INTELLIGENT CONTROL BASED MULTILEVEL AND MULTIBAND DIGITAL IMAGE THRESHOLDING USING FUZZY ENTROPY AND PARTICLE SWARM OPTIMIZATION

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Abstract—Thresholding is a fundamental image segmentation approach which transforms grayscale images into binary images. For more complicated cases than bi-level thresholding, extension is necessary in order to fulfill multilevel and multiband image thresholding. In multilevel thresholding, computation complexity increases significantly. In multilband thresholding, multiple dimension optimization should be made across different color frequency bands. Balanced histogram thresholding is widely accomplished in automatic image thresholding. Fuzzy entropy is thus introduced as the objective function to solve the optimal thresholds, where the image histogram is partitioned into various objects via fuzzy partition, so that diverse fuzzy membership functions can be assigned. Fuzzy entropy based optimization has been formulated accordingly. To speed up the convergence rate and shorten the computation time, particle swarm optimization based intelligent control schemes have been proposed to optimize thresholds rapidly. Comparisons of the role of fuzzy membership assignment on the thresholding quality are also made. Superior outcomes on multilevel and multiband thresholding are reached.

Index Terms— Fuzzy Entropy, Fuzzy Membership Function, Particle Swarm Optimization, Intelligent Control, Multilevel Thresholding, Multiband Thresholding

I. INTRODUCTION

The segmentation technique is used to separate images into regions or identifies contours for classification, such as thresholding and clustering. Thresholding is a natural and popular method to classify the bright and dark regions of objects by generating binary images based on the grayscale images. The Otsu's method acts as the typical means to produce automatic image thresholding with the assumption of two classes of the histogram based pixels available representing either foreground or background. However, it is sensitive to noises whose resulting pixels could be also inconsistent. In addition, there could be other modes except for foreground and background, so extension to multi-level thresholding is needed. Multilevel thresholding involves in reducing a gray scale image into several distinct regions. The 2D histogram based multilevel thresholding approach is presented for clear separation between objects, which is better than the 1D histogram based techniques. The method incorporates the 2D histogram for the generalized multilevel thresholding using the Tsallis entropy. Differential evolution is also used to improve computational efficiency. However there is no sound proof of convergence [2]. The discrete entropy of the grayscale image is a statistical measure of randomness which can be used to characterize source and segmented images. It is also applied to evaluate the K-means clustering of both underwater and aerial images [3]. Fuzzy logic is widely applied for system analysis in various fields of signal and image processing. For example, the nonlinear fuzzy control approach is exploited to filter out the rapidly varying noises when the linear filtering is far from enough. Fuzzy k-means clustering is also implemented on the biomedical sample differentiation [4-5]. An entropy function defined in terms of the relationship between a probability partition and a fuzzy c-partition is proposed to measure compatibility in image thresholding. An efficient scheme for three-level thresholding is deduced with respect to maximum entropy. The experiment results show that the proposed method gives the best performance in three-level thresholding using fuzzy partition [6]. Furthermore, the fuzzy entropy based fuzzy C-means clustering is introduced to locate vague boundaries that any crisp clustering can hardly achieve. Both sharp contours and distinctive regions can be attained using optimal fuzzy entropy based fuzzy C-means clustering [7]. To accelerate the convergence rate of image thresholding using performance index in terms of Shannon entropy, Tsallis entropy or fuzzy entropy, comprehensive nature evolution inspired global optimization techniques have to be introduced such as genetic algorithms (GA), particle swarm optimization (PSO), ant colony optimization (ACO) [1, 8-13].

Genetic algorithms are employed for both signal and image processing. Biomedical sample differentiation using Raman spectroscopy is used to characterize various samples for decision-making and medical diagnosis. Raman spectra are relatively weak signals whose features are inevitably affected by noises and background fluorescence during the calibration process. In order to identify optimal baseline spectra against diverse instrumentation and environmental conditions, genetic algorithms are applied to locate the optimal baselines [8]. The simulated binary crossover based genetic algorithm multilevel

thresholding is also used for the segmentation of medical brain images. The weighted magnetic resonance imaging inputs are selected for segmentation and optimal multilevel thresholds are reached by maximizing the entropy. The performance of genetic algorithm multilevel thresholding is better which has more consistent performance than existing methods [9].

