APPLICATION OF QUADRATIC EQUATIONS, (AREA OF SECTOR), PROPERTIES OF CIRCLE, TRIGONOMETRY AND DILUTION IN PRODUCTION ENGINEERING AND MECHANICAL ENGINEERING

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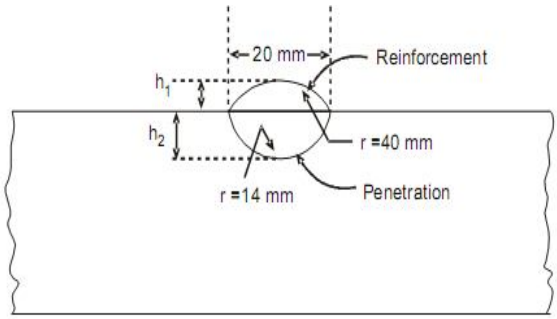
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Abstract- In engineering, welding plays important role. Without use of external heat we get success in welding of Gold and Silver only till today but in future the use of temperature in welding may be reduced considerably. By the application of heat source we are depositing the liquid metal at the joint. The depth up to which the material is liquified is called penetration and there will be raised part of the plane of parent metal called reinforcement. In ideal cases the dilution is considered as 100%. In this paper we are going to use the quadratic equations, area of a sector (properties of circle), trigonometry and dilution in the welding concept (for production engineering and mechanical engineering).

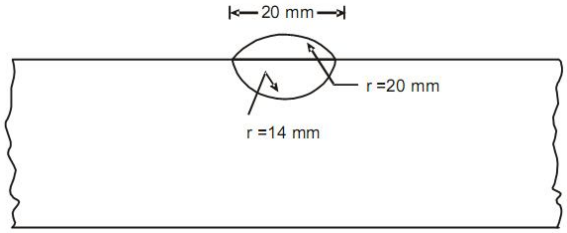
Key Words- External heat, considerably, liquified, penetration, reinforcement, dilution.

I. INTRODUCTION

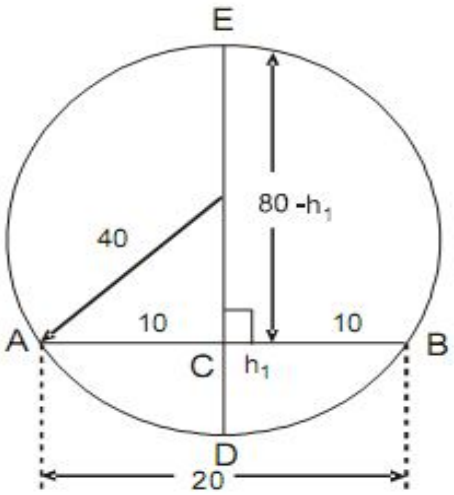
Let take an example, the cross-section of weld bead is shown in below fig. The profile of the bead and the fusion zone are taken circular for convenience. Bead width and radius of the curvature of circular profiles are given in for the figure. Find



(i) the bead height
(ii) penetration
(iii) dilution %



According to Figure :
Reinforcement is a part of circular of radius 40mm we known that perpendicular drawn from the centre on the chord bisects the chord because of the properties of circle.
In circle, AB = 20 mm



$$\therefore \quad \frac{28 \pm \sqrt{784-400}}{2} = \frac{28 \pm \sqrt{384}}{2}$$

$$\therefore \quad AC = \frac{1}{2} \times AB \Rightarrow \frac{1}{2} \times 20 = 10 \text{ mm}$$

and BC = 10 mm

We have

$AC \times BC = DC \times EC$

$\Rightarrow \quad 10 \times 10 = h_1 \times (80 - h_1)$

$\Rightarrow \quad 100 = h_1 \times (80 - h_1)$

$\Rightarrow \quad 100 = -h_1^2 + 80h_1$

or $\quad h_1^2 - 80h_1 + 100 = 0 \qquad ...(1)$

Equation (i) is called an quadratic equation,

Let us find the roots of h_1

$\therefore$ Here $a = 1, b = -80, c = 100$

$\therefore$ According to formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad h_1 = \frac{80 \pm \sqrt{(80)^2 - 4(1)(100)}}{2(1)}$$

$$\therefore \; h_1 = \frac{-80 \pm \sqrt{6400-400}}{2} = \frac{80 \pm \sqrt{6000}}{2}$$

$$\Rightarrow \quad h_1 = \frac{80 \pm 72.45}{2}$$

$$h_1 = \frac{80+72.45}{2} = \frac{152.45}{2} = 76.22$$

$$h_1 = \frac{80-72.45}{2} = \frac{7.55}{2} = 3.77$$

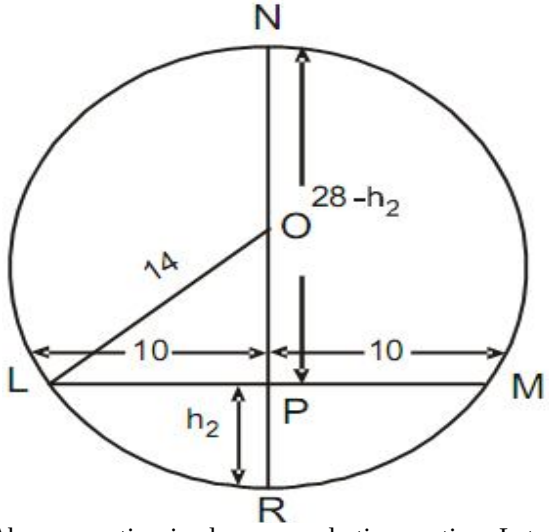
Here h_1 = 76.22 mm, 3.77 mm
These are roots of h_1
Similarly, penetration is also a part of the circle of radius 14 mm.
Using the properties of circle, we have

