EFFECT OF MAGNETIC FIELD ON NON-NEWTONIAN BLOOD IN STENOSED ARTERY

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Abstract - In this paper, the effect of magnetic field on blood is considered in stenosed artery under influence of an oscillating pressure gradient in the axial direction and the periodic body acceleration.. Approximate solutions to the blood velocity, particles velocity and the shear stress are obtained in series forms. For the variation of the magnetic field intensity, the magnitude of maximum value of fluid velocity becomes higher in the case of lower values of the Hartman number, therefore, in the case of weak magnetic fields. This behavior is due of the Lorentz's force which opposes the motion of an electrically conducting fluid.

I. INTRODUCTION

Several authors have studied the blood flow through artery with stenosis using Newtonian fluid model or non-Newtonian fluid models [1-4]. Nagarani and Sarojamma [5] have analyzed blood flow through a stenosed artery under effect of body acceleration using the Casson fluid model. They found that the yield stress of fluid and the body acceleration strongly influence the flow rate in a stenosed artery. The effects of stenosis length on resistance to flow of the blood considered as a Bingham fluid, through an artery with multiple stenoses have been studied by Yadav and Kumar [6]. They obtained that the flow resistance decreases if the shape parameter increases and, the flow resistance increases if increases the height / length of stenosis. Srikanth [9] has studied the pulsatile blood flow in a multiple stenotic artery using the micropolar fluid model and the couple-stress fluid model. Siddiqui et. al. [7] has considered the blood flow through a stenosed artery with body acceleration and oscillating pressure gradient, using the Bingham plastic fluid model. They obtained analytic solution to the blood velocity, flow rate, shear stress and effective blood viscosity. The study reveals that the blood velocity and the flow rate increase with the body acceleration, while the effective blood viscosity and shear stress decrease. Applications to the targeting of a magnetic drug are an important subject of research in medical treatment methods for the effective targeting and delivery of drugs to a specific target area [8-10]. Furlani and Ng [11] have studied a mathematical model for transport of the therapeutic magnetic nanoparticles in the human microvasculature. The aim of this paper is to study the blood flow along with magnetic particles through an arterial stenosis under the influence of a magnetic field perpendicular to the flow direction, an oscillating pressure waveform in the axial direction and the body acceleration. We assume that the blood is flowing in the axial direction and the magnetic particles are uniformly distributed throughout in the blood. Using the suitable non-dimensional variables, we have formulated the dimensionless initial-boundary value problem which governs the flow. Approximate solutions for the axial fluid velocity, particles velocity and shear stress are obtained for small values of the Womersley's parameter. In order to eliminate this restrictive condition, we have determined an analytical solution using the Laplace and finite Hankel transforms. These analytical solutions are new in the literature and can be used to study the fluid flow parameters for any value of the Womersley parameter.

II. MATHEMATICAL MODEL FORMULATION

We consider the pulsatile blood flow along with magnetic particles through a stenosed artery with slip velocity at the constricted wall. The blood is considered as a non-Newtonian fluid of Bingham plastic model [10, 17, 26-28], in the presence of an applied magnetic field, an oscillating pressure gradient in the axial direction and periodic body acceleration. The magnetic particles are flowing along the axis of the artery and are uniformly distributed throughout in the blood.

2.1Model equations

We suppose that the stenosis of the arterial wall (Fig. 1) is a function of the axial coordinate z, given by Eq. (1)