The aim of an effective segmentation is to separate objects from background and to differentiate pixels with similar values to enhance the contrast. The Otsu's method and Kittler's method are two major exhaustive thresholding methods. A new hybrid optimization scheme is proposed based on the ant colony algorithm with both Otsu's and Kittler's methods to render the optimal thresholding technique more applicable and effective. It is concluded that the ACS-Otsu outperforms ACS-Kittler algorithm on both CPU time and image quality in most cases [10]. To avoid edge suppression and feature deformity, ACO is also implemented on edge and contour detection against false detection, from which more intrinsic information will be extracted. The evolutionary computation oriented ACO scheme is a promising approach for feature capturing without necessity of smoothing filters. The adaptive contour tracking scheme is coordinated with the enhanced ACO schemes. Technology integration results in true edge representation together with sufficiently connected linkages [11].

As a stochastic population based evolutionary algorithm, particle swarm optimization simulates social optimization, where the swarm has been modeled by particles in multidimensional space with a position and a velocity. Compression on digital images and videos plays important roles in data compression. PSO is proposed to enhance the quality of wavelet based image compression. Objective measures are also defined to evaluate the integration of the wavelet technologies and PSO optimization schemes [12]. PSO has also been applied to thresholding that converts the gray scale into the binary image. In order to determine thresholds, the histogram of the image is subject to analysis. The optimal thresholds are attained by either minimizing or maximizing an objective function that is a function of thresholds. Particle swarm optimization (PSO) is presented for multilevel thresholding whose performance is superior to genetic algorithms [13].

II. MULTILEVEL AND MULTIBAND IMAGE THRESHOLDING

Thresholding classifies the pixels of an image into multiple classes with respect to the intensity levels of pixels. Bi-level thresholding is the simplest way of image segmentation, which assigns those pixels with gray scales above a threshold as one and those below it as zero, so that binary images can be produced. Both parametric and non-parametric thresholding criteria are able to bring about bi-level thresholding. Various approaches can be applied to thresholding such as clustering based, entropy based, histogram shape based and object based methods. The pioneering Otsu's method employs the variance of region homogeneity as the measure, whose optimization is reached when the within-class variance of two sets of pixels is minimized. However its assumption is that only two classes of the image histograms of the similar size are available. In most practices, multilevel thresholding is a must and the bi-level thresholding should be extendable to multilevel thresholding. It divides image pixels into several classes, each of which solely consists of the pixels within as a specified intensity range defined by two thresholds. With remarkable increment of computation time, automatic thresholding should be carried out together with some intelligent approaches. In order to achieve optimal multilevel thresholding, an exhaustive search could be unavoidable to globally search for a set of optimal thresholds among all possible subsets. The performance index has to be defined well in advance. In the meanwhile, true color multiband images are more frequently being taken than the grayscale images, thus multiband thresholding also acts as another challenge of image segmentation. The true color images are comprised of red, green and blue components (RGB). Each of three major components has the similar intensity distribution as the grayscale image, which ranges between 0 and 255. For this reason, separate thresholds can be assigned to each component before a true color image can be reconstructed by combining three thresholded components.

III. FUZZY LOGIC AND FUZZY ENTROPY

Fuzzy logic serves as the intelligent control technique to handle complex systems whose structure and measure-ment are hard to quantify. The fuzzy membership function is illustrated mostly as a curve showing the mapping from a universe of discourse (crisp input) to degree of membership (fuzzy output) between 0 and 1. The degree of membership provides the quantified value of the crisp input in a fuzzy set. The vagueness attribute of the fuzzy sets as a whole indicates that precision is independent of either simple or complex structure of membership functions. Most typical fuzzy membership functions will be assigned in the case studies using new image thresholding approaches as below.

Due to simplicity, straight line membership functions are widely chosen, including both the triangular membership function and trapezoidal membership function.

