ADVANTAGE OF MAKE-TO-STOCK STRATEGY BASED ON LINEAR MIXED-EFFECT MODEL

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Abstract- In the past few decades, demand forecasting becomes relatively difficult because of the rapid changes of world economic environment. In this research, the make-to-stock (MTS) production strategy is applied as an illustration to explain that forecasting plays an essential role in business management. We also suggest that linear mixed-effect (LME) model could be used as a tool for prediction and against environment complexity. Data analysis is based on a real data of order quantity demand from an international display company operating in the industry field, and the company needs accurate demand forecasting before adopting MTS strategy. The forecasting result from LME model is compared to the common used approaches, times series model, exponential smoothing and linear model. The LME model has the smallest average prediction errors. Furthermore, multiple items in the data are regarded as a random effect in the LME model, so that the demands of items can be predicted simultaneously by using one LME model. However, the other approaches need to split the data into different item categories, and predict the item demand by establishing model for each item. This feature also demonstrates the practicability of the LME model in real business operation.

Index Terms- forecasting, linear mixed-effect model, make-to-stock, order demand, production strategy

I. INTRODUCTION

Demand forecasting is crucial for supply chain management. Production planning, inventory management, and manufacturing scheduling are typically formulated according to short- and long-term expected demand [1]. To reduce the occurrence of delivery delays caused by the "crowding out" effect of manufacturing processes, contemporary enterprises have gradually changed their production patterns from make-to-order (MTO) to make-to-stock (MTS), and increasingly fewer enterprises are using the MTO production strategy [2 , 3]. The MTO production involves commencing product production only after the customer places the order. The MTS production pattern entails a stocking-up production, in which a company manufactures products and stores them in inventory before customer orders are received. Subsequently, the company sells its stock as customer places orders. If a company receives orders requesting a high mix of products but in low volumes, it must be capable of forecasting their order demand accurately before attempting an MTS production strategy. Accordingly, the advantages of the MTS production strategy—including quick delivery, arranging a long-term manufacturing schedule, reducing the stock levels, and stabilizing product prices—can be realized. Worldwide, variation in customer demand has forced many manufacturers to adopt a high-mix low-volume production model. However, this type of enterprise is not as efficient as a low-mix high-volume enterprise. Therefore, determining how high-mix low-volume enterprises can enhance their business operation performance urgently requires a solution. Hence, accurately forecasting order demand is a fundamental to successfully applying the MTS production strategy to a high-mix low-volume business operation model. Because inaccurate demand forecast is a concern for high-mix low-volume enterprises, the MTO production strategy is typically adopted. However, this production pattern increases financial risks and requires a long delivery time, making centralized production difficult, which subjects production lines to frequent changes, resulting in high operating costs and low product quality. Complex operations are the primary cause of human error and low job satisfaction. Therefore, if the inefficiency of the high-mix low-volume business operation model cannot be solved, then, despite a high business revenue, business operation costs would increase rapidly, product quality would reduce, and employee job satisfaction and customer satisfaction would decrease, which result in that business development would stagnate. Therefore, the forecasting method proposed in this study can provide a crucial basis for transitioning from using the MTO to the MTS production, and may offer a viable solution for improving the business operation performance of high-mix low-volume enterprises. The application and improvement of the proposed forecasting method can assist researchers with understanding the characteristics of business operations and construct related business operation models. Forecasting ability depends on crucial information and reliable forecasting methods. In recent years, demand forecasting has become increasingly complex, primarily because the global economic environment has gradually changed. The underlying reasons for this change can be explained in terms of the following four dimensions: volatility, uncertainty, complexity, and ambiguity (VUCA) [4 , 5 , 6], all of which have been shown to influence

