SLIDING SECTOR CONTROLLER FOR MISSILE AUTOPILOT DESIGN

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Abstract - The paper introduces a design method for sliding sector controller for a class of nonlinear systems. For this purpose, nonlinear sliding surfaces are formed from the solutions of the so-called State-Dependent Riccati Equation (SDRE) based sliding sector controller (SSC) method. Therefore, nonlinear sliding sectors are built which are the sectors covering the nonlinear sliding surfaces. The SSC is designed such that the nonlinear system states are forced to inside the sliding sector and track the desired states. The method is illustrated by applying it to a nonlinear missile model in order to design an acceleration autopilot.

Keywords - Sliding Surfaces, Robust Control, Sliding Sector Control, Nonlinear Systems, Missile Autopilot.

I. INTRODUCTION

Nonlinear Control Methods (NMC) are getting more popular and these methods are applied to numerous systems [1]–[4]. Sliding Mode Control (SMC) is one of the robust control techniques which is insensitive to some kinds of model errors, disturbances, and uncertainties. For that reason various studies have been carried out on SMC [5]–[7]. Although SMC has robustness property, it has the so-called chattering problem which is studied by researchers in order to eliminate this undesired phenomenon appeared in the SMC (see [[8], [9]]). As a desire of canceling chattering problem, a new method is proposed, which is called Sliding Sector Controller (SSC),by various researchersfor linear and nonlinear systems [10], [11]. Rather than forcing the system trajectories on a hyperplane as in the case of SMC, the system trajectories, in the SSC, are bounded inside a sector. Therefore, SSC creates smoother control signal, as the system is free inside the sector. Because of that, no control action is needed inside the sector ([12]). SSCs are applied on different models such as an inverted pendulum, missile, etc. ([8], [13]).
In this paper, a State-Dependent Riccati Equation (SDRE) based SSC is proposed for a missile autopilot. SDRE method ensures a solution to the design of a nonlinear system controller by freezing the nonlinear system at some certain time periods ([12]). Researchers ([13]–[16]) carried out various studies on state-dependent nonlinear control, sliding mode and sliding sector control on the missile. This paper combines the SDRE and SSC in order to have an effective design methodology for a general nonlinear dynamical system and extends the method for a missile autopilot design.

The paper is organized as follows; in section 2, SSC design method for nonlinear systems is reviewed. Missile dynamics and SSC Design are introduced in Section 3. Section 4 presents the simulation results. The conclusions are given in Section 5.

II. SSC DESIGN METHOD FOR NONLINEAR SYSTEMS

In this section, the mathematical backgrounds of the SDRE based SSC technique is reviewed. Consider the following types of single input nonlinear systems,

$$\dot{x}(t) = f(x,t) + g(x,t)u(t) \quad (1)$$

where $x \in R^n$ and $u \in R^1$ are state and input vectors and $f(x,t) \in R^n$ and $g(x,t) \in R^n$ are nonlinear functions. The system may be interpreted as follows,

$$\dot{x}(t) = A(x,t)x + B(x,t)u(t) \quad (2)$$

where $A(x,t) \in R^{n\times n}$ and $B(x,t) \in R^{1\times n}$ are State Dependent Coefficient (SDC) matrices. It is assumed that $\big(A(x,t), B(x,t)\big)$ is a point-wise controllable pair for all x and t.
The sliding sector design for Linear Time Invariant systems is well-defined in ([11]). To start with the design, the methodology is extended to nonlinear systems in this section. Consider the following candidate Lyapunov function for the nonlinear system,

$$V(t) = x^T(t)P(x,t)x(t) > 0 \quad (3)$$

When states to be controlled enter the sector without control effort,

$$\frac{d}{dt}(V(t)) = \frac{d}{dt}(x^T(t)P(x,t)x(t)) < -x^T(t)R(x,t)x(t) \quad (4)$$

The sliding sector, thatsatisfies the condition above, can be defined as,

$$\delta(x,t) = \{x \,||s(x,t)| \le \delta(x,t)\} \quad (5)$$

For the stability,

$$P(x,t)A(x,t) + A^T(x,t)P(x,t) - P(x,t)B(x,t)B^T(x,t)P(x,t) = -\bar{Q}(x,t) \quad (6)$$

the State-Dependent Riccati Equation (SDRE) equation should be solvedfor a positive definite P matrix.