A. Triangular Membership Function

The triangular membership function is defined as (1) or (2), which has 3 parameters of a lower limit a, an upper limit c, and an apex point b, where $a < b < c$.

$$\mu_{\Delta}(x; a, b, c)=\begin{cases} 0 & x \le a \\ (x-a)/(b-a) & a \le x \le b \\ (c-x)/(c-b) & b \le x \le c \\ 0 & x \ge c \end{cases}$$

(1)

$$\mu_{\Delta}(x; a, b, c)=\max[\min(\frac{x-a}{b-a}, \frac{c-x}{c-b}), 0]$$

(2)

B. Trapezoidal Membership Function

The trapezoidal membership function is defined as (3) or (4), which has 4 parameters of a lower limit a, an upper limit d, a lower apex limit b, and an upper apex limit c, where a < b < c < d. The triangular membership function is a special case of the trapezoidal membership function (b = c).

$$\mu_{T}(x; a, b, c, d)=\begin{cases} 0 & x \le a \\ (x-a)/(b-a) & a \le x \le b \\ 1 & b \le x \le c \\ (d-x)/(d-c) & c \le x \le d \\ 0 & x \ge d \end{cases}$$

(3)

$$\mu_{T}(x; a, b, c, d)=\max[\min(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}), 0]$$

(4)

Zero values occur frequently in (1) – (4). Low fuzzy values actually are more appropriate than zeros in description. The alternative is to use smooth and concise membership functions to avoid zeros. The symmetrical Gaussian and Bell membership functions are easy means to specify fuzzy sets.

C. Simplified Gaussian Membership Function

The simplified Gaussian membership function is defined as (5), which has 2 parameters of a central value c and a standard deviation k > 0. The larger k is, the wider the curve is, and vice versa.

$$\mu_{G}(x; c, k)=e^{-\frac{(x-c)^2}{2k^2}}$$

(5)

D. Bell Shape Membership Function

The bell shape membership function is defined as (6), which has 3 parameters of a central value c, an exponential parameter b, and a scaling parameter a.

$$\mu_{B}(x; a, b, c)=\frac{1}{|\frac{x-c}{a}|^{2b}+1}$$

(6)

From (5) and (6), neither Gaussian nor bell membership function could produce nonsymmetrical curves with respect to center line (x=c). Sometimes flexibility could be lost. On the other hand, some special types of asymmetric fuzzy membership functions could also be formulated.

E. Sigmoidal Membership Functions

The individual sigmoidal membership function is defined as (7), which has 2 parameters of a central value c and a scaling parameter k. The sign of parameter k will determine if the curve is left open or right open. Given two sigmoidal membership functions $\mu_s(x; k_1, c_1)$ and $\mu_s(x; k_2, c_2)$, the close forms of sigmoidal membership functions can also be made using difference or product of $\mu_s(x; k_1, c_1)$ and $\mu_s(x; k_2, c_2)$.

$$\mu_{S}(x; k, c)=\frac{1}{1+e^{-k(x-c)}}$$

(7)

F. Polynomial Based π Membership Function

Depend on the actual order of a univariate polynomial, there could be a variety of polynomial based membership functions. A typical π shape fuzzy membership function will be used in the case study. It reaches the zero at both ends while it has a concave curve (π) between 0 and 1 within two ends, which is formulated either symmetric or asymmetric.

G. Fuzzy Entropy via Membership Functions

The discrete entropy of the digital image is defined as a fitness function in evolution computing. It is based on the probability distributions of the grayscale image histogram with the intensity level between 0 and 255. When the fuzzy logic is introduced, the fuzzy entropy in terms of a single fuzzy membership function μ is defined as (8), which is corresponding to single level thresholding. The fuzzy entropy represents the entropy of a fuzzy set that loosely depicts information of uncertainty. For 8-bit grayscale images, there are $N = 2^8 = 256$ intensity levels. μ is referred to as a type of fuzzy membership function being selected.

$$H(\mu) = -\sum_{i=1}^{N}\mu_i \log_2(\mu_i)$$

(8)