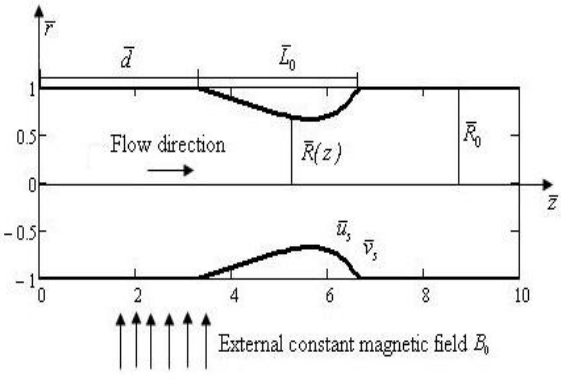


Fig. 1 Geometry of stenosis

$$\frac{\bar{R}(\bar{z})}{\bar{R}(0)} = \begin{cases} 1-\bar{\psi}\left[\bar{L}_0^{m-1}(\bar{z}-\bar{d})-(\bar{z}-\bar{d})^m\right], & \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0 \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

where $\bar{R}(\bar{z})$ is the radius of the artery in the stenosed region, $\bar{R}(0)=\bar{R}_0$ is the radius of the normal artery, $\bar{L}_0$ is the length of the stenosis, $\bar{d}$ is the region of the stenosis and $\bar{\psi} = \left(\frac{\xi_s}{\bar{R}(0)\bar{L}_0^m}\right)\frac{m^{m/(m-1)}}{(m-1)}$ is the maximum height of the stenosis at $\bar{z} = \bar{d} + \left(\bar{L}_0 / m^{m/(m-1)}\right)$ such that $\xi_s / \bar{R}(0) < 1.$

The considered model for the blood motion consists of Navier-Stokes equations, the constitutive equations of the Bingham plastic fluid and the following Maxwell's equations of the magnetic field:

$$\nabla\cdot\mathbf{B}=0,\ \nabla\times\mathbf{B}=\mu_0\mathbf{J},\ \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \quad (2)$$

where $\mathbf{B}$ is the magnetic flux intensity, μ_0 is the magnetic permeability, $\mathbf{E}$ is the electric field intensity and $\mathbf{J}$ is the current density given by

$$\mathbf{J}=\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B}), \quad (3)$$

σ being the electrical conductivity and $\mathbf{v}$ is the velocity field

The electromagnetic body force $\mathbf{F}_{em}$ is defined as

$$\mathbf{F}_{em} = \mathbf{J}\times\mathbf{B} = \sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B})\times\mathbf{B} = \sigma B_0^2\,\bar{u}(r,t)\mathbf{k}, \quad (4)$$

where $\mathbf{k}$ is the unite vector of the z-direction and $\mathbf{v}=\bar{u}(r,t)\mathbf{k}$ is the axial velocity of the blood. The electromagnetic body force $\mathbf{F}_{em}$ is inserted in the momentum Eq. (5).

In the assumption that the ratio of the maximum height of the stenosis and the radius of the non-stenotic region, respectively, the ratio of the radius of non-stenotic region and the length of stenosis are much smaller than the unity, the governing equation of motion and stress tensor are given by [8]:

Momentum equation

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}}\right) = -\left(\frac{\partial\bar{p}}{\partial\bar{z}}\right) - \left(\frac{1}{\bar{r}}\right)\frac{\partial}{\partial\bar{r}}\left(\bar{r}\bar{\tau}\right) + \bar{G}(\bar{t}) + KN(\bar{v}-\bar{u}) - \sigma B_0^2\bar{u}, \quad (5)$$

The constitutive Bingham plastic model stress tensor $\bar{\tau}$ of the form [10]

$$\bar{\tau} = \bar{\tau}_y - \mu\left(\frac{\partial\bar{u}}{\partial\bar{r}}\right); \quad \text{for } \bar{\tau}\ge\bar{\tau}_y \quad (6)$$

$$\frac{\partial\bar{u}}{\partial\bar{r}} = 0, \quad \text{for } \bar{\tau}<\bar{\tau}_y \quad (7)$$

where $\bar{u}$ is the axial velocity of blood, $\bar{v}$ is the velocity of particles, $\bar{t}$ is the time, $\bar{p}$ is the pressure, $\bar{\tau}$ is the shear stress, $\bar{G}(\bar{t})$ is the body acceleration, B_0 is the externally applied constant magnetic field, $\bar{\tau}_y$ is the yield stress, ρ is the density of blood, μ is the dynamic viscosity of the blood, K is the Stokes constant, N is the number of magnetic particles per unit volume and $\frac{KN}{\rho}(\bar{v}-\bar{u})$ is the interaction between motion of blood and magnetic particles. We considered the Reynolds number of the relative velocity is small; therefore the force between blood and magnetic particles is proportional to the relative velocity.

