NEW QUALITATIVE METHOD TO INVESTIGATE THE VIBRATION OF COMPOSITE RECTANGULAR PLATES UNDER THE EFFECT OF DIMENSION AND RIGIDITIES RATIO OF BOTH PERPENDICULAR SIDES

[1]HAMADA SAMY, [2]BENMANSOUR TOUFIK

[1,2]Department of Mechanical Engineering - University Constantine 1 - Algeria
E-mail: [1]samy11.hamada@gmail.com, [2]toufik.benmansour@yahoo.fr

Abstract—In this work we propose a qualitative method that allows rapid expertise of the vibration behavior of composite rectangular plates taking into account the dimensions ratio (a/b), and the effect of the rigidity ratio (D_X/D_Y) along the two axes of the plate (x,y). The comparative analysis of two or more composite rectangular plates which differ in their boundary conditions (ex: SSCC / SCSC) is intended to systematically verify the frequency curves and ensure their relative proper positions with regards to their modal progression.From previous work, the qualitative method applied to the isotropic rectangular plates with the support conditions (SSCC/ SCSC) and based only on the effect of the dimensions ratio (a/b) showed a good concordance with the results obtained from the FE method. In our previous works, we proposed a study based on a dimension ratio (a/b>1) and on varying the rigidity ratios (D_X/D_Y) = (2, 1, (2:3), (1:2), (1:3), (1:4)).In this work we propose to globalize and validate the qualitative method by studying the case where we have a dimension ratio (a/b<1) and by taking into account the effect of the rigidity ratio (D_X/D_Y) = (2, 1, (2:3), (1:2), (1:3), (1:4)). The comparative and qualitative analysis of frequency responses allowed the justification of the position and the threshold of the respective modal frequencies of the two plates (SSCC/ SCSC) in accordance with the results obtained from the numerical and analytical methods in use. The first six frequency modes are obtained from the results achieved by developing a MATLAB program that checks these results against those obtained by the FEM using the code of calculation ANSYS.

Keywords—Free Vibration, Thin Plates, composite, Rigidity Ratio, Equivalent Method, Rayleigh's Method, Finite Element Method.

I. INTRODUCTION

The use of orthotropic plates has become more common in several fields of structural engineering such as aeronautical, aerospace, civil and sports equipment. That is why the study of such structures to develop an exact and confident conception is important in the case where the properties of materials andlimit conditions of anorthotropic plate are known. The finite element method (FEM) provides a complete solution to the problem of evaluation of vibration modes and dynamic responses. When it comes to the selection of dimensions and the properties of the material, it is useful to have a simplified method to calculate all the modal frequencies of orthotropic plates.

In the last decades, different studies have been elaborated to solve the problem of free vibration of orthotropic plates which can be found summarized and collected in the Leissa and Bert[1-9] series of papers on the dynamic behavior of composite and sandwichplates. Due to its high versatility and conceptual simplicity, the Rayleigh-Ritz method[10] is considered to be the most popular method to obtain an approximate solution for the frequencies of an orthotropic rectangular plate. It began with the initial paper of Hearmon[11] (1946) who initiated the study of some particular cases. In 1959[12], an approximategeneral solution is proposed by Hearmon for the free vibration of orthotropic plates applying the Rayleigh method. His approach is an extended development of Warburton[13] work for all vibration modes of isotropic plates with any boundary conditions combination: free, simply supported, orclamped. Among the works based on the Rayleigh-Ritz method, the contribution of Marongoni et al[14] is used. This contribution is a combination of the Rayleigh-Ritz upper bound and the decomposition method proposed by Bazely[15] for lower bound evaluation. For the case where one or more free edges are used, an extended analysis is done by Rossi et al[16]. To overcome the difficulty arising from the case of free edge constraint condition where the classic Rayleigh-Ritz formulation is useless, Rossi et al used an optimized formulation proposed by Laura et al[17] which shows excellent agreement with the FEM calculation.

Gorman[18-19] applied the superposition method, developed for isotropic plates, for the case of clamped orthotropic plates, which showed a good consistency of the method including theeigenvaluescalculatedforawiderangeofplate geometries and orthotropic materials available for engineering use. The attention of several researchers who search the differential equation which satisfies exactly all plates and boundary conditions with a satisfying degree of precision was given to this method.In fact, Li[20] showed that it is desirable also to analyze the forced vibration of orthotropic plates, while Mossou and

Novit[21] used the method to provide a reliable nondestructive testing method to determine elastic constants of an orthotropic material forming a rectangular vibration free plate.

