ON GENERALIZED N-SYSTEMS IN NEAR-RINGS

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Abstract - In this study, we define n_c-system andn_3-system which are generalizations of n-systemsin rings by extending c-semiprimeand 3-semiprime N-ideal concepts in near-rings. We investigate the connection of ann_c-system(n_3-system)andthe c-semiprime (3-semiprime) ideal.

Keywords - M-system, N-system, Near-Ring, Prime Ideal.

I. INTRODUCTION

A special kind of generalized rings, near-rings, was first introduced by Dickson [1] in 1905. Dickson proved the existence of near objects that satisfies the distributive property in one direction. Near-rings are algebraic structures such that first operation on the ring is not required to be commutative and distribution of second operation over first operation is one direction [2]. Because of these two properties, many concepts known in rings differed in near-rings. In particular, whereas there is only one type of prime definition in rings, there are many definitions in near-rings about primes. First studies on primeness have been carried out by many researchers [3-10].

Juglal, Groenewald and Lee [12] generalized the various notions of primeness that were defined in near-ring to the near-ring module.Severaldefinitions of primeness have been proposed for near-rings and near-ring modules [11-21].

In rings, there is only a one definition for$m$-system. However, in near-rings, there are many definitions of m-system for near-ring and near-ring modules. First, we give these definitions and relations between m-systems and the prime ideals. Likewise, there is a definition of$n$-system in rings.

In this study, we define n_c-system and n_3-system which are generalizations of n-systemsin rings by extending c-semiprime and 3-semiprime N-ideal concepts in near-rings. We investigate the connection of an n_c-system(n_3-system) and the c-semiprime (3-semiprime) ideal.

II. PRELIMINARY DEFINITONS

Nwill always denote aright near-ring. We assume that the reader is familiar with the basic definitions of right near-ring, zero-symmetric near-ring and ideal.

A. Prime Ideals

Definition 1:

We recall from Holcombe [7], Groenewald [11] and Booth et al. [9] that an ideal $P \lhd N$ is v-prime, if for $A, B \subseteq N$ with:

A, Bideals of N if $v = 0$
A, Bleft ideals of N if $v = 1$
A, B N-subgroups of Nif $v = 2$,

the inclusion $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

An ideal P of a near-ring N is called a 3-prime ideal if for all $a, b \in N$, $aNb \subseteq P$ implies $a \in P$ or $b \in P$ .If for all$a, b \in N$, $ab \in P$ implies $a \in P$ or $b \in P$, then $P \lhd N$ is called a completely prime ideal [8]. $P \lhd N$ is called an equiprime ideal, if $a \in \mathrm{N}\backslash P$ and $x, y \in N$ are such that $anx - any \in P$ for all $n \in N$, then $x - y \in P$ [9].If the zero ideal of N is v-prime$(v = 0,1,2,3, c, e)$, then$N$ is called a v-prime near-ring.It is already known that if $P \lhd N$, then P is completely prime $\Longrightarrow P$ is 3-prime $\Longrightarrow P$ is 2-prime, P is 1-prime $\Longrightarrow P$ is 0-prime and P is 2-prime $\Longrightarrow P$ is 1-prime when N is zero-symmetric. Furthermore, any equiprime ideal is 3-prime [13].

B. N-groups and N-ideals

Definition2:
Let $(\Gamma, +)$be a group and$a, b \in N, \gamma \in \Gamma$,
Given:

$$\delta: N \times \Gamma \to \Gamma$$
$$(a, \gamma) \to a\gamma$$

if $(a + b)\gamma = a\gamma + b\gamma$ and $(ab)\gamma = a(b\gamma)$ is satisfied, thenthe pair $(\Gamma, \delta)$is called an*N-group*(near-ring module).
A normal subgroup P of an N -group Γ is called an N *–ideal*of $(P \lhd_N \Gamma)$ if $\forall \gamma \in \Gamma, \forall p \in P, \forall n \in N: n(\gamma + p) - n\gamma \in P$[2, 21].

Throughout this paper, Nwill denote a zero-symmetric near-ring and Γ an N-group.