The sliding surface can be determined from equation (6) given a positive semi-definite $\overline{Q}(x,t)$matrix. Now consider the sliding surface function defined by

$$s(x,t) = S(x,t)x(t) \quad (7)$$

$S(x,t)$vector (state dependent time-varying slope vector) can be calculatedfrom,

$$S(x,t) = B^T(x,t)P(x,t) \quad (8)$$

While the system is on the sliding surface,

$$s(x,t) = B^T(x,t)P(x,t)x(t) = 0 \quad (9)$$

Then the derivative of the Lyapunov Equation will be,

$$\dot{V}(t) = -x^T(t)\overline{Q}(x,t)x(t) < -x^T(t)R(x,t)x(t) \quad (10)$$

The sector is divided into two parts, inner and outer, to eliminate the chattering problems that may occur in the sliding sector boundaries. After the completion of the design of the sliding sector, the controller that forces the system trajectories into the sliding sector can be determined from,

$$u(t) = -\sigma(s(x,t),\delta(x,t))\left[\left(S(x,t)B(x,t)\right)^{-1}(S(x,t)A(x,t)x(t) + Ks(x,t) + \dot{S}(t)x(t))\right] \quad (11)$$

III. MISSILE DYNAMICS AND SSC DESIGN

The missile dynamics in this study is a generic missile dynamics and has been used in various studies ([17]).The missile dynamics used in this study are moving at a speed of 3 Mach at a height of 20000 ft.and the missile is assumed in the cruise phase.

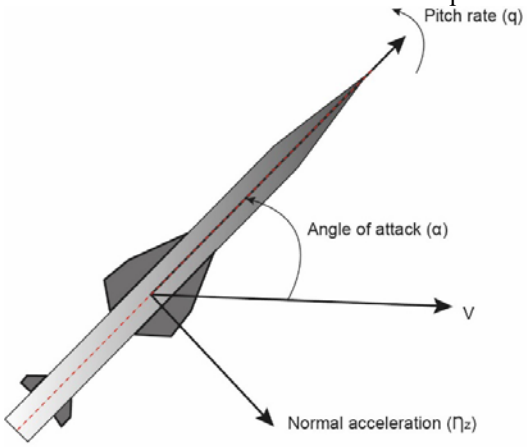

Fig. 1. Airframe Dynamics of the Missile

The mathematical model of the missile dynamics used is as follows (Fig 1),

$$\dot{\alpha} = f\frac{g\left(\cos\left(\frac{\alpha}{f}\right)\right)}{WV}F + q \quad (12)$$

$$\dot{q} = f\frac{M}{I_{yy}} \quad (13)$$

$\alpha =$ angle of attack, degree

$q =$ pitch rate, deg/s

$f =$ radians $-$ to $-$ degrees conversion coefficient, $(180/pi)$

$g =$ acceleration of gravity, $(32.2\ ft/s^2)$

$W =$ weight, $(450\ lb)$

$V =$ speed, $(3109.3\ ft/s)$

$F = C_zQS =$ normal force, lb

$M = C_mQSD =$ pitch moment, $ft-lb$

$I_{yy} =$ pitch moment of inertia, $(182.5\ slug-ft^2)$

$Q =$ dynamic pressure, $(6132.8, lb/ft^2)$

$S =$ reference area, $(0.44ft^2)$

$D =$ reference diameter, $(0.75\ ft)$

$C_m =$ coefficient of pitch moment of inertia,

$C_z =$ coefficient of normal force,

$$C_z = 0.000103\alpha 3 - 0.00945\alpha|\alpha| - 0.170\alpha - 0.034\delta$$

$$C_m = 0.000215\alpha 3 - 0.0195\alpha|\alpha| + 0.051\alpha - 0.206\delta$$

$\delta =$ fin deflection, degree

$\eta_z =$ The normal acceleration of the missile

$\eta_z^* =$ desired normal acceleration

The dynamical equations of the missile system can then be represented in the state-space form as follows,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ q \\ \eta_z \end{bmatrix}$$