Sakata and Hosokawa[22] introduced a solution in a double trigonometric series to solvethe forced vibrationproblem of clamped orthotropic plates using an iterative method for analyzing free vibration. In reference[23], Sakata et al applied an iterative method deduced from a differential equation obtained for a plate already studied in isotropic cases[24], in order to obtain natural frequencies of orthotropic rectangular plates with accurate results. In references [25], [26] and [27] methods to calculate the frequencies of anisotropic plates are proposed. Finally, Chen[28]proposesan approach based on finite differentialequations and numerical integration to calculate the fundamental vibration frequency of an orthotropicplate. The methods described above involve a considerable effort of calculations that is why they are not desirable for:

- ✓ a quick approximate method.
- ✓ a preliminary project design or a fast final general check of accuracy

A simplified method for evaluating natural frequencies of an orthotropic plateisproposedinthis workbasedonaformula developed in a general form and using results obtained by researchers who have approached this problem. It is possible to access higher modal frequencies basedon a general expression of fundamental frequency of a plate suggested by Hearmon and taking into account the boundary conditions and using a formulation of Rayleigh method. This reduces the problem of evaluating any frequency of an orthotropic plate on the basis of the fundamental frequency of an equivalent plate associated with the real one. Sakata[29] used this approach to study higher natural frequencies in the case of an isotropic rectangular plate simply supported on both sides, parallel to the y-axis, and retained against restraint rotation along the other parallel side to the x-axis.

- ✓ **Qualitative method of dynamic analysis of rectangular plates:**

The qualitative method is essentially based on the theory of long plates which behave like an equivalent beam. The behavior in this case, is dominated by the privileged small snse of rectangular plates.

In this method, a quick comparative study of both upper and lower rigidities and consequently the extreme lower and upper frequencies is done by a simple observation and on the basis of fixed conditions as well as the dimensions ratio. In all cases, dynamic investigation is based on some evaluating criteria such as boundary conditions effect, plate dimensions ratio effect, material effects and mode number effect. A confrontation with the finite element method is done to verify the accuracy of the proposed method.

II. BACKGROUND THEORY

The determination of frequencies, outside the simply supported case[30], presents difficulties in the integration of differential equations of dynamics motion of 4th order. That is why we need to use approximate methods.

An orthotropic material is characterized by the fact that its elastic mechanical properties have two symmetrical plans, thus only four independent elastic constants specifically E1, E2, G12, ν12 are considered.

The coefficient ν21 can be determined using the equation: $E_1/E_2 = \upsilon_{12}/\upsilon_{21}$ (1)

Using the hypotheses of Love-Kirchoff which neglect the effect of the shear forces and the rotational inertia, and introducing the parameters:

$$D_1 = E_1 h^3/12\mu, \quad D_2 = E_2 h^3/12\mu, \quad D_{12} = G_{12} h^3/12$$
$$\mu = 1 - \upsilon_{21}\upsilon_{12}, \quad 2H = \upsilon_{21} D_1 + \upsilon_{12} D_2 + 4D_{12} \quad (2)$$

With reference to figure -1- [31], the equation of the motion follows:

$$D_1 (\partial^4 w/\partial x^4)(x, y, t) + 2H (\partial^4 w/\partial x^2 \partial y^2)(x, y, t) + D_1 (\partial^4 w/\partial y^4)(x, y, t) + \Lambda^4 W = 0 \quad (5)$$

With: $\Lambda^4 = \omega\sqrt{\rho h}$ (6)

Equation (5) must be solved to satisfy the following limit conditions:

$$(7)\begin{cases} M = 0, \quad R = 0, \quad Free\ side\ (F) \\ M = 0, \quad W = 0, \quad For\ simply\ side\ (S) \\ W = 0, \quad \partial W/\partial x \ (or\ \partial W/\partial x) = 0, \quad For\ clamped\ edge\ (C) \end{cases}$$

