NUMERICAL SOLUTIONS OF OPTIMAL CONTROL PROBLEMS FOR MICROWAVE HEATING

[1]AYDIN YILMAZ, [2]IBRAHIM MAHARIQ, [3]FIKRIYE YILMAZ

[1,2]University of Turkish Aeronautical Association, Department of Electrical and Electronics Engineering, Turkey
[3] Gazi University Department of Mathematics, Turkey
E-mail: [1]aydinyilmaz76@gmail.com, [2] imahariq@thk.edu.tr, [3] yfikriye@gmail.com

Abstract- In this work, we study numerical solutions of optimal control problem for a microwave heating. Microwave heating problem is modeled by Helmholtz equation coupled with a heat equation. At first, the necessary optimality conditions are obtained. Finite element method is used to discretize the state and the adjoint state equations. It is the first time that optimal control for the microwave heating is solved numerically. Numerical application is presented.

Keywords- Optimal Control, Helmholtz Equation, Finite Element, Microwave Heating.

I. INTRODUCTION

Optimal control problems have been an interest in recent decades. There are several applications in physics, finance and engineering science. One of the applications from electromagnetics is the optimal control of microwave heating. Uniform distribution of the heat produced by microwaves has been challenging recently. In [9], a suitable applied electric field was selected and they formulated an optimal control problem governed by Maxwell's equations coupled with nonlinear heat conduction with a source generated by microwaves.
In this paper, the numerical solution of the optimal control problem of microwave heating is studied. As for constraint equations, we consider the Helmholtz equation coupled with a heat equation at equilibrium. We control the applied field to get a desired temperature. Here, the function based approach optimize-then-discretize is followed to get the optimality conditions. Our aim is solve the modeled problem numerically. So that, we use finite element approach for space discretization. Freefem++ is used to get the numerical results [3].

The organization of the paper is as follows: Firstly, optimal control problem is formulated. Then, the first order optimality conditions are obtained by Lagrange method. The finite element method is used for the discretization of the optimal control problem. Finally, we conclude our study with a numerical example..

II. PROBLEM FORMULATION

We define Ω be a bounded polygonal domain in $\mathbb{R}^d$ with $d = 2,3$ and its Lipschitz boundary be $\Sigma_1 \cup \Sigma_2 = \partial\Omega$. We consider the following optimal control problem:

$$\min_{(v,u)} \mathbb{J}(v,u) = \frac{1}{2}\int_\Omega (v(x) - v_d(x))^2 dx + \frac{\alpha}{2}\int_{\Sigma_2} \big(u(x)\big)^2 ds \qquad (1)$$

subject to

$$\begin{aligned} -\nabla^2 y - \omega^2 y &= 0 \ \text{in } \Omega, \\ \frac{\partial y}{\partial n} &= u(x) \text{ on } \Sigma_2, \\ y &= 0 \text{ on } \Sigma_1, \\ y &= g(x) \text{ on } \Sigma_2, \\ -\Delta v &= \frac{1}{2}\beta v^2 \text{ in } \Omega, \\ v &= 0 \text{ on } \Sigma_1 \cup \Sigma_2, \end{aligned} \qquad (2)$$

where the function $g(x)$ is given, $y : \Omega \to \mathbb{R}^d$ is the state variable and u is the control variable. ω denotes the frequency of the harmonic oscillations. Here, $\alpha > 0$ stands for the regularization parameter and v_d is the desired temperature.

We use the standard notations for Sobolev and Lebesgue spaces, their norms and inner products. We let be $Y = H^1(\Omega)$, $Y = H_0^1(\Omega)$, and the control space $U = L^2(\Omega)$.

III. OPTIMALITY CONDITIONS

In this section, we will state the optimality conditions of the control problem of Helmholtz equation coupled with a heat quation. We use Lagrange approach. We have the following optimality conditions:

$$\begin{aligned} -\Delta w &= v(x) - v_d(x) \text{ in } \Omega, \\ w &= 0 \text{ on } \Sigma_1 \cup \Sigma_2, \\ \nabla^2 p + \omega^2 p &= ypw \ \text{in } \Omega, \\ \frac{\partial p}{\partial n} &= 0 \text{ on } \Sigma_2, \\ p &= 0 \text{ on } \Sigma_1, \end{aligned}$$

and

$$(\mathrm{Re}(\alpha u + p), \bar{u} - u)_{\Sigma_2} \geq 0 \ \forall \bar{u} \in U.$$

