I_{zzuf}+I_{zzur}+M_{uf}l_f^2+M_{ur}l_r^2\end{bmatrix}$$

Assuming Constant tire contact rotational motion for an un-sprung body about any axis next to z axis can be neglected.

$$I_{tot,roll\ center} = RI_{sb}R^T + I_{UNSPRUNG}$$

$$\sum M = \frac{dH}{dt} = \frac{\partial H}{\partial t} + \overrightarrow{\omega_c} \times \vec{H} + Added\ Terms$$

Added Terms: Due to the moment created by the acceleration of masses.

The procedure to get to the momentum equation is same as previous Appendix. Since, terms are too many to be exposed simultaneously, MATLAB has been implemented to get to the result of $\frac{\partial H}{\partial t} + \overrightarrow{\omega_c} \times \vec{H}$ and then added terms will be added to it. Needless to say, it has been assumed to have symmetric geometry in all planes.

Added Terms:

Moment due to the acceleration of front un-sprung mass

$$\vec{a}_{uf} = \left(-\dot{\psi}\vec{V}_{0y}\right)\vec{\imath} + \left(\dot{\vec{V}}_{0y} + \dot{\psi}\vec{V}_{0x} + l_f\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{r}_{uf}$$

$$\vec{r}_{uf} \times M_{uf}\vec{a}_{uf} = \left(M_{uf}h_{uf}\dot{\psi}\vec{V}_{0y}\right)\vec{\jmath} + M_{uf}\left(l_f\dot{\vec{V}}_{0y} + l_f\dot{\psi}\vec{V}_{0x} + {l_f}^2\ddot{\psi}\right)\vec{k} + M_{uf}\left(h_{uf}\dot{\vec{V}}_{0y} + h_{uf}\dot{\psi}\vec{V}_{0x} + h_{uf}l_f\ddot{\psi}\right)\vec{\imath}$$

Moment due to the acceleration of rear un-sprung mass

$$\vec{a}_{ur} = \left(-\dot{\psi}\vec{V}_{0y}\right)\vec{\imath} + \left(\dot{\vec{V}}_{0y} + \dot{\psi}\vec{V}_{0x} - l_r\ddot{\psi}\right)\vec{\jmath}$$

$$\vec{r}_{ur}$$

$$\vec{r}_{uf} \times M_{ur}\vec{a}_{uf} = \left(M_{ur}h_{ur}\dot{\psi}\vec{V}_{0y}\right)\vec{\jmath} + M_{ur}\left(-l_r\dot{\vec{V}}_{0y} - l_r\dot{\psi}\vec{V}_{0x} + {l_r}^2\ddot{\psi}\right)\vec{k} + M_{ur}\left(-h_{ur}\dot{\vec{V}}_{0y} - h_{ur}\dot{\psi}\vec{V}_{0x} + h_{ur}l_r\ddot{\psi}\right)\vec{\imath}$$

Moment due to the acceleration of sprung mass

$$\vec{a}_s = l_s\ddot{\theta}\vec{k} + (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})\vec{\jmath}$$

$$\overrightarrow{r_s} = (l_s cos\theta - h_s cos\phi sin\theta)\vec{\imath} + (h_s sin\phi)\vec{\jmath} + (-l_s sin\theta - h_s cos\phi cos\theta)\vec{k}$$

Moment due to the acceleration of masses

X Direction:

$$M_{uf}\left(h_{uf}\dot{\vec{V}}_{0y} + h_{uf}\dot{\psi}\vec{V}_{0x} + h_{uf}l_f\ddot{\psi}\right) + M_{ur}\left(-h_{ur}\dot{\vec{V}}_{0y} - h_{ur}\dot{\psi}\vec{V}_{0x} + h_{ur}l_r\ddot{\psi}\right) M_s\left(l_s\ddot{\theta}(h_s\phi) + (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})(l_s\theta\right.$$

Y Direction:

$$M_{uf}h_{uf}\dot{\psi}\vec{V}_{0y} + M_{ur}h_{ur}\dot{\psi}\vec{V}_{0y} - M_s l_s\ddot{\theta}(l_s$$

Z Direction:

$$M_{uf}\left(l_f\dot{\vec{V}}_{0y} + l_f\dot{\psi}\vec{V}_{0x} + {l_f}^2\ddot{\psi}\right) + M_{ur}\left(-l_r\dot{\vec{V}}_{0y} - l_r\dot{\psi}\vec{V}_{0x} + {l_r}^2\ddot{\psi}\right) + M_s(l_s - h_s\theta)\,(\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta$$