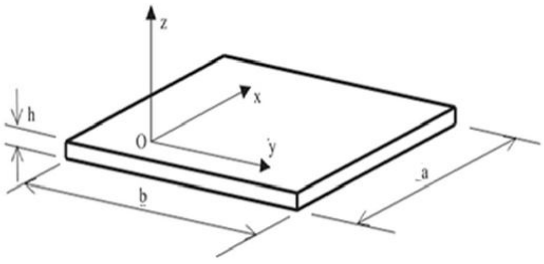


Figure 1: Geometry of the Model adopted in the paper

Mechanical properties	ISO	ORTHO
E_X (MPa)	1E+10	1.21e5
E_Y (MPa)	1E+10	8600
E_Z (MPa)		8600
ν_{XY}	0.2	0.27
ν_{YZ}		0.4
ν_{XZ}		0.27
G_{XY} (MPa)	4.17E+09	4700
G_{YZ} (MPa)		3100
G_{XZ} (MPa)		4700
ρ (Kg/m³)	7800	1490

Table 1: Mechanical properties

	Model 1	Model 2	Model 3
L_x (m)	3	3	3
L_y (m)	4	2	1.2
Thickness (m)	0.01	0.01	0.01

Table 2: geometrical properties

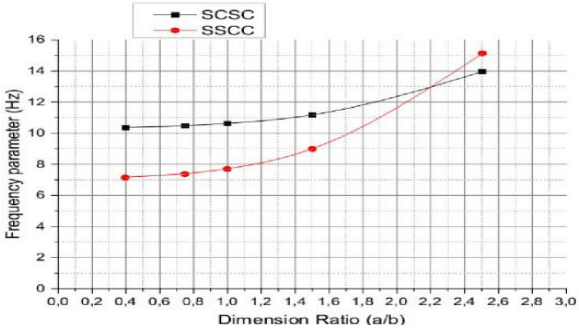
III. DISCUSSION OF SOME RESULTS.

A. Finite Elements Method: The response according to finite element analysis is based on some models of rectangular meshing with (20X20), (40X40), (80X80) and (120X120). The case of (80X80) has been taken as a reference of comparison[32]. The precision of the method of investigation is based on the study of a plate made of one isotropic material and one orthotropic material.

B. Dynamic Investigation: Dynamic investigation is based on some evaluation criteria including dimension ratio effect, limit conditions effect, material effect and modal sequence effect. According to those factors, analysis has permitted to apprehend some aspects of plate vibratory behavior. The influence of the increase in the dimension ratio (Lx/Ly) indicates a parabolic augmentation. Long plates in onedirectionoranotherandregardless ofmaterialreduce the contribution of rigidities in the length direction which reduces vibration behavior to that of a beam supported on two elongated boards with fixity conditions of the same edges and the width direction. A qualitative dynamic method has been proposed, based on the concept of long plates, which are dominated by the behavior of oriented beams according to the width direction of rectangular plates supposed infinite.

- **Influence of the dimension ratio (a/b) and the fixities conditions onthebehaviorof freevibration of orthotropic rectangular plates.**

We have two composite plates subjected to the limit conditions (SCSC) and (SSCC); figure -2- shows the influence of dimensions ratio and boundary conditions on the curves for two cases of fixities. We noted that at a certain common ratio (a/b 2.2) the level of frequency curves was inverted, at this point; thetwo plates behave identically even thoughthe fixities conditions are different. This interesting remark means that there is a limit of the ratio (a/b) for which the two types of rectangular plates remain insensitive to the effect of boundary conditions.



We should mention also that this common ratio (a/b) changes with the modal number of the same plates with the same fixities conditions.

- **Influence of rigidities ratio (E1/E2) effect and the fixities conditions onthebehaviorof freevibration of orthotropic rectangular plates.**

The two plates are subjected to the following limit conditions:

• SCSC • SSCC

Beside the dimension ratio, we introduce different rigidity ratios (Dx/Dy) = (2, 1, (2:3), (1:2), (1:3), (1:4)) to verify the frequency curves and ensure their relative proper positions progressively to their modal progressions.

We noticed a rollover of frequency factors for the first three modes N1, N2, N3. In order to justify and validate this behavior, a qualitative analysis exploiting also Hearmon's approximate method which is useful for upper modes, is used.