IV. FINITE ELEMENT DISCRETIZATION

In order to state the finite element discretization of the problem, we let $Y^h \subseteq Y$, $V^h \subseteq V$ and $U^h \subseteq U$ be the finite element spaces. Find $y^h \in Y^h$, $v^h \in V^h$ and $u^h \in U^h$ such that

$$\min_{(v^h,u^h)} \mathbb{J}(v^h,u^h) = \frac{1}{2}\|v^h - v_d\|_\Omega^2 + \frac{\alpha}{2}\|u^h\|_{\Sigma_2}^2$$

subject to

$$-(\nabla y^h, \nabla q^h) + (\omega y^h, q^h) = 0 \quad \forall\, q^h \in Y^h,$$
$$(\nabla v^h, \nabla l) - \frac{1}{2}(\beta y^h y^h, q^h) = 0 \quad \forall\, l^h \in V^h,$$
$$(\nabla w^h, \nabla m^h) + (v^h - v_d, m^h) = 0 \quad \forall\, m^h \in V^h,$$
$$-(\nabla p^h, \nabla r^h) + (\beta y^h w^h, r^h) = 0 \quad \forall\, r^h \in Y^h.$$

V. IMPLEMENTATION AND NUMERICAL EXAMPLE

In this section, we will carry out some numerical experiments.

We use nonlinear conjugate gradient method to solve the optimization problem. All computations are carried out with public license finite element software package Freefem++ [3].

We choose a tolerance as $\varepsilon = 10^{-5}$ for the Newton linearization. As a line search strategy, Armijo's rule is used. The gradient descent algorithm is stopped if two successive iterates are close enough, that is $\|u^h(n) - u^h(n-1)\|^2_{\Sigma_2} \leq tol = 10^{-7}$.

EXAMPLE

In this example, we consider a model of a microwave cooker. We have a control parameter to obtain a desired temperature in the domain. We consider a rectangular oven where the wave is emitted by part of the upper wall. Boundary of the domain is made up a part of Σ_1 and and the other part is $\Sigma_2 = [c, d]$ [3]. We choose the regularization parameter $\alpha = 0.01$. Boundary function is given by

$$g(x) = \sin\pi\left(\frac{y-c}{d-c}\right).$$

.

In the following figures, we present the state, the adjoint state, the temperature and the adjoint counterpart of the temperature are presented.