The periodic body acceleration $\bar{G}(\bar{t})$ and pressure gradient $\frac{\partial\bar{p}}{\partial\bar{z}}(\bar{z},\bar{t})$ for $t>0$, which appears in Eq. (5), have the expression of the following forms [10]:

$$\bar{G}(\bar{t}) = B_0\cos(\bar{\omega}_b\bar{t}+\varphi), \quad (8)$$

$$-\frac{\partial\bar{p}}{\partial\bar{z}}(\bar{z},\bar{t}) = A_0 + A_1\cos(\bar{\omega}_p\bar{t}), \quad (9)$$

where $\bar{\omega}_b = 2\pi\bar{f}_b$ is the frequency, $\bar{f}_b$ is the pulse rate frequency, φ is the lead angle of the body acceleration with respect to the pressure gradient, $\bar{\omega}_p = 2\pi\bar{f}_p$ is the heart pressure frequency, $\bar{f}_p$ is the pulse rate frequency, A_0 is the amplitude of the steady state pressure gradient, and A_1 is the amplitude of the oscillating pressure gradient.

The motion of nanoparticles in the vascular system is governed by a number of forces [16]. In the current investigation, we consider the fluidic force and the unsteady flow analysis are implemented. Therefore, the motion of magnetic single particle is governed by Newton's law for dynamic of particle motion

$$M\frac{\partial\bar{v}}{\partial t} = \sum F_{\text{ext}}, \quad (10)$$

where M is the mass of the single nanoparticles and $\sum F_{\text{ext}} = F_m + F_b + F_{\text{blood}}$ represents all the external forces exerted on the particle from which F_m is the magnetic force, F_b is the buoyancy force and F_{blood} is the fluidic force.

The fluidic force experience by a particle, assuming a Reynolds number $\text{Re}\le 1$, is predicted using Stokes's law for viscous drag on a sphere

$$F_{\text{blood}} = -6\pi\mu E_{\text{hyd,p}}(\bar{v}-\bar{u}), \quad (11)$$

where $E_{\text{hyd, p}}$ is the effective hydrodynamic radius of the particle, $\bar{v}$ is the particle velocity, the $\mathcal{L}$ sign indicates that the fluidic force acts opposite to the direction of the particles motion. We predict particle motion when only the force on the particle is fluidic and no buoyancy force

$$M\frac{\partial \bar{v}}{\partial t} = K(\bar{u}-\bar{v}), \quad (12)$$

where $K = 6\pi\mu E_{\text{hyd, p}}$ is the Stokes's constant.

2.2 Initial and boundary conditions

To the proposed problem, we consider the following initial and boundary condition

$$\bar{u} = \bar{v} = 0, \quad \text{at} \quad \bar{t} = 0, \quad (13)$$

$$\bar{u} = \bar{u}_s \text{ and } \bar{v} = \bar{v}_s \text{ at } \bar{r} = \bar{R}(\bar{z}), \quad (14)$$

$$\bar{\tau} \text{ is finite}, \quad \text{at } \bar{r} = 0, \quad (15)$$

where $\bar{u}_s$, $\bar{v}_s$ are the slip velocities at the stenotic walls (see Fig. 1)