C. Prime N-ideals

Definition3:
Let $P \lhd_N \Gamma$ be such that $N\Gamma \nsubseteq P$.Then P is called [12]:

a) **0-prime** if for an ideal A of N and an N-ideal $B$of Γ, $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
b) **1-prime** if for a left ideal A of N and an N-ideal $B$of Γ, $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
c) **2-prime** if for a left N-subgroup A of N and an N -subgroup B of Γ , $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
d) **3-prime** if $nN\gamma \subseteq P$ implies that $n\Gamma \subseteq P$ or $\gamma \in P$ for $n \in N$and $\gamma \in \Gamma$.
e) **completely prime (c-prime)** if$n\gamma \in P$ implies that $n\Gamma \subseteq P$ or $\gamma \in P$ for $n \in N$ and $\gamma \in \Gamma$.

$\Gamma$is said to be av-prime N-group $(v = 0,1,2,3, c)$ if$N\Gamma \neq 0$ and 0 is a v-prime N-ideal of Γ.

We state the following results from [12] which will be used later in the paper.

Proposition 1:

Let $P \lhd_N \Gamma$. Then P is completely prime $\Longrightarrow$ P is 3-prime $\Longrightarrow$ P is 2-prime $\Longrightarrow$ P is 0-prime.

Corollary 1:

If Γ is an N-group, then Γ is completely prime $\Longrightarrow$ Γ is 3-prime $\Longrightarrow$ Γ is 2-prime $\Longrightarrow$ Γ is 0-prime.

As in the case of prime N-ideals, the concept of semiprimeness is generalized from N to N-group Γ.

Definition4:

Let $P \lhd_N \Gamma$ be such that $N\Gamma \nsubseteq P$.Then P is called [12]:

a) **0-semiprime** if $A^2\Gamma \subseteq P$ implies that $A\Gamma \subseteq P$ for all ideals A of N;
b) **1-semiprime** if $A^2\Gamma \subseteq P$ implies that $A\Gamma \subseteq P$ for all left ideals A of N;
c) **2-semiprime** if $A^2\Gamma \subseteq P$ implies that $A\Gamma \subseteq P$ for all N-subgroups A of N;
d) **3-semiprime** if $aN a\gamma \subseteq P$ implies that $a\gamma \in P$ for all $a \in N$ and $\gamma \in \Gamma$;
e) **Completelysemiprime(c-semiprime)** if $a^2\gamma \in P$ implies that $a\gamma \in P$ for all $a \in N$ and $\gamma \in \Gamma$.

$\Gamma$is said to be a v-semiprime N-group $(v = 0,1,2,3,c)$ if $N\Gamma \neq 0$ and 0 is a v-semiprime N-ideal of Γ.

Proposition2:

Let P be an N-ideal of Γ. Then P is completely semiprime $\Longrightarrow$ P is 3-semiprime $\Longrightarrow$ P is 2-semiprime $\Longrightarrow$ P is 1-semiprime $\Longrightarrow$ P is 0-semiprime.

D. M-systems

An m-system is used to characterize prime ideals.

In this part we first give m-systems in rings and then in near-rings and near-ring modules.

Definition5:

A nonempty subset T of R is said to be an m-system (generalized multiplicative system) if $c, d \in T \Longrightarrow cxd \in T$ forsome $x \in R$[18].

Proposition 3:

Let R be a ring and $P \lhd R$. $P$is a prime ideal of Rif and only if $R\backslash P$ is an m-system.

M-systems in near-rings has been defined similarly to the$m$-systems in rings as following:

Definition6:

Let N be a near-ring and $X \subseteq N$, then X is called an m-*system* if for all $a, b \in X$, there exists $a_1 \in \langle a \rangle$ and $b_1 \in \langle b \rangle$ such that $a_1 b_1 \in X$ [2].

Example1:

The empty set $\emptyset$ and near-ring N are m-systems.

For all $n \in N$, the set $\{n, n^2, \dots\}$ is an m-system.

Proposition 4:

Let N be a near-ring and $P \lhd N$. P is a prime ideal of Nif and only if $N\backslash P$ is an m-system.

Furthermore, there are many definitions in near-rings about m-systems[19].

Definition7:

Let $I \lhd N$ and $G \subseteq N$ be such that $G \cap I = \{0\} \vee \emptyset$.

