OPINION DYNAMICS IN THE HEGSELMANN-KRAUSE MODEL ON THE CIRCLE: ANALYSIS AND SIMULATION

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Abstract– Over the last decades, the problem of synchronization has been solving by many researchers. Opinion formation is one of the critical issues in sociology. By opinion formation we mean the changing opinions in a group of people even if they have different opinions at the beginning. In this senior thesis we consider Hegselmann-Krause model on a segment and modify this model on the circle. The study and analysis of consensus algorithms are based on matrix and graph theory. We will establish that there may be several results: either a group of people always achieves a consensus or there is a clustering of opinions, that is, in each cluster people hold the same opinion.Examples that support theoretical assumptions are given. We also developed programs on Python and Mathematica Wolfram which help to find out how many different opinions we have as a result. These technical tools plot graphs to see the general trend of solution to specific problems.

Key words - opinion dynamics, Hegselmann-Krause model, bounded confidence, consensus problem, matrix theory.

I. INTRODUCTION

The problem of synchronization's opinion of people has a long history in social science. In the modern world, each person has his own opinion, which determines his behavior in different situations. Moreover, the individual's positions can change under the influence of other people. This situation is especially characteristic when a group of people have to make a common decision. In the process of discussing the problem, the following outcomes may arise: (1) a group of people agree and make a general decision; (2) a group of people is divided into subgroups with different opinions. It is important to predict people's opinions and to apply these results in sociology [1] – [8].

The task of consensus is widely used to solve practical problems in various fields. A model in which objects change their mind under the influence of other people is of particular interest to scientists [4].This model was first published in 1997 Hegselmann and Krause.

The primary objective in this paper is to analyze the Hegselmann-Krause model with limited confidence on the circle. We will consider this approach as a special kind of consensus algorithm.

In this senior thesis we will describe in detail the Hegselmann-Krause model which emulates a discussion among a group of idealised agents who are assumed to be willing to compromise, but only with those whose opinions are sufficiently close to their own. The Hegselmann-Krause model will be modified and viewed on the circle. Moreover, we will analyze in detail the model with limited confidence and compare results. Furthermore, we will develop programs on the programming language Wolfram Mathematica that help to solve numerically and analytically the consensus problem and to build trajectories of each agent of our group.

This paper is organized as follows. Firstly, we consider some previous results of this problem. Secondly, it will be described the methods and approaches applicable to this problem. The last part of this senior thesis is the presentation of results.

II. LITERATURE REVIEW

In the past decades, there has been a rapidly growing interest of studying the synchronization problem and their practical applications in social science. Researchers and scientists have been trying to decribe all possible aspects of this problem, consider all connections of agents in group.

In the scientific literature, we can find the problem of coordination the execution of several processes and the same states of agents. There are a lot of different approaches to solve this problem. EdvinWedin shows that consensus is not guaranteed even for a continuum of agents and a regular opinion function by proving a conjecture by Hendrickx et al. [3]. Also, he considers cases in which he investigates what happens when agents hold one dimensional opinion which are equidistant on an interval. Furthermore, he determines the freezing time of a system where the initial distances between the agents equal the radius of confidence.

The noisy Hegselmann-Krause model is the special case of consensus problem. According to [4] it can be clearly seen that individuals can change spontaneously with certain probability their opinion to another one selected randomly inside the opinion space with different rules. Researchers study the effects of noise in this systemand apply Hegselmann-Krause's rule if and only if when the random jump does not occur. They analyze in detail two protocols. The first case is whenagents can perform jumps only inside a small interval located around the current opinion. Another case is when agents can change

their states doing opinion random jumps inside the whole opinion space.

In [5] SeyedRasoulEtesami and Tamer Basar consider the Hegselmann-Krause model for opinion dynamics. They study the evolution of the system under various settings: synchronous, asynchronous, homogeneous, and heterogeneous, and show that in general the termination time depends on the number of agents involved in the dynamics. They analyze homogeneous and heterogeneous multidimensional Hegselmann-Krause dynamicsand provide a necessary condition for finite time convergence of such dynamics and reaching of consensus. Furthermore, researchers study the asynchronous Hegselmann-Krause model and compare this approach with synchronous model.

Guiyuan Fu and Weidong Zhang focus on in their article a slightly modified Hegselmann-Krause model which is proposed based on non-Bayesian rules. They investigate the continuous opinion dynamics in the population level with group-based heterogeneous confidences [6]. They consider three different cases where all agents are divided into three subgroups: open-minded-, moderate-minded-, and close-minded-subgroups, and each subgroup has their own confident level. The result of their work is the simulations on the complete network and finding dependency of reaching synchronization from group topology.