The dimensionless forms of the governing equations are obtained by considering the following dimensionless variables:

$$u = \frac{\bar{u}}{A_0\bar{R}_0^2/4\mu}, \quad v = \frac{\bar{v}}{A_0\bar{R}_0^2/4\mu}, \quad \theta = \frac{\bar{\tau}_y}{A_0\bar{R}_0/2}, \quad R(z) = \frac{\bar{R}(z)}{\bar{R}_0}$$

$$t = \frac{\bar{t}}{\bar{w}_p}, \quad A = \frac{A_1}{A_0}, \quad \xi_s = \frac{\bar{\xi}_s}{\bar{R}_0}, \quad B = \frac{B_0}{A_0}, \quad r = \frac{\bar{r}}{\bar{R}_0}, \quad (16)$$

$$u_s = \frac{\bar{u}_s}{A_0\bar{R}_0^2/4\mu}, \quad v_s = \frac{\bar{v}_s}{A_0\bar{R}_0^2/4\mu}, \quad \omega = \frac{\bar{w}_b}{\bar{w}_p}, \quad \tau = \frac{\bar{\tau}}{A_0\bar{R}_0/2}.$$

Eqs. (5), (12) and (7) becomes:

$$\alpha^2\frac{\partial u}{\partial t} = f(t) + R_1(v-u) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau) - Ha^2u, \quad (17)$$

$$\alpha^2\frac{\partial v}{\partial t} = \frac{1}{G}(u-v), \quad (18)$$

$$\tau = \theta - \frac{1}{2}\frac{\partial u}{\partial r}; \quad \text{for } \tau \geq \theta \quad (19)$$

$$\frac{\partial u}{\partial r} = 0, \quad \text{for } \tau < \theta \quad (20)$$

The dimensionless initial and boundary conditions are:

$$u = v = 0, \quad \text{at} \quad t = 0, \quad (21)$$

$$u = u_s, \quad v = v_s, \quad \text{at} \quad r = R(z) \quad (22)$$

$$\tau \text{ is finite}, \quad \text{at } r = 0, \quad (23)$$

where $\alpha^2 = \frac{\bar{w}_p\bar{R}_0^2\rho}{\mu}$, α is the Womersley frequency parameter, $Ha = \sqrt{\frac{\sigma}{\mu}}B_0\bar{R}_0$ is the Hartmann number, $R_1 = \frac{KN\bar{R}_0^2}{\mu}$ is the particle concentration parameter, $G = \frac{M\mu}{\rho\bar{R}_0^2K}$ is the particle mass parameter and $f(t) = 4[(1 + A\cos t) + B\cos(\omega t + \varphi)]$,

It is important to point out that, if the particle concentration parameter is zero ($R_1 = 0$) and no magnetic field effect ($Ha=0$), then Eq. (17) represent the classical model which was given by [10].

III. SOLUTION TECHNIQUE

Let the blood velocity u, the magnetic particles velocity v and the shear stress τ expressed in the forms:

$$u(z,r,t) = u_0(z,r,t) + \alpha^2u_1(z,r,t) + \ldots \quad (24)$$

$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2\tau_1(z,r,t) + \ldots \quad (25)$$

$$v(z,r,t) = v_0(z,r,t) + \alpha^2v_1(z,r,t) + \ldots \quad (26)$$

Substituting u, v and τ from Eqs. (24)-(26) into Eqs. (17)-(19) and equating the coefficients of α^0 and α^2, respectively, we obtained:

$$f(t) + R_1(v_0 - u_0) - Ha^2u_0 - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_0) = 0, \quad (27)$$

$$u_0 - v_0 = 0, \quad (28)$$

$$\tau_0 = \theta - \frac{1}{2}\frac{\partial u_0}{\partial r}. \quad (29)$$

and

$$\frac{\partial u_0}{\partial t} = R_1(v_1 - u_1) - Ha^2u_1 - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_1), \quad (30)$$

$$G\frac{\partial v_0}{\partial t} = u_1 - v_1, \quad (31)$$

$$\tau_1 = -\frac{1}{2}\frac{\partial u_1}{\partial r}. \quad (32)$$

Substituting Eqs. (28) and (29) into Eq. (27), we obtain

$$\frac{2}{r}\frac{\partial}{\partial r}\left[r\left(\theta - \frac{1}{2}\frac{\partial u_0}{\partial r}\right)\right] + Ha^2 u_0 - f(t) = 0, \quad (33)$$