After simplifications:

$$\begin{Bmatrix}\sin\phi\approx\phi & \cos\phi\approx 1 & \sin\theta\approx\theta & \cos\theta\approx 1\\ \phi\theta\approx 0 & \dot{\phi}\dot{\theta}\approx 0 & \dot{\phi}^2\approx 0 & \dot{\theta}^2\approx 0\end{Bmatrix}$$

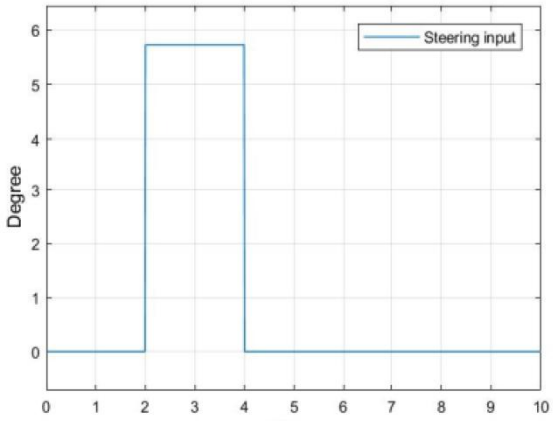
$$M_x = \ddot{\phi}(I_{xx} + M_s * h_s * h_s) + \ddot{\psi}\theta(I_{xx} - I_{zz} + M_s * h_s * h_s - M_s * l_s * l_s) + M_{uf}\left(h_{uf}\dot{\vec{V}}_{0y} + h_{uf}\dot{\psi}\vec{V}_{0x} + h_{uf}l_f\ddot{\psi}\right) + M_{ur}\left(-h_{ur}\dot{\vec{V}}_{0y} - h_{ur}\dot{\psi}\vec{V}_{0x} + h_{ur}l_r\ddot{\psi}\right) + M_s\left(l_s\ddot{\theta}(h_s\phi) + (\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x})(l_s\theta + h_s)\right) = c * \left(F_{zfl} - F_{zrf}\right) + d * (F_{zlr} - F_{zrr}) + h_o * (F_{ylf} + F_{yfr} + F_{yrl} + F_{yrr})$$

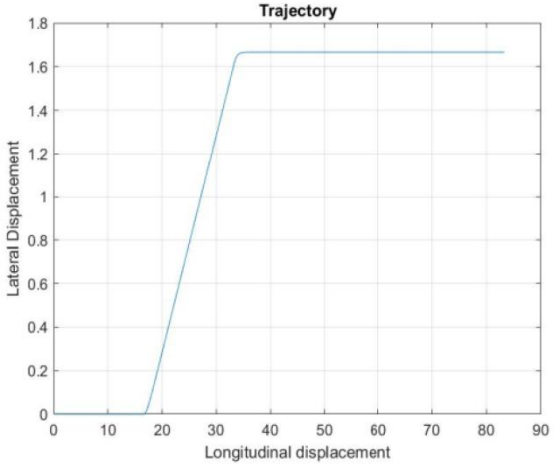
$$M_y = \ddot{\theta}(I_{yy} + M_s * h_s * h_s + M_s * l_s * l_s) + \ddot{\psi}\phi(I_{zz} - I_{yy} - M_s * h_s * h_s) + M_{uf}h_{uf}\dot{\psi}\vec{V}_{0y} + M_{ur}h_{ur}\dot{\psi}\vec{V}_{0y} - M_s l_s\ddot{\theta}(l_s - h_s\theta) = a * \left(F_{zfl} + F_{zfr}\right) - b * (F_{zrl} + F_{zrr})$$