For multilevel thresholding, the fuzzy entropy fitness function has to be redefined. Assuming that the triangular fuzzy membership function μ_{Δ} in (2) is utilized, the fuzzy entropy of 2-level and 3-level thresholding should follow the similar idea. However, single membership function should be substituted by 2 fuzzy membership functions (Dark, Bright) and 3 fuzzy membership functions (Dark, Central, Bright) separately. Let the degrees of fuzziness be $\mu_{\Delta D}$ and $\mu_{\Delta B}$ in 2-level case, and $\mu_{\Delta D}$, $\mu_{\Delta C}$ and $\mu_{\Delta B}$ in 3-level case, with the histogram p(k) at intensity level k. Then fuzzy membership functions ($\mu_{\Delta D}$, $\mu_{\Delta C}$ and $\mu_{\Delta B}$) can be formulated.

$$\mu_{\Delta D}(k; a_1, b_1, c_1)=\max[\min(\frac{k-a_1}{b_1-a_1}, \frac{c_1-k}{c_1-b_1}), 0]$$

(9)

$$\mu_{\Delta B}(k; a_2, b_2, c_2)=\max[\min(\frac{k-a_2}{b_2-a_2}, \frac{c_2-k}{c_2-b_2}), 0]$$

(10)

$$\mu_{\Delta C}(k; a_0, b_0, c_0)=\max[\min(\frac{k-a_0}{b_0-a_0}, \frac{c_0-k}{c_0-b_0}), 0]$$

(11)

When the range of intensity level is defined as [0, 255], fuzzy entropies of 1-level thresholding, 2-level thresholding, and 3-level thresholding are defined as (12)-(14).

$$H(a_0, b_0, c_0) = -\sum_{k=0}^{255} p_k \mu_{\Delta C}(k) \log_2[p_k \mu_{\Delta C}(k)] \quad (12)$$

$$H(a_1, b_1, c_1, a_2, b_2, c_2) = -\sum_{k=0}^{255} p_k \mu_{\Delta D}(k) \log_2[p_k \mu_{\Delta D}(k)] - \sum_{k=0}^{255} p_k \mu_{\Delta B}(k) \log_2[p_k \mu_{\Delta B}(k)] \quad (13)$$

$$H(a_1, b_1, c_1, a_2, b_2, c_2, a_0, b_0, c_0) = -\sum_{k=0}^{255} p_k \mu_{\Delta D}(k) \log_2[p_k \mu_{\Delta D}(k)] - \sum_{k=0}^{255} p_k \mu_{\Delta C}(k) \log_2[p_k \mu_{\Delta C}(k)] - \sum_{k=0}^{255} p_k \mu_{\Delta B}(k) \log_2[p_k \mu_{\Delta B}(k)] \quad (14)$$

where $0 \leq a < b < c \leq 255$ should be satisfied in (12) – (14). For a specified level, optimal sets of thresholds should be computed for each of three color components (Red, Green and Blue), followed by reconstructing the true color thresholded images. To achieve the fast convergence rate, intelligence computation approaches could be applied, so relatively simple but powerful particle swarm optimization has been selected in this study.

IV. PARTICLE SWARM OPTIMIZATION (PSO)

The fitness function is defined as the multilevel fuzzy entropy with respect to different fuzzy memberships. A set of parameters in each case is formulated as a vector. The objective is to maximize the fuzzy entropy, so that the entropy function takes a candidate solution (a set of thresholds) as argument in the form of a vector to be optimized. The total number of parameters will determine the dimension of a search space in PSO. In a d-dimensional search space, the optimization problem is formulated whose ith particle of the swarm has the position and velocity vectors denoted as $X_i = (X_{i1}, X_{i2}, \ldots, X_{id})$ and $V_i = (V_{i1}, V_{i2}, \ldots, V_{id})$. At iteration k, the position vector, velocity vector, local best location reached by particle i, global best location reached by other neighbors are expressed as X_i^k, V_i^k, P_{besti}^k, G_{besti}^k, then the position and velocity of each particle in the next iteration are updated by (15) and (16).