In this case$(G, I)$is called an:

i. m_1-system if$x \in N\backslash I$ and$g \in G \Rightarrow \langle g\rangle\langle x\rangle \cap G \neq \emptyset$.
ii. m_2-system if $x \in N\backslash I$ and $g \in G \Rightarrow [x]_N[g]_N \cap G \neq \emptyset$.
iii. m_3-system if$x \in N\backslash I$ and$g \in G \Rightarrow xNg \cap G \neq \emptyset$.
iv. m_e-system if $x, y \in N$, $a \in G, x - y \in N\backslash I \Rightarrow anx - any \in G$ for some $n \in N$.

Proposition 5:

An ideal$P$ of N is v-prime$(v = 1,2,3,e) \Leftrightarrow (N\backslash P,\ P)$ is an m_v-system.

Likewise, an m-system can be defined and used tocharacterize prime N-ideals.

Definition8:

Let $X \subseteq \Gamma$ where Γ is an N-group. Then X is called an:

a) m_0-system if $0 \neq n \in N$ and $0 \neq x \in X$ implies that $n\langle x\rangle_N \cap X \neq \emptyset$.(Recall that $\langle x\rangle_N$ is the N-ideal of Γ generated by x).
b) m_2-system if $0 \neq n \in N$ and $0 \neq x \in X$ implies that $n[x]_N \cap X \neq \emptyset$. (Recall that $\langle x\rangle_N$ is the N-subgroup of Γ generated by x).
c) m_3-system if $0 \neq n \in N$ and $0 \neq x \in X$ implies that $nNx \cap X \neq \emptyset$.
d) m_c-system if $0 \neq n \in N$ and $0 \neq x \in X$ implies that $nx \in X$ [17].

Proposition 6:

Let $v = 0,2,3,$ and$P \lhd_N \Gamma$. Then P is a v-prime if and only if $\Gamma\backslash P$ is an m_v-system [12].

Thus far, we only provided background information (i. e. definitions and propositions). In this section, we first introduce two new definitions which we calledn_c-system and n_3-system. Then we prove that if $\Gamma - P$ is an n_c-system (n_3-system), then P is c-semiprime(3-semiprime)N-ideal.

III. N-SYSTEMS

An m-system was used to characterize prime ideals. Likewise, an n-system can be defined and used to characterizesemiprime ideals. Firstly, we give the definition of n-system in rings.

Definition9:

Anonempty subset X of R is said to be an n-system if $0 \notin X$ and if for each $a \in X$ there is an$r \in R$ such that $ara \in X$[18].

Proposition7:

Aproper ideal P of R is semiprime if and only if $R\backslash P$ is an n-system[18].

Now we are ready to introduce the n_c-system and n_3-system.

Definition10:

Let Γ be an N-group and $X \subseteq \Gamma$. If $0 \neq n \in N$ and $0 \neq x \in X$ implies that $n^2x \in X$ then X is called ann_c-system.

Proposition8:

Let $P \lhd_N \Gamma$. If $\Gamma - P$ is an n_c-system, then P is c-semiprime$N$-ideal.

Proof:

Assume, $\Gamma - P$ is ann_c-system. $0 \neq n \in N$and$0 \neq x \in \Gamma$ such that $n^2x \in P$. Suppose that $x \notin P$. Then, $0 \neq x \in \Gamma - P$. Since $\Gamma - P$ is ann_c-system, $n^2x \in \Gamma - P$. This implies that $n^2x \in P$. Therefore, $x \in P$. Since P is an N-ideal, $nx \in P$. So, P is c-semiprime.

Definition11:

Let Γ be an N-group and $X \subseteq \Gamma$. If $0 \neq n \in N$ and $0 \neq x \in X$ implies that $nNnx \cap X \neq \emptyset$, then X is called an n_3-system.

Proposition9:

Let $P \lhd_N \Gamma$. If $\Gamma - P$ is an n_3-system, then P is 3-semiprime$N$-ideal.

Proof:

Suppose that $\Gamma - P$ is an n_3-system. $0 \neq n \in N$ and $0 \neq x \in \Gamma$ such that $nNnx \subseteq P$. Suppose that $x \notin P$. Then, $0 \neq x \in \Gamma - P$. Since $\Gamma - P$ is an n_3-system, $nNnx \cap \Gamma - P \neq \emptyset$. Then, there existsan $a \in nNnx \cap \Gamma - P$. Thus, $a \in nNnx$ and $a \in \Gamma - P$. Hence, $a \notin P$ which contradicts with $nNnx \subseteq P$. So, $x \in P$. Since P is an N-ideal, $nx \in P$. Therefore, P is 3-semiprime.

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