The control input is the fin deflection,

$$u = \delta \quad (14)$$

State space version of the equation is,

$$\dot{x_1} = f\frac{g\left(\cos\left(\frac{x_1}{f}\right)\right)}{WV}F + q \quad (15)$$

$$\dot{x_2} = f\frac{M}{I_{yy}}()$$

$$\dot{e} = \eta_z - \eta_z^* \quad (16)$$

These equations can be represented by the State-Dependent Coefficient (SDC) matrix form as;

$$\dot{x}(t) = A(x)x + B(x)u \quad (17)$$

where

$$A = \begin{bmatrix} a_{11}(x) & 1 & 0 \\ a_{21}(x) & 0 & 0 \\ a_{31}(x) & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_1(x) \\ b_2 \\ b_3 \end{bmatrix}$$

The state dependent coefficients are as follows;

$$a_{11}(x) = \frac{fgqcos\left(\frac{x_1}{f}\right)}{wv}[0.000103x_1^2 - 0.00945|x_1| - 0.170] \quad (18)$$

$$a_{21}(x) = \frac{fqsd}{I_{yy}}[0.00215x_1^2 - 0.0195|x_1| + 0.051] \quad (19)$$

$$a_{31}(x) = \frac{qs[0.000103x_1^2 - 0.00945195|x_1| - 0.170]}{w} \quad (20)$$

$$b_1(x) = \frac{f\cos(x_1/f)qsg(-0.034)}{wv} \quad (21)$$

$$b_2 = \frac{fqsd(-0.206)}{I_{yy}} \quad (22)$$

$$b_3 = \frac{(-0.20310874)qs}{w} \quad (23)$$

By using the SSC equation given by (11), the SDRE based SSC is defined by

$$u(t) = -\sigma(s(x,t),\delta(x,t))\left[\left(S(x,t)B(x,t)\right)^{-1}(S(x,t)A(x,t)x(t) + Ks(x,t) + \dot{S}(t)x(t))\right] \quad (24)$$

where,

$\sigma(s(x,t),\delta(x,t)) = 0$ if the system states are inside the inner sector and $\sigma(s(x,t),\delta(x,t)) = 1$ if the system states are outside of the outer sector. The control input is kept as it is, if the system states are between inner and outer sectors.

IV. SIMULATION RESULTS FOR NONLINEAR MISSILE DYNAMICS

Nonlinear missile plant dynamics and SDRE based sliding sector controller simulations are performed in Simulink environment.

For the simulation, the weighting matrices are selected as follows,

$$Q = \mathrm{diag}[50 \quad 5 \quad 3750 \quad 2500] \quad (25)$$

$$R = 0.5 \quad (26)$$

Figures 2-6 show the simulation results of SDRE based sliding sector controller effects on the missile dynamics. The initial condition is taken as,

$$x_0^T = [0 \quad 0 \quad 0 \quad 0]$$

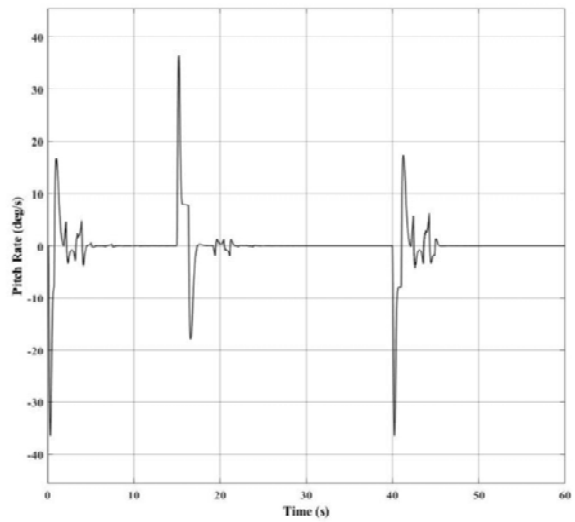

Fig. 2Pitch Rate vs.Time with SSC

The system states, namely Pitch Rate, Angle of Attack and Normal Acceleration, are depicted in Figures 2, 3 and 4 respectively. The Sliding Sector Controller input, which is the fin deflection, is given in Figure 5.