Making the change of function

$$Ha^2 u_0 - f(t) = 2F_0(r,z,t), \quad (34)$$

Eq. (33) becomes

$$r\frac{\partial^2 F_0}{\partial r^2} + \frac{\partial F_0}{\partial r} - Ha^2 r F_0 = \theta Ha^2. \quad (35)$$

We seek the solution of Eq. (35) in the form of power series as

$$F_0 = \sum_{n=0}^{\infty} C_n(z,t) r^n. \quad (36)$$

It follows that, the general terms for C_{2n+1} and C_{2n+2} can be immediately followed, respectively as:

$$C_{2n+1} = (2Ha)^{2n}\left(\frac{n!}{(2n+1)!}\right)^2 C_1 = (2Ha)^{2n}\left(\frac{n!}{(2n+1)!}\right)^2 \theta Ha^2, \quad (37)$$

$$C_{2n+2} = \left(\frac{Ha}{2}\right)^{2(n+1)} \frac{C_0}{[(n+1)!]^2} \quad n = 0,1,2,..., \quad (38)$$

The function $F_0(r,z,t)$ is expressed as:

$$\begin{aligned} F_0(r,z,t) &= C_0 + \sum_{n=0}^{\infty} C_{2n+1} r^{2n+1} + \sum_{n=0}^{\infty} C_{2n+2} r^{2n+2} \\ &= C_0 + \theta Ha^2 \sum_{n=0}^{\infty} (2Ha)^{2n}\left(\frac{n!}{(2n+1)!}\right) r^{2n+1} \quad (39) \\ &\quad + C_0 \sum_{n=0}^{\infty}\left(\frac{Ha}{2}\right)^{2(n+1)} \frac{r^{2n+2}}{[(n+1)!]^2}, \end{aligned}$$

which can be written in the simpler form

$$F_0(r,z,t) = C_0(z,t) I_0(H_a r) + \frac{\pi\theta}{2}\sum_{n=0}^{\infty} \frac{1}{\left[\Gamma\left(n+\frac{3}{2}\right)\right]^2}\left(\frac{rHa}{2}\right)^{2n+1}. \quad (40)$$

The solution u_0 can be expressed as

$$u_0 = \frac{2F_0(r,z,t) + f(t)}{Ha^2}. \quad (41)$$

Using the boundary condition $u_0 = u_s$ at $r = R(z)$, we obtain

$$C_0(z,t) = \frac{1}{2}\left[\frac{Ha^2 u_s - f(t) - \pi\theta Ha^2 b(z)}{I_0(H_a R(z))}\right], \quad (42)$$

with $b(z) = \sum_{n=0}^{\infty} \frac{1}{\left[\Gamma\left(n+\frac{3}{2}\right)\right]^2}\left[\frac{H_a R(z)}{2}\right]^{2n+1}$ and I_0 is the modified Bessel function of order zero.

The solution of v_0 and τ_0 can be readily expressed as

$$v_0 = u_0, \quad (43)$$

$$\tau_0 = \theta - \frac{1}{Ha^2}\frac{\partial F_0(r,z,t)}{\partial r}. \quad (44)$$

Following the same procedure for obtaining u_0, v_0 and τ_0, the u_1, v_1 and τ_1 can be achieved similarly.