$$V_i^{k+1} = V_i^k + \{w_1 * rand_1^k (P_{besti}^k - X_i^k) + w_2 * rand_2^k * (G_{besti}^k - X_i^k)\} \quad (15)$$

$$X_{id}^{k+1} = X_{id}^k + V_{id}^{k+1} \quad (16)$$

where $w_1 > 0$ is the cognitive parameter that pulls each particle towards local best position; $w_2 > 0$ is a social parameter that pushes the particle towards the global best position; $rand_1$ and $rand_2$ are two separately generated uniformly distributed random variables between an interval [0,1]. The inertia weight (w_0) could also be adopted in order to ensure the convergence of the PSO algorithm as shown in (17). A larger inertia weight favors global exploration and a smaller inertia weight favors local exploration instead.

$$V_i^{k+1} = w_0 * V_i^k + \{w_1 * rand_1^k (P_{besti}^k - X_i^k) + w_2 * rand_2^k * (G_{besti}^k - X_i^k)\} \quad (17)$$

where w_0 is chosen to be 1 in the case studies. Assuming that d-parameters are used to optimize the fuzzy entropy, then the location of ith particle is a vector in the form of $X_i = (X_{i1}, X_{i2}, \ldots, X_{id})$. After initializing a swarm of a specified population size, particles are randomly generated between the lower and upper limits of each threshold value. The fitness function for the initial particles is then formulated and calculated. The set of parameters that leads to maximal fuzzy entropy turns out to be the local best position, while the optima among all local best positions within a population size is the global best position. The velocity vector is also initialized and then updated at each iteration via stochastically generated factors so as to further update the position vector globally. The global best position vector won't be updated unless the latter iteration will produce greater fuzzy entropy than the former. When termination criteria are reached, the final global best position vector will be the optimal solution. Without presence of evolution operators of crossover and mutation, PSO is easy to implement with fewer numbers of parameters than other evolution computation methods such as genetic algorithms. As the swarm iterates, fitness of the global best improves. Moving through the multi-dimensinal hyperspace is guided by the fuzzy entropy fitness function, with population converging to the best fitness value eventually.

V. CASE STUDIES OF MULTILEVEL AND MULTIBAND THRESHOLDING

In Figs. 1-3, the 1-level, 2-level and 3-level multiband thresholding optimization outcomes are shown with bell membership functions, where the resulting RGB, red, blue and green components are all obtained. It depicts that multilevel thresholding will generate better visual appeals than single level thresholding. New technology integration of fuzzy logic and PSO works well in each case. Comparisons can also be made among effects of image thresholding using various fuzzy membership functions.

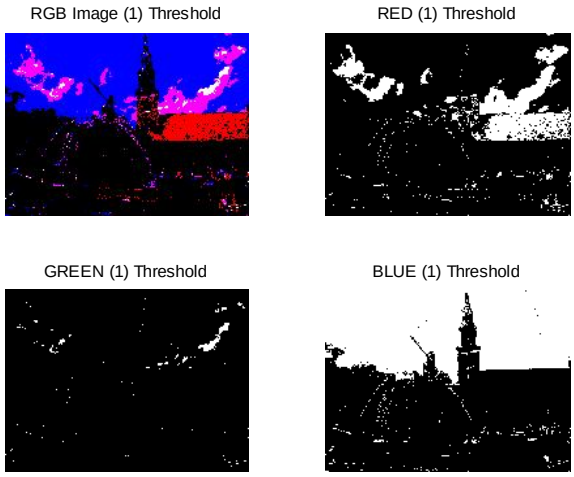


Fig. 1 Multiband 1-Level Image Thresholding (Bell)