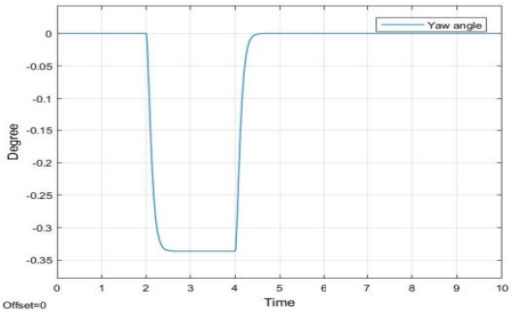
$$M_z = \left(I_{zzuf} + I_{zzur} + m_{uf} * {l_f}^2 + m_{ur} * {l_r}^2 + I_{zz} + M_s * l_s * l_s\right) * \ddot{\psi} + \phi\ddot{\theta}\left(I_{zz} - I_{yy} - M_s * h_s * h_s\right) + \ddot{\phi}\theta(I_{xx} - I_{zz} - M_s * h_s * h_s + M_{uf}\left(l_f\dot{\vec{V}}_{0y} + l_f\dot{\psi}\vec{V}_{0x} + {l_f}^2\ddot{\psi}\right) + M_{ur}\left(-l_r\dot{\vec{V}}_{0y} - l_r\dot{\psi}\vec{V}_{0x} + {l_r}^2\ddot{\psi}\right) + M_s(l_s - h_s\theta)\,(\dot{V}_{0y} + h_s\ddot{\phi} + l_s\ddot{\psi} - h_s\ddot{\psi}\theta + \dot{\psi}V_{0x}) = l_f\left[F_{ylf} + F_{yfr}\right] - l_r\left[F_{yrl} + F_{yrr}\right]$$

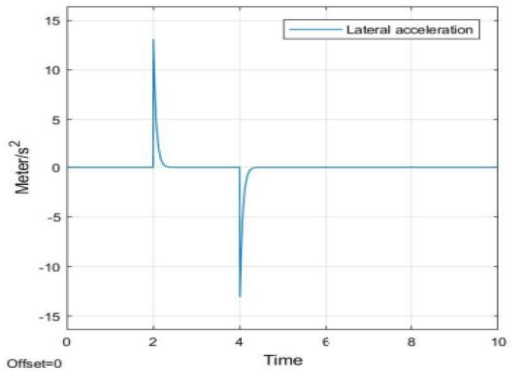
IV. RESULTS

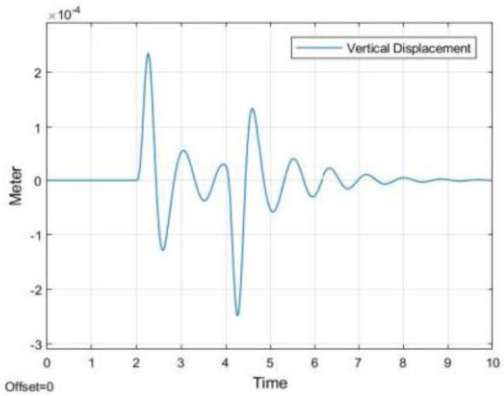
Data: Constant velocity at 30 km/hr Steering Input as a windowed step with 5.7 degree 4 WS RC Equation