Substituting Eqs. (31) and (32) into Eq. (30), we obtain:

$$r\frac{\partial^2 u_1}{\partial r^2} + \frac{\partial u_1}{\partial r} - Ha^2 u_1 r = \left[1 - \sum_{n=0}^{\infty}\left(\frac{Ha}{2}\right)^{2(n+1)} \frac{r^{2n+2}}{[(n+1)!]^2}\right] h(t) r, \quad (45)$$

where

$$h(t) = \frac{4(1+RG)}{Ha^2}\frac{df(t)}{dt}. \quad (46)$$

The solution of the problem (45), (46) with the boundary condition $u_1(R(z),z,t) = 0$ is

$$\begin{aligned} u_1(r,z,t) &= \frac{h(t)}{4} r^2 + D_0(z,t) + D_0(z,t)\sum_{n=1}^{\infty}\left(\frac{Ha}{2}\right)^{2n+2} \frac{r^{2n+2}}{((n+1)!)^2} \\ &\quad + \sum_{n=1}^{\infty} D_{2n+1} r^{2n+1}, \; D_0 = -\frac{\frac{h(t)}{4}R^2(z) + \sum_{n=1}^{\infty} D_{2n+1} R^{2n+1}(z)}{1 + \sum_{n=0}^{\infty}\left(\frac{Ha}{2}\right)^{2n+2}\frac{R^{2n+2}(z)}{((n+1)!)^2}}. \end{aligned} \quad (47)$$

The solution of v_1 and τ_1 are immediately follow as

$$v_1 = u_1 - G\frac{\partial v_0}{\partial t}, \; \tau_1 = -\frac{1}{2}\frac{\partial u_1}{\partial r}. \quad (48)$$

IV. RESULTS AND DISCUSSION

We have investigated the mathematical model that represents non-Newtonian flow of the blood through a stenosed artery, in the presence of an external magnetic field perpendicular to the flow direction. Periodic body acceleration and an oscillating pressure gradient along the flow direction are considered by the function $f(t)$, periodic in time (Fig. 2).

Fig. 3 exhibits how the shear stress is perturbed at the different Hartman numbers Ha, different radial positions and also, at different axial positions of the stenosed artery. It is observed from Fig. 5 that, the shear stress consistently decreases in the presence of a magnetic field with increasing intensity. The peak stress occurs at the throat of the stenosis. The shear stress distribution plays an important role, because, the deterioration of the arterial wall is related to the generation of tension on the arterial walls.

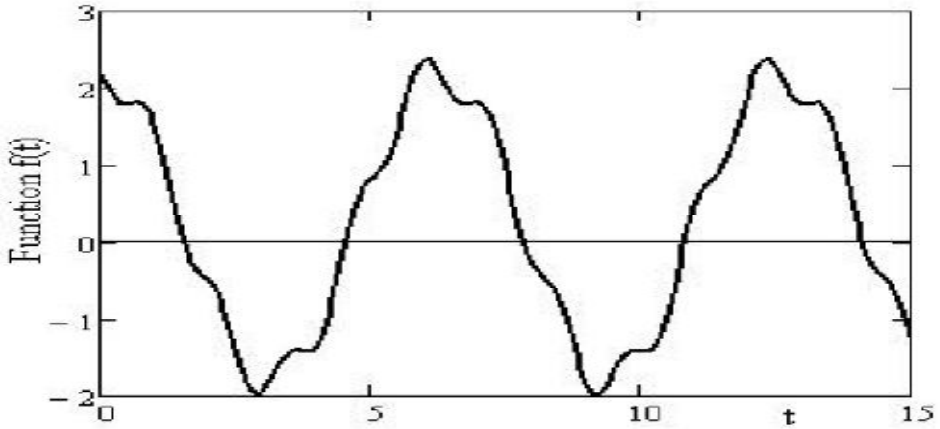


Fig. 2. Function f(t) – the oscillating pressure gradient and body acceleration z,r,t) for time t = 4 and for different values of Hartman number Ha