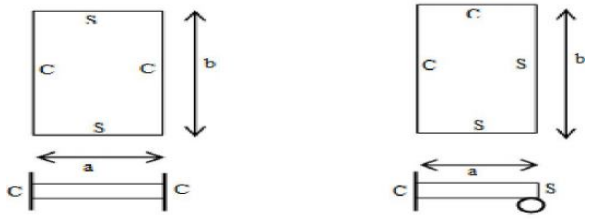
• Justification and validating of N1(ω11):

If we focus on the plate (a/b) = (3/4), we can observe, based on the theory of infinite plate, that the behavior should be dominated by the small side (Lx=a).

Two types of contributions will be discussed:

Plate type SCSC: The vibration behavior being still under the control of the small side along the (x) direction, we get a vibration contribution of a beam which has both sides clamped [SCSC ⟶ CC]. (Figure 4) shows how this beam is restrained.

Plate type SSCC: We get a vibration contribution of a beam having both clamped and roller supports [SSCC ⟶ CS] (Figure 4)



$$a/b = 3/4 \Rightarrow L_x$$
$$\Rightarrow (2S) < (C+S) \Rightarrow (\omega_{11})_{SCSC} < (\omega_{11})_{SSCC}$$

figure4 : Qualitative analysis (mode 1)

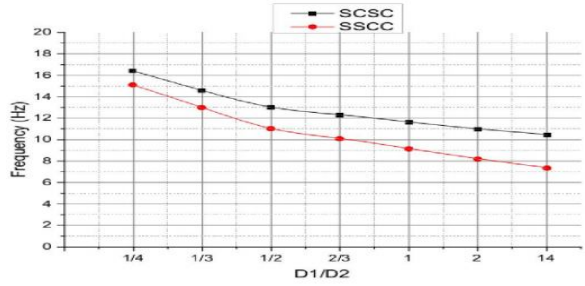


figure5 : effect of rigidity ratio on evolution of frequencies (mode 1)

Now let us compare the two cases:

Plate SCSC: Mode N1 (ω11) involve ⟶ 2 Clamps ⟶ 2C

Plate SSCC: Mode N1 (ω11) involve ⟶ 1 Clamp + 1 Roller ⟶ C+S

Based on a qualitative analysis, we can say that since we have (C+S) < (2C), we should have ω11)SSCC<(ω11)SCSC because there is less restraint on plate SCSC than on plate SSCC. This result is consistent with that given by the FEM analysis (figure 3).

Now, by introducing the rigidity ratios to see their influence on the behavior of the plate with the same restraint

conditions, we can observe from the graph (figure 5) that the order of the curves is respected so that we always have (ω11)SSCC<(ω11)SCSC, which means that the rigidity ratios have no effect on the behavior of the plate in the first mode N1.

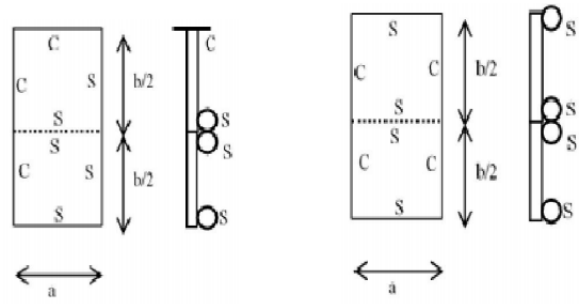
- Justification and validating of N2 (ω12):

Let us focus now on the sub plate (a/b')=(a/(b/2)). For the first plate (SCSC), we get vibration contribution of two types of the same beam [SCSC ⟶2 (SS)]. Hence, for the second plate (SSCC), we also get vibration contribution of two beams, one having clamped and roller supports, and the other having pin support at one end and roller support at the other [SSCC ⟶(CS)+(SS)]. (Figure 6)

Finally, let's compare the two cases:

Plate SCSC: Mode N2 (ω12) involve ⟶4 Roller ⟶4S

Plate SSCC: Mode N2 (ω12) involve ⟶1 Clamp + 3 Roller ⟶ C+3S



$a/b' = 3/2 \Rightarrow L_y$

$\Rightarrow (4S) < (C+3S) \Rightarrow (\omega_{12})_{SCSC} < (\omega_{12})_{SSCC}$

figure6 : Qualitative analysis (mode 2)