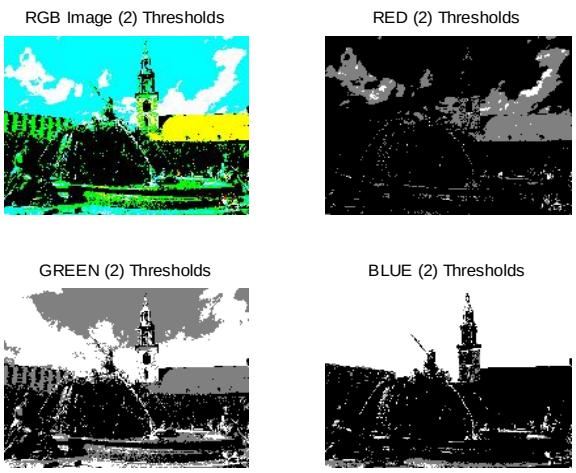


Fig. 2 Multiband 2-Level Image Thresholding (Bell)

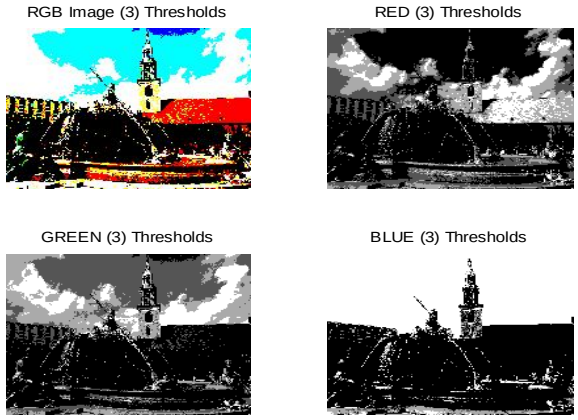


Fig. 3 Multiband 3-Level Image Thresholding (Bell)

From Fig. 4 to Fig. 10, the source image is reduced to thresholded images using the 1-level, 2-level and 3-level multiband thresholding, respectively. It is shown that relatively simple and symmetric membership functions (e.g. Triangular) generate higher quality than relatively complex and irregular membership functions (e.g. Sigmoidal). In Figs. 4-5, outcome comparisons between the triangular membership function and difference of sigmoidal function are made, where the former is better than the latter. If sigmoidal and product of sigmoidal functions are adopted, the results could be even worse as being observed.

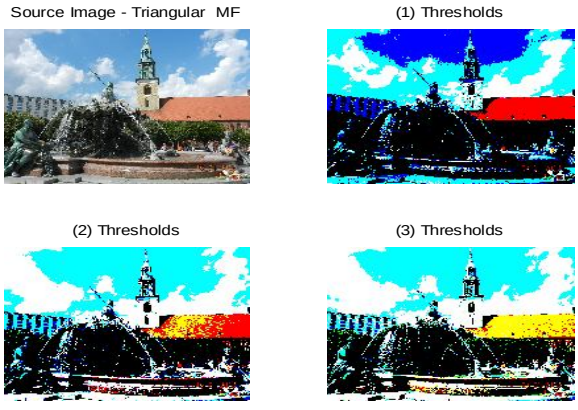


Fig. 4 Multiband 3-Level Image Thresholding (Triangular)

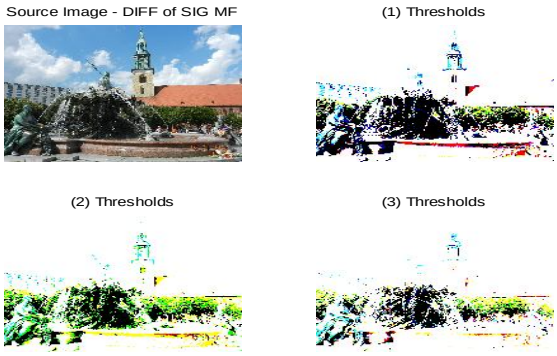


Fig. 5 Multiband 3-Level Image Thresholding (DIFF of SIG)

In Figs. 6-8, when numerous parameters are involved in a relatively complex membership function (e.g., Trapezoidal, Gaussian, Bell, π Shape), there is no significant difference between multilevel thresholding at 2-level, 3-level and even higher levels. It demonstrates the vital role of the proposed intelligent control based scheme in multiple dimension and multivariate optimization.