EdvinWedin, Peter Hegarty analyze Hegselmann-Krause model and proved in their paper an example of a regular opinion function which always retains opinions which are separated into two trends even if it evolves in accordance with the discrete-time Hegselmann-Krause bounded confidence dynamics [7].

III. METHODS

In the Hegselmann-Krausemodel the interaction of people occurs in large groups. Each individual of the group expresses their opinion, listens to the opinions of others, and then adjusts their opinion about those people who are in the area of his trust. It can be that some people correlate with other and this fact make model more difficult [3], [4], [6].

The most convenient tool to illustrate this problem is the graph in which each node is an agent, the edge of the graph is the connection between the agents, and the trust interval can be specified by weights on the edges.

Let n be the number of agents in the group, we denote the group as $V = \{1,2,\dots,n\}$. The agent's opinion and the analyst's opinion, which should make a prediction, are expressed by a real number and belongs to the space $\mathbb{R}$. For each agent i, where $i \in V$, we denote his opinion at time t, as $x_i(t) \in \mathbb{R}$. Given the limited confidence of each agent, we introduce the set $\varepsilon = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$.Each agent i interacts only with its neighbors whose opinions differ from its own no more than a certain confidence level ε_i, then the discrete-time Hegselmann-Krause model is described by equation (1):

$$x_i(t+1) = \frac{1}{|N_i(t)|} \sum_{j \in N_i(t)} (x_j(t) - x_i(t)), \quad (1)$$

where $N_i(t) = \{j \in V | |x_i(t) - x_j(t)| \le \varepsilon\}$ is set of neighbors of agent i at time t, and $|N_i(t)|$–the cardinality $N_i(t)$. Obviously, equation (1) indicates that agent i is always a neighbor and updates his opinion averaging all the opinions of his neighbors with the same weights $\frac{1}{|N_i(t)|}$ [2], [6].

To solve the consensus problems with discrete time, it is important to set the initial state and a step. This step means the time after which it is necessary to know the following state of the system. In this case it is convenient to use the recursion method [2]. We should define the matrix $A(t)$ (2):

$$A_{ij} = \begin{cases} \frac{1}{|N_i(t)|} & i\ j \epsilon N_i(t), \\ 0 & \text{else} \end{cases} \quad (2)$$

The following system (3) describe our model:

$$x(t+1) = \begin{cases} A(t)x(t) \\ x(0) = a \end{cases} \quad (3)$$

The study of consensus algorithm is based on analysis of properties matrix $A(t)$.

There are a lot of methods to define the matrix $A(t)$. Each approach needs to be carefully considered and analyzed. Hegselmann-Krause model describes most accurately the practical tasks of finding a consensus.

In my senior thesis I modify this Hegselmann-Krause algorithm and consider it on a circle. It can be proved that solving of the synchronization problem on a circle will help create a more accurate mathematical model for describing of a large number of important tasks.

IV. RESULTS

As stated in the introduction, the main aim of this thesis is to modify the classic Hegselmann-Krause model and to analyze the consensus problem on a circle.

In this paper, the properties of the deterministic and stochastic models of Hegselmann-Krause models on a circle were analyzed. We compare results of this algorithm on a circle and on a segment. As a result, we established which properties of model on a circle are the same in model on a segment. We proved that each problem can be solved: either a group of people always achieves a result nor there is a clustering of opinions, that is, in each cluster people hold the same opinion.

We also developed an algorithm of reaching consensuson the programming language Python. It solves the given consensus problem and outputs the intermediate results on the screen. This program recursively simulates the trajectories of the synchronization and graphically displays them on the graphs.

The main part of my research was done analytically and numerically using the Wolfram Mathematica software.

CONCLUSION

In this paper, one of the most popular and practical Hegselmann-Krause model of the dynamics opinions is considered. This mathematical model can be applied to solve a lot of practical problems in various disciplines, including social sciences, mathematics, physics, biology, and engineering.

The article covers important aspects of the consensus model. Firstly, we constructed a Hegselmann-Krause model on a circle, study the basic data consensus algorithms. Secondly, we made a comparative analysis of the behavior of the algorithm with limited confidence on the circle and on the line. Moreover, we proved that even the individual's positions change under the influence of other people, consensus can be reached.

We made simulationalgorithms with limited confidence. The programs for implementing an algorithm with limited confidence both for deterministic and stochastic case have been developed on the programming languages Python and Wolfram Mathematica. They facilitate solving the problems of finding a consensus and illustrate them in detail on the graph.

In the future it is planned to research in detail the properties of deterministic models and compare them with stochastic models.

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