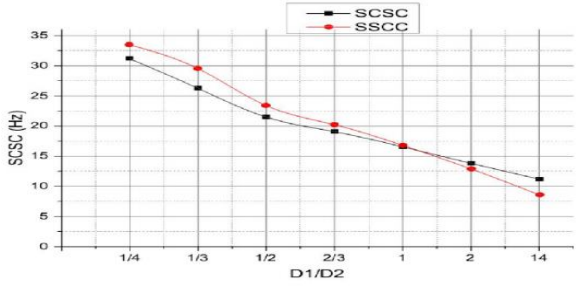


figure7 : effect of rigidity ratio on evolution of frequencies (mode 2)

Based on a qualitative analysis, and since we have (4S)<(C+3S), we can say that we should have (ω12)SCSC<(ω12)SSCC . This result is consistent with that given by the FEM analysis (figure 3). Now, by introducing different rigidity ratios, and by observing the graph (figure 7), we can say that between a ratio (Dx/Dy)=0.25 and a ratio (Dx/Dy)=1, and a dimension ratio (a/b=0.75), the rigidity ratio changes have no effect on the behavior of the plate. Hence, for a ratio (Dx/Dy) greater then (1) and the same dimension ratio (a/b=0.75) the behavior of the two plates changes and the behavior is reversed and becomes dominated by the side (Lx=a). In this case we have (2C+2S)<(4C) which means that (ω12)SSCC<(ω12)SCSC.

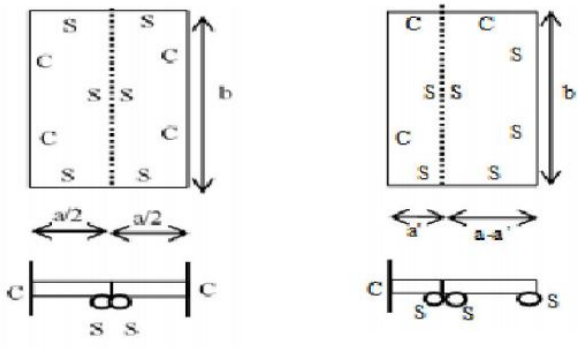
- Justification and validating of N3 (ω21):

The qualitative analysis of mode (N3) for both plate cases (SCSC) and (SSCC) and focusing on the sub plate (a'/b)=(a/2)/b, shows that the behavior should be dominated by the small side (Lx=a).

By comparing the two cases, we have:

Plate SCSC: Mode N3 (ω21) involve ⟶2 Clamps +2 Roller ⟶2C+2S

Plate SSCC: Mode N3 (ω21) involve ⟶1 Clamp + 3 Roller ⟶C+3S



$a'/b = (a/2)/b = 6/4 \Rightarrow L_x$

$\Rightarrow (C+3S) < (2C+2S) \Rightarrow (\omega_{21})_{SSCC} < (\omega_{21})_{SCSC}$

figure8 : Qualitative analysis (mode 3)

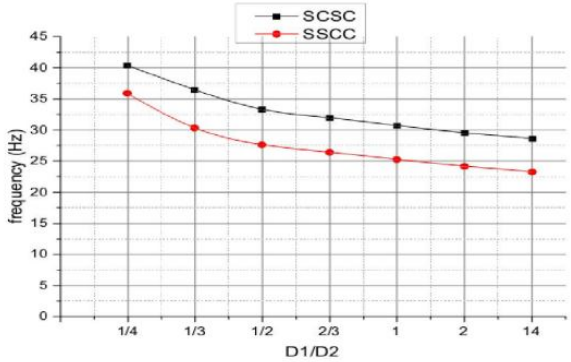


figure9 : effect of rigidity ratio on evolution of frequencies (mode 3)

Since we have (C+3S) < (2C+2S), we can say that we should have (ω21)SSCC < (ω21)SCSC . This result is consistent with that given by the FEM analysis (figure 3).