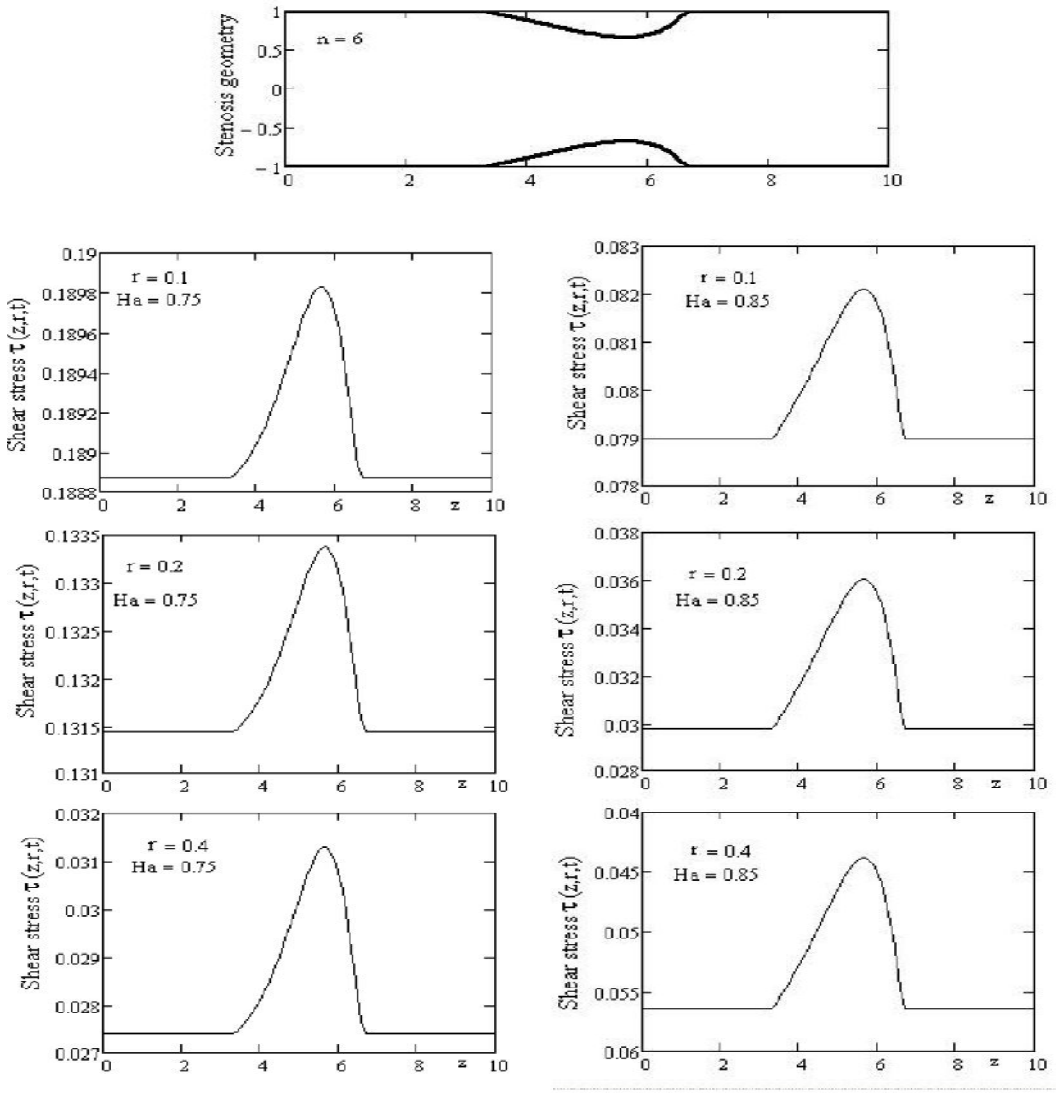


Fig. 3. Profiles of the shear stress τ(z,r,t) for time t = 1, θ = 0.75 and for different values of Hartman number Ha

CONCLUSIONS

The main results of the current study are concise as follows:

- the blood velocity and magnetic particles velocity can be controlled by adjusting the magnetic parameter..
- the shears stress has variation from a constant in the area without stenosis and much higher in the layers located close to the longitudinal axis of the artery
- the peak velocity and peak shear stress occur at the throat of the stenosis.

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