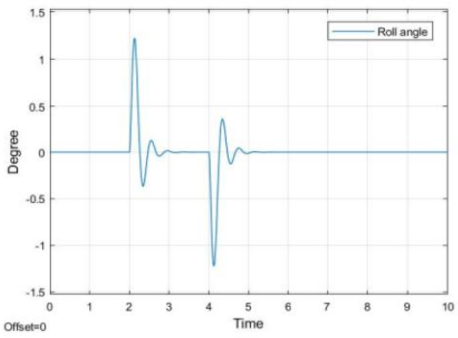


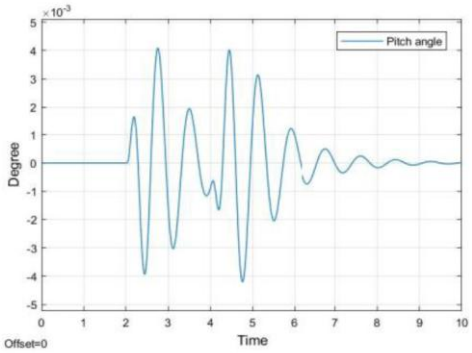










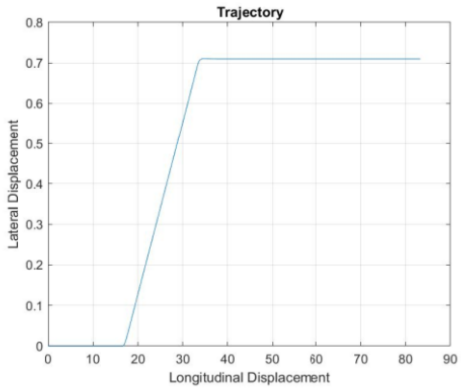


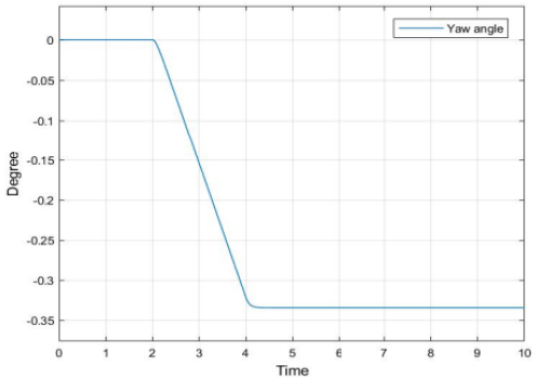
Data:

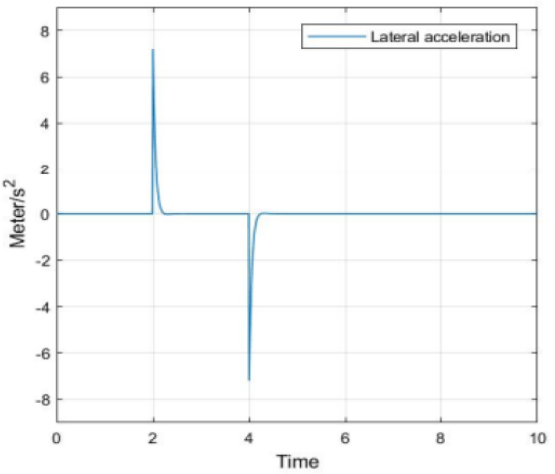
Constant velocity at 30 km/hr

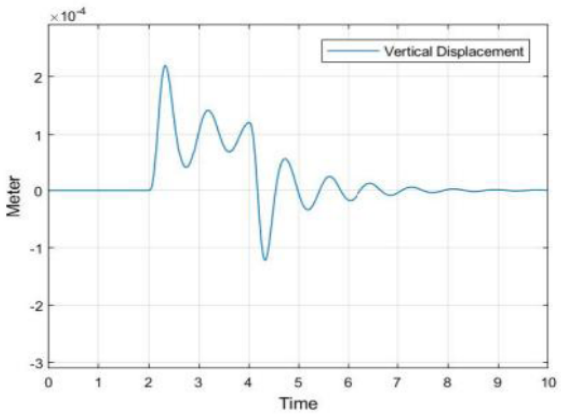
Steering Input as a windowed step with 5.7 degree

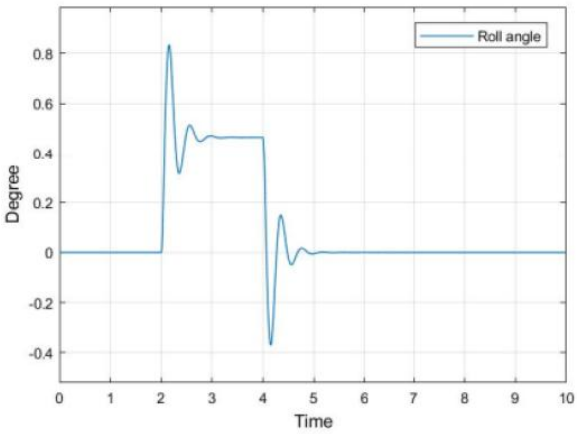
2 WS RC Equation

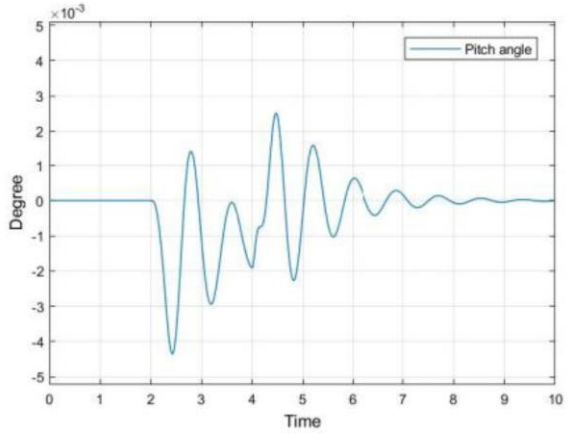












V. CONCLUSION

Yaw angle Diagram shows that a 4 WS will be shifted rather than being dragged as is the case in 2 WS. However, Trajectory shows a larger amount lateral displacement in 4WS.

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