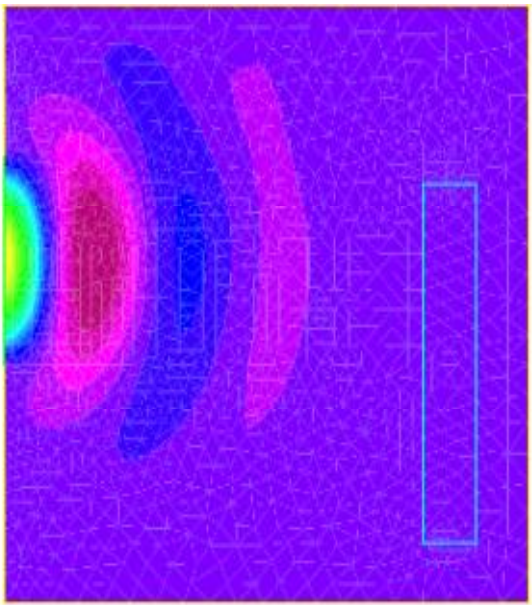


Fig1. Real part of the state

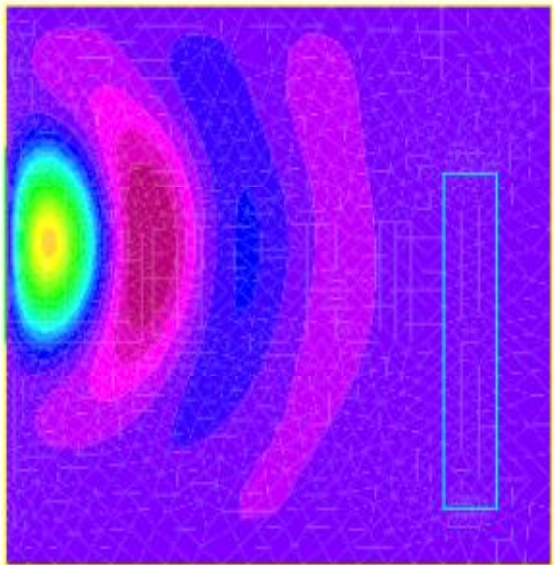


Fig2. Imaginary part of the state

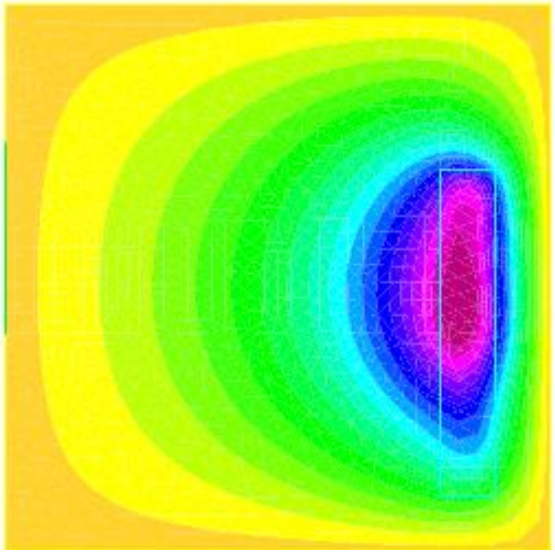


Fig3. Temperature

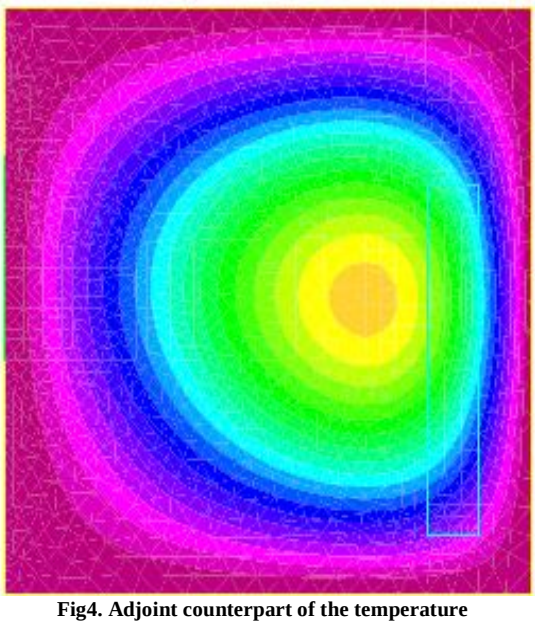


Fig4. Adjoint counterpart of the temperature

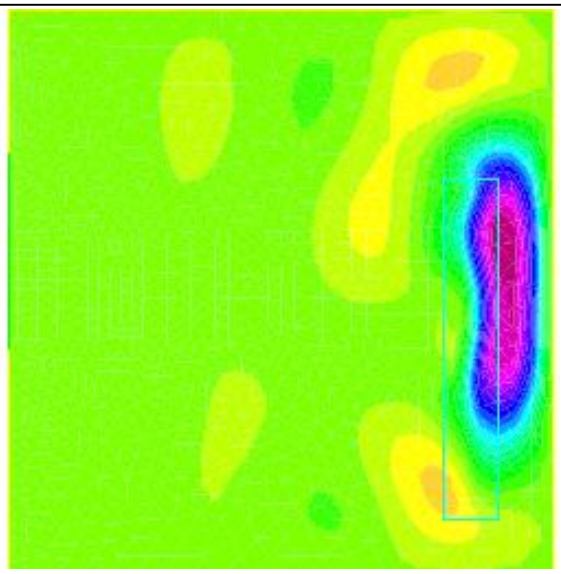


Fig5. Real part of the adjoint state

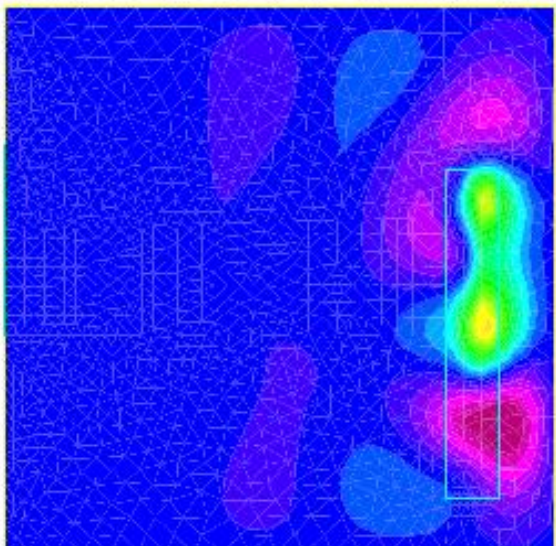


Fig6. Imaginary part of the adjoint state

CONCLUSION AND OUTLOOK

In this work, we have formulated an optimal boundary control of the microwave heating. We have obtained the optimality conditions by using the Lagrangian function. We have given the numerical results obtained by using freefem++ package.

As a future work, we shall consider error analysis for the control problems of the Helmholtz equation. Also, domain decomposition method can be used to solve optimal control of the microwave heating problem.

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