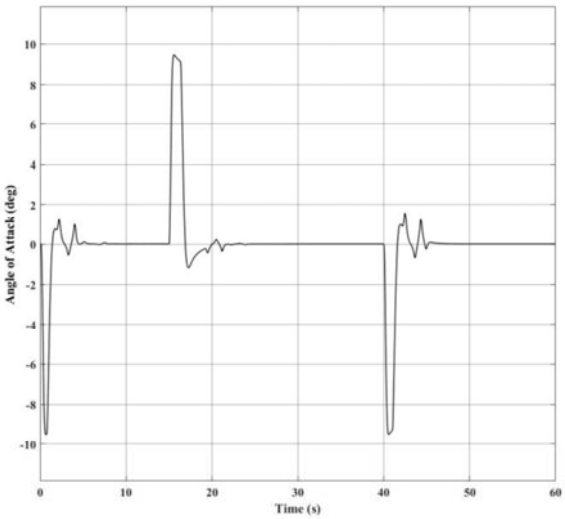

Fig. 3Angle of Attack vs.Time with SSC

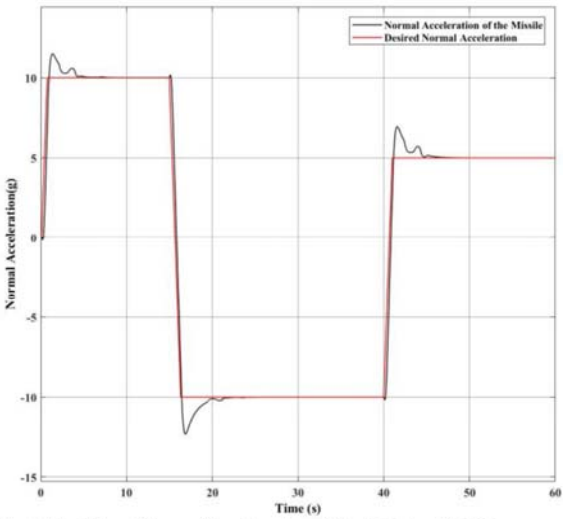

Fig. 4. Tracking of Normal Accelaration of the Missile with SSC

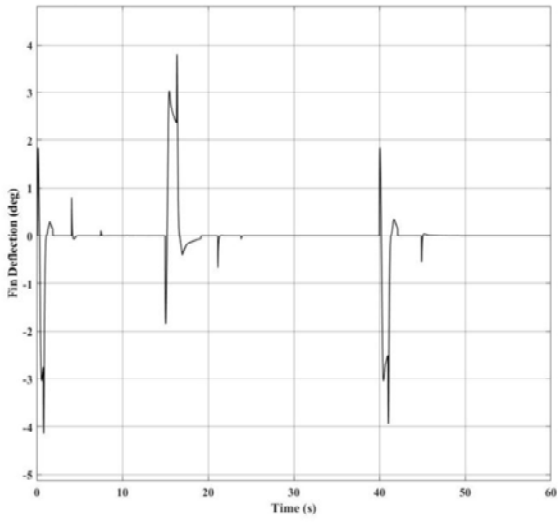

Fig. 5. Control Input vs Time with SSC

It is seen from Figure 6 that the system trajectories are kept between inner and outer sliding sectors. The overshoots that observed on the figures 2-4occur from the desired acceleration inputs. For the tracking

control, step function was created input at 0.1^{th},15th and 40th seconds and controller give a good response to these step inputs. Sliding surface is inside the sectors. It shows that SSC work well for the missile autopilot.

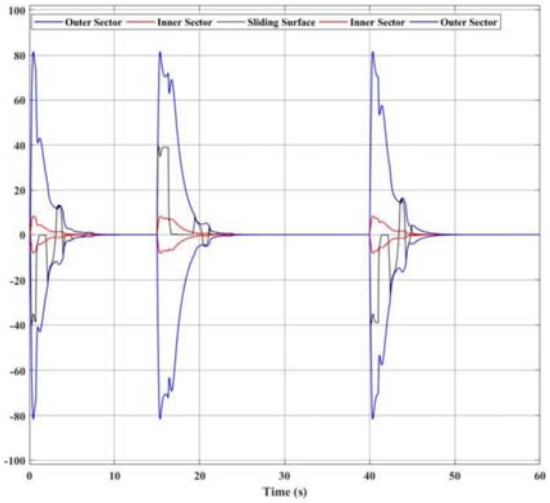

Fig. 6. Sliding Surface and Sectors

CONCLUSION

In this paper, State-Dependent Riccati Equation based Sliding Sector Controlleris designed and applied to a nonlinear missile dynamics and it is observed that thechattering phenomenon is eliminated by using theproposed nonlinear sliding sector controller. The application of the method is shown with an example.

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