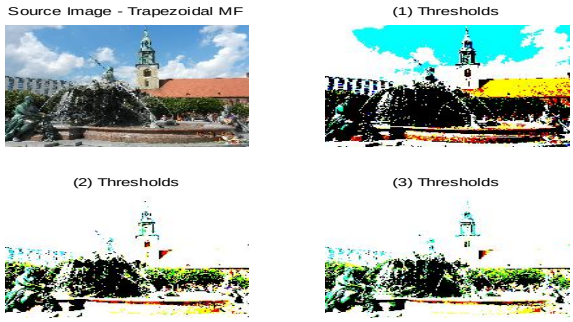


Fig. 6 Multiband 3-Level Image Thresholding (Trapezoidal)

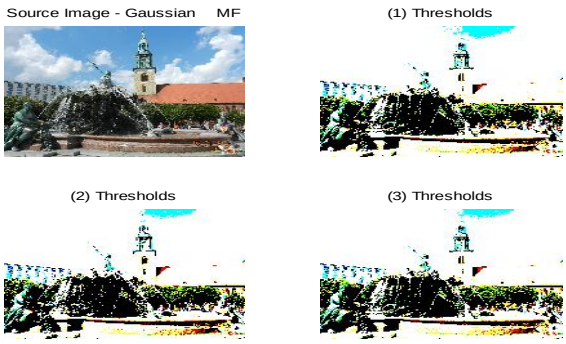


Fig. 7 Multiband 3-Level Image Thresholding (Gaussian)

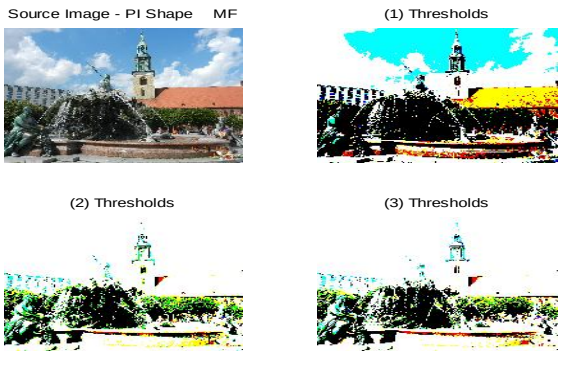


Fig. 8 Multiband Image Thresholding (π Shape)

However, if most parameters are fixed, then larger level thresholding is superior to smaller level thresholding. Now in Fig. 9, both the scaling and exponential parameters are constants, with the central value as the sole parameter of an individual Bell fuzzy membership function, then significant differences can be observed among diverse levels of thresholding.

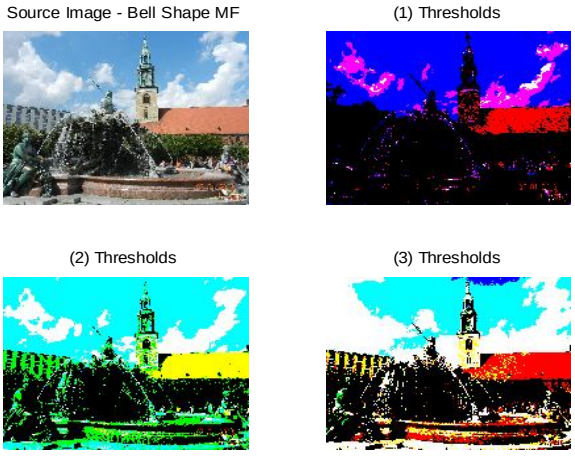


Fig. 9 Multiband 3-Level Image Thresholding (Bell)

To further verify if it could be unnecessary to adopt high level thresholding with sufficient fuzzy parameters in multiple dimensional optimization, implementation of Gaussian fuzzy membership function on several blurry or low dynamic range images are also conducted. The results are shown in Figs. 10-12, where Gaussian membership function has been applied. The same conclusion has been made easily. Once again, due to occurrence of numerous parameters in the corresponding multivariate optimization problem, a larger level thresholding has not produced much better results than a smaller level thresholding. But the computation time increases dramatically at the same time. Therefore, when the tradeoff is to be made between computational complexity and thresholding quality, the larger level thresholding should not be the best choice if a plenty of parameters show up as the argument of the fitness function being optimized.