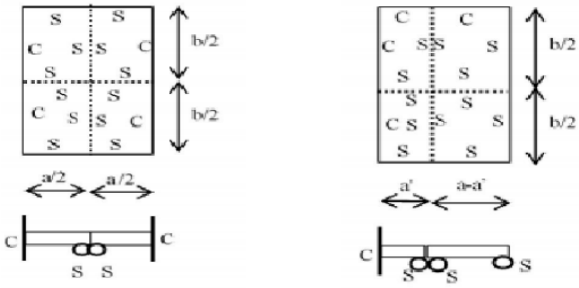
Now, to see the influence of rigidity ratios on the behavior of the plate, and by observing the graph (figure 7), since the order of the curves is respected and the comparison (ω21)SSCC< (ω21)SCSC is valid, we can deduce that the rigidity ratio changes don't affect the behavior of the plate in the mode N3.

• Justification and validating of N4 (ω22):

Let's compare the two cases:

Plate SCSC: Mode N4 (ω22) involve ⟶ 4 Clamps + 4 Roller ⟶ 4C + 4S

Plate SSCC: Mode N4 (ω22) involve ⟶ 2 Clamps + 6 Roller ⟶ 2C + 6S



$$a'/b' = (a/2)/(b/2) = 3/4 \Rightarrow L_x$$

$$\Rightarrow (C+3S) < (2C+2S) \Rightarrow (\omega_{22})_{SSCC} < (\omega_{22})_{SCSC}$$

figure 10 : Qualitative analysis (mode 4)

Based on a qualitative analysis and since we have (2C+6S)<(2C+4S), we should have (ω22)SSCC<(ω22)SCSC because there is less restraint in plate (SSCC) than plate (SCSC). This result is confirmed by the FEM analysis (figure 3).

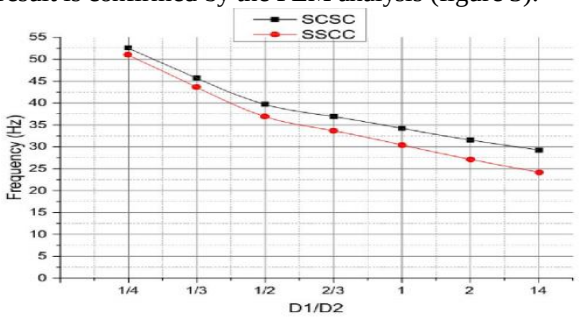


figure 11 : effect of rigidity ratio on evolution of frequencies (mode 4)

By introducing the rigidity ratio effect and by observing the graph (figure 11), we can say that the rigidity ratio has no effect on the behavior of the plate since the order of the curves is respected which means that we always have (ω22)SSCC< (ω22)SCSC.

• Justification and validating of N5 (ω31):

By focusing on the sub plate (a"/b)=((a/3)/b) and using qualitative analysis, we notice that the behavior is

dominated by the small side (Lx=a).

We should have (ω31)SSCC<(ω31)SCSC since we have (C+5S)<(2C+4S)

Plate SCSC: Mode N4 (ω31) involve ⟶2Clamps + 4 Roller ⟶2C+4S

Plate SSCC: Mode N4 (ω31) involve ⟶1 Clamp + 5 Roller ⟶2C+6S

This result is confirmed by the FEM analysis (figure 3).

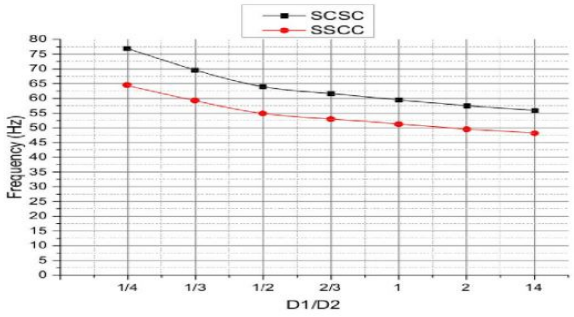
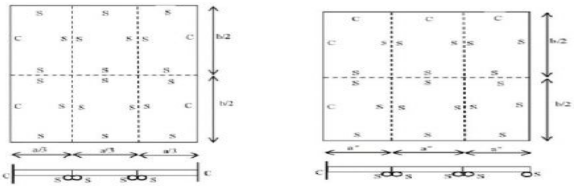


figure 13 : effect of rigidity ratio on evolution of frequencies (mode 5)

Now, by introducing the rigidity ratio effect and by observing the graph (figure 13), we can say that the rigidity ratio changes don't affect the behavior of the plate.