$(LP) \times (PM) = (P \times R) \times (P \times N)$

$10 \times 10 = h_2 \times (28 - h_2)$ $[\because h_2 = P \times R]$

$[\because P \times N = 28 - h_2]$

$$\Rightarrow \quad 100 = 28\ h_2 - h_2^2$$
$$\text{or} \quad 28\ h_2 - h_2^2 - 100 = 0$$
$$\text{or} \quad -(h_2^2 - 28h_2 + 100) = 0$$
$$\text{or} \quad h_2^2 - 28h_2 + 100 = 0$$



Above equation is also an quadratic equation, Let us find the roots of h_2.

$$\therefore \text{ Here } a = 1, b = -28, c = 100$$

$$\therefore \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(100)}}{2 \times 1}$$

$$\therefore \quad \frac{28 \pm \sqrt{784 - 400}}{2} = \frac{28 \pm \sqrt{384}}{2}$$

$$h_2 = \frac{28 \pm 19.60}{2} \Rightarrow \frac{28 + 19.60}{2} = 23.8 \text{mm}$$

$$h_2 = \frac{28 - 19.60}{2} = \frac{8.4}{2} = 4.2 \text{ mm}$$

According to first condition,

Bead height = $h_1 + h_2 = 3.77 + 4.2 = 7.97$ mm

According to second condition,

Penetration = h_2 = 4.2 mm Answer.

According to third condition,

$$\text{Percentage dilution} = \left[\frac{A_P}{A_P + A_R}\right] \times 100$$

where A_P = Penetration area

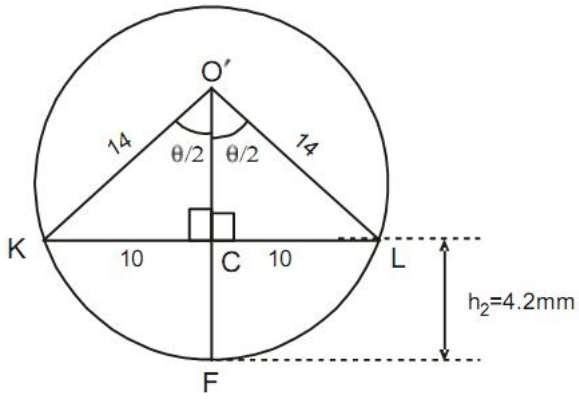
A_R = Reinforcement area

Because penetration is a part of circle of radius 14 mm.
Using the triangle $\Delta O'LC$,

$$\sin\frac{\theta}{2} = \frac{EL}{O'L} = \frac{10}{14}$$

$$\text{or} \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{10}{14}\right)$$

$$\theta = 91.16^\circ$$



$$\text{Now area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{91.16}{360} \times \frac{22}{7} \times (14)^2$$

$$= \frac{91.16}{360} \times \frac{22}{7} \times 196$$

$$= \frac{393081.92}{2520} = 155.98 \text{mm}^2$$

Area of $\Delta KO'L$, $= \frac{1}{2} \times$ Base $\times$ Perpendicular

or $= \frac{1}{2} \times KL \times O'E$

$= \frac{1}{2} \times 20 \times 9.8$

$[\because O'E = O'F - FE = 14 - 4.2 = 9.8 \text{ mm}]$

Penetration area = area of sector (O'KFL)

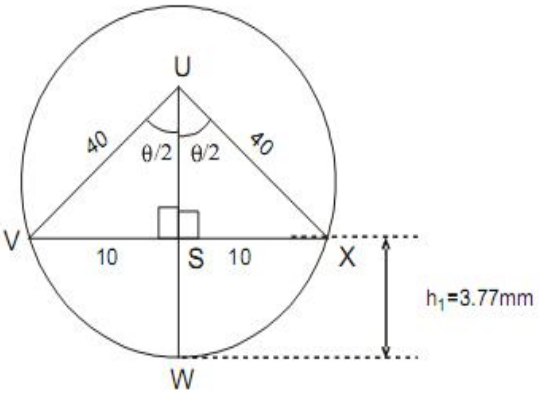
– Area of $\Delta KO'L$

$A_p = 155.98 - 98 = 57.98 \text{ mm}^2$

$\therefore \quad A_p = 57.98 \text{ mm}^2$

Reinforcement is a part of circle of radius 40 mm.,

Using ΔUXS,



$$\sin\frac{\theta}{2} = \frac{SX}{UX} = \frac{10}{40} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{1}{4}\right)$$

$$\theta = 28.95^\circ$$

$$\text{Area of Sector (UVWX)} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{28.95}{360} \times \frac{22}{7} \times (40)^2$$

$$= \frac{28.95}{360} \times \frac{22}{7} \times 1600$$

$$= \frac{1019040}{2520} = 404.38 \text{ mm}$$

$$\text{Area of } \Delta UVX = \frac{1}{2} \times \text{Base} \times \text{Perpendicular}$$

$$= \frac{1}{2} \times VX \times SU$$

$$= \frac{1}{2} \times 20 \times 36.23$$

$[\because SU = UW - SW = 40 - 3.77 = 36.23 \text{ mm}]$

$= 362.3 \text{ mm}^2$

Now, Reinforment area

(A_R) = (Area of Sector UVWX) – (Area of Δ UVX)

$A_R = 404.38 - 362.3$

$\therefore \quad A_R = 42.08 \text{ mm}^2$

$$\text{Percentage of dilution} = \left[\frac{A_P}{A_P + A_R}\right] \times 100$$

$$= \left[\frac{57.98}{57.98 + 42.08}\right] \times 100$$

$$= \frac{5798}{100.06} = 57.94$$

We get % dilution 57.94%

Conclusion : From above discussion, we conclude that mathematics is a core subject in any branch of engineering.

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