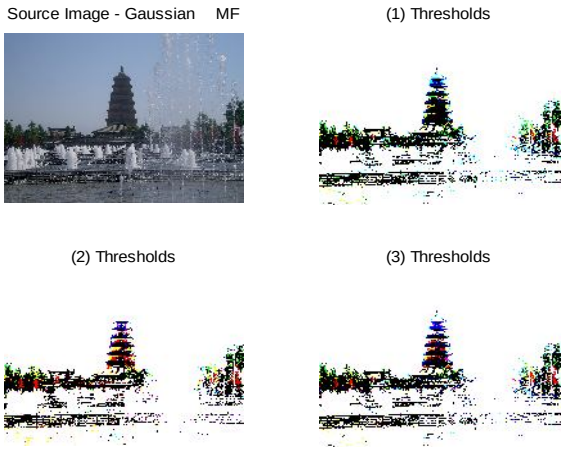


Fig. 10 Multiband 3-Level Image Thresholding (Gaussian)

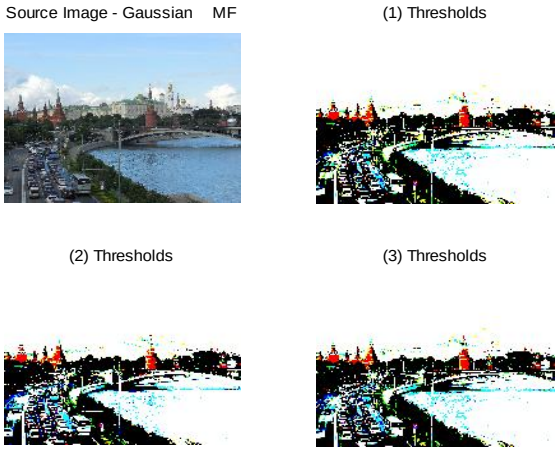


Fig. 11 Multiband 3-Level Image Thresholding (Gaussian)

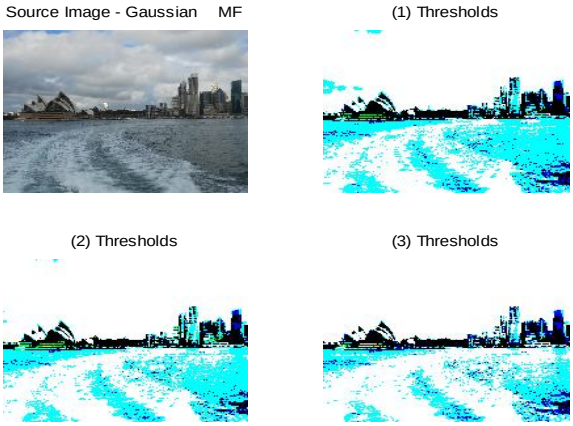


Fig. 12 Multiband 3-Level Image Thresholding (Gaussian)

CONCLUSION

Integration of fuzzy logic and particle swarm optimization has been implemented on the multilevel and multiband image thresholding. Multilevel thresholding divides image pixels into multiple classes. The pixels in any individual class have gray scales within a limited scope constrained by several specified thresholds. For true color image multiband thresholding, each of three RGB components is treated similarly as the gray scale component independently. The thresholded components are then combined to reconstruct multiband thresholding images. Instead of the Shannon entropy, the fuzzy entropy based algorithm is presented to accomplish more consistent performance in image thresholding, where several fuzzy membership functions are defined for comparison purposes. It involves the maximization of sum of entropies for each individual class. Some simple fuzzy membership functions give rise to good quality results. To accelerate the convergence rate in order to avoid curse of dimensionality, particle swarm optimization is then exploited to shorten computation time. PSO is relatively simpler to handle with fewer parameters than some other stochastic computation intelligence based approaches. The robust performance of the

proposed approach is verified in numerical simulations.

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