• Justification and validating of N6 (ω32):

If we focus on the sub plate (a"/b')=((a/3)/(b/2)) and using qualitative analysis, we notice that the behavior should be dominated by the small side (Lx=a),



$$a''/b' = (a/3)/(b/2) = 1/2 \Rightarrow L_x$$

$$\Rightarrow (C+5S) < (2C+4S) \Rightarrow (\omega_{32})_{SSCC} < (\omega_{32})_{SCSC}$$

figure 14 : Qualitative analysis (mode 6)

Two types of contribution will be discussed:

Plate SCSC: Mode N6 (ω32) involve ⟶4 Clamped + 8 Roller ⟶ 4C+8S

Plate SSCC: Mode N4 (ω31) involve ⟶2 Clamps + 10 Roller ⟶ 2C+10S

Since we have (2C+10S) < (4C+8S) we should have (ω32)SSCC<(ω32)SCSC. This result is confirmed by the FEM analysis (figure -3-).

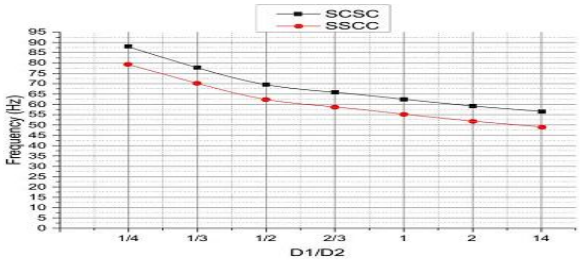


figure 15 : effect of rigidity ratio on evolution of frequencies (mode 6)

Now, by observing the graph (figure 15) and by introducing rigidity ratio effect, we notice that the order of curves is respected which means that

$(\omega 32)SSCC<(\omega 32)SCSC$, and we can say that the rigidity ratio doesn't affect the behavior of the plate.

CONCLUSION

This investigation using the qualitative method was carried out to verify and to ascertain the relative position of the frequency curves.

The main goal of this investigation was to see if the changes in the rigidity ratio affect the general behavior of an orthotropic plate which have a dimension ratio ($a/b<1$).

As conclusion, and by referring to previous works[33] when the case of an orthotropic plate with a dimension ratio ($a/b>1$), we can say that the rigidity ratio changes have no effect on the general behavior of rectangular orthotropic plates.

However, we should mention that for the mode number 3 and at a specified rigidity ratio for the two cases ($a/b<1$) and ($a/b>1$), the order of the curves change, this observations will be discussed in future works.

rectangular orthotropic plates. However we should mention that at a certain rigidity ratio ($0.7 \div 0.2$) and for the mode number 3, this ratio changes the order of the curves.

REFERENCES

[1]. A.W. Leissa, Recent research in plate vibrations: complicating effects, Shock and Vibration Digest9 (11) (1977) 21–35.

[2]. A.W. Leissa, Plate vibration research 1976–1980: complicating effects, Shock and Vibration Digest13 (10) (1981) 19–36.

[3]. A.W. Leissa, Recent research in plate vibrations. 1981–1985—Part II: complicating effects, Shock and Vibration Digest19 (3) (1987) 10–24.

[4]. C.W. Bert, Dynamics of composite and sandwich panels—Part I and II, Shock and Vibration Digest8 (11) (1976) 15–24.

[5]. C.W. Bert, Recent research in composite and sandwich plate dynamics, Shock and Vibration Digest11 (10) (1979) 13–23.

[6]. C.W. Bert, Research on dynamics of composite and sandwich plates, Shock and Vibration Digest14 (10) (1982) 17–34.

[7]. C.W. Bert, Research on dynamic behaviour of composite and sandwich plates—Part IV, Shock and Vibration Digest17 (11) (1985) 3–25.

[8]. C.W. Bert, Research on dynamic behavior of composite and sandwich plates. V—Part I, Shock and Vibration Digest23 (6) (1991) 3–14.

[9]. C.W. Bert, Research on dynamic behavior of composite and sandwich plates. V—Part II, Shock and Vibration Digest23 (7) (1991) 9–21.

[10]. L. Meirovitch, Elements of Vibration Analysis, McGraw-Hill, New York, 1986.

[11]. R.F.S. Hearmon, The fundamental frequency of vibration of rectangular wood and plywood plates, Proceedings of the Physical Society of London, vol. 58, 1946, pp. 78–92.

[12]. R.F.S. Hearmon, The frequency of flexural vibration of rectangular orthotropic plates with clamped or supported edges, Journal of Applied Mechanics26 (1959) 537–540.

[13]. G.B. Warburton, The vibration of rectangular plates, Proceedings of the Institution of Mechanical Engineers, vol. 168, 1954, pp. 371–384.

[14]. R.D. Marangoni, L.M. Cook, N. Basavanhally, Upper and lower bounds to the natural frequencies of vibration of clamped rectangular orthotropic plates, International Journal of Solids and Structures14 (1978) 611–623.

[15]. N.W. Bazely, D.W. Fox, J.T. Stadter, Upper and lower bounds for frequencies of rectangular clamped plates, Technical Memo. TG-626, Applied Physics Laboratory, The John Hopkins University, Baltimore, May, 1965.

[16]. R.E. Rossi, D.V. Bambill, P.A.A. Laura, Vibrations of a rectangular orthotropic plate with a free edge: comparison of analytical and numerical results, Ocean Engineering25 (7) (1998) 521–527.

[17]. P.A.A. Laura, D.V. Bambill, V.A. Jederlinic, K. Rodriguez, P. Diaz, Rayleigh's optimization concept and the use of sinusoidal coordinate functions, Journal of Sound and Vibration200 (1997) 557–561.

[18]. D.J. Gorman, Accurate free vibration analysis of clamped orthotropic plates by the method of superposition, Journal of Sound and Vibration140 (3) (1990) 391–411.

[19]. D.J. Gorman, Free Vibration Analysis of Rectangular Plates, Elsevier, North Holland, New York, 1982.

[20]. N. Li, Forced vibration analysis of the clamped orthotropic rectangular plate by the superposition method, Journal of Sound and Vibration158 (2) (1992) 307–316.

[21]. F. Moussu, M. Nivoit, Determination of elastic constants of orthotropic plates by modal analysis/method of superposition, Journal of Sound and Vibration165 (1) (1993) 149–163.

[22]. T. Sakata, K. Hosokawa, Vibrations of clamped orthotropic rectangular plate, Journal of Sound and Vibration125 (3) (1988) 429–439.

[23]. T. Sakata, K. Takahashi, R.B. Bhat, Natural frequencies of orthotropic rectangular plates obtained by iterative reduction of the partial differential equation, Journal of Sound and Vibration189 (1) (1996) 89–101.

[24]. R.B. Bhat, J. Singh, G. Mundkur, Plate characteristic functions and natural frequencies of vibration of plates by iterative reduction of partial differential equation, Journal of Vibration and Acoustics115 (2) (1993) 177–181.

[25]. I.A. Veres, L.P. Kollar, Approximate analysis of mid-plane symmetric rectangular composite plate, Journal of Composite Materials36 (6)(2002) 673–684.

[26]. J.M. Whitney, Structural Analysis of Laminated Anisotropic Plates, Technomic Publishing Co., Dayton, OH, USA, 1987.

[27]. S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, New York, 1959.

[28]. Y.Z. Chen, Evaluation of fundamental vibration frequency of an orthotropic bending plate by using an iterative approach, Computer Methods in Applied Mechanics and Engineering161 (1998) 289–296.

[29]. T. Sakata, A note on nodal lines of rectangular plates, Journal of Applied Mechanics45 (1978) 691–693.

[30]. Leissa A.W "Vibration of Plates", NASA SP-160, Scientific and technical information Division of NASA, Washington, US, DC, 1969.

[31]. Biancolini M.E, Butti. C, Reccia. L, "Approximation solution for free vibration of thin orthotropic plates". Journal of sound and vibration 2005.

[32]. A.Sekhri, T. Benmansour, H. Cheridi, "Equivalents methods to investigate free vibration of isotropic and orthotropic thin rectangular plate with non-homogeneous supports", International Review of Mechanical Engineering, Vol5, N7, 2011.

[33]. S. Hamada, T. Benmansour " Analysis of complex structures by equivalent methods: application to orthotropic plates and the effect of rigidity ratio", 5th International Symposium on Technology aeronautics ACMA2014, 23-26 April 2014 Marrakech, Morocco.

★★★