demand forecasting [7]. Volatility means that new products are rapidly developed, product lifecycles are shortened, customer preferences change suddenly, and organizations are frequently restructured; consequently, historical data diminishes in value. Uncertainty refers to unknown factors that cause sudden shifts in demand, and these factors are generally regarded as outliers or interferences. Complexity means that the interaction of these influential factors cannot be modelled easily, and ambiguity refers to fuzzy events and situations that cannot be quantifiably defined, leading to the loss of key influential factors. In summary, according to the influence of economics on demand forecasting, developing a reliable forecasting method requires analyzing whether historical data can contribute to demand forecasting, and whether the effects of influential factors can be identified. To meet the requirements of modern forecasting methodologies, this study proposed using linear mixed-effect models to perform forecasting. Linear mixed-effect models have been extensively developed and widely applied in various fields. However, no study has used this model to forecasting in business operation. Linear mixed-effect models are characterized by the inclusion of temporal factors and explanatory variables and the analysis of their significance. Accordingly, crucial influential factors can be identified to forecast demand. These characteristics fulfill the requirements of modern forecasting methodologies and can be used as the basis for companies to improve their operation efficiency and to develop competitive advantages. The following sections explore the influences of the MTO and MTS production strategies on business operation as well as the role of forecasting in the MTS strategy, provides a review of the literature on forecasting methodologies, and summarizes the strengths and weaknesses of commonly used forecasting methods. In addition, the proposed linear mixed-effect model as well as a method for model parameter estimation are introduced. Subsequently, the order demand of a manufacturer in central Taiwan is forecasted using product type as a crucial explanatory variable. Specifically, the linear mixed-effect model is applied to forecast the order demand for 20 individual product types. A 1-year forecast of monthly demand is reported, and three types of forecast errors are used to assess the forecasting ability of the model. The results show that the forecasting ability of the linear mixed-effect model in an empirical analysis is superior to those of a linear forecasting model, exponential smoothing method, and time-series forecasting method.

II. LITERATURE REVIEW

A. Influences of the MTO and MTS on Business Operations

Modern production strategies primarily involve two main production patterns: the MTO (based on customer orders), and the MTS (based on production capacity) [8]. From the perspective of customers, one competitive advantage of the MTS production is short delivery time and quick response [9]. Therefore, identifying the types of products that are specifically suitable for the MTS production pattern or both MTS and MTO patterns is a favored research topic in management science [8].

Regarding the influences of the MTO and MTS production strategies on business operations, Hendry and Kingsman [10] showed that the MTS and MTO production strategies are mostly used for manufacturing standard and customized products, respectively. Regarding the attributes of orders, order demand for MTS products is generally predictable, whereas that for MTO products is irregular and unpredictable. Concerning production planning, MTS production lines operate according to forecast results, and the production line schedule can be adjusted easily. However, the schedule of MTO production lines is determined based on recent order demand, and long-term manufacturing schedules are difficult to determine. In terms of product delivery, enterprises that adopt the MTS production strategy can ensure rapid product delivery, thus maintaining high customer satisfaction. The MTO production pattern requires long delivery times, and enterprises adopting this strategy must communicate with customers to achieve consensus regarding product delivery time. Concerning product price, compared with prices of products produced adopting the MTO strategy, the prices of MTS-produced products are relatively more stable. Soman, van Donk, and Gaalman [8] indicated that the MTO production pattern is effective for handling orders requesting high-mix customized products; the production planning for the MTO strategy must prioritize meeting order demands, while production effectiveness is determined according to crucial elements in the orders (e.g., the expected delivery volume and number of delayed delivery days). The goal of a company that manufactures MTO products is to shorten product delivery times; production efficiency emphasizes the importance of capability planning, orders that are lost due to problems with manufacturing processes, and on-time product delivery. By contrast, the MTS production pattern is effective for handling uniform product specifications and less customized products, where production planning is determined based on product demand forecasting and production effectiveness is production-oriented. Therefore, the goal of a company manufacturing MTS products is to enhance product availability, and its production efficiency emphasizes the importance of inventory policy, finished goods inventory, one-off or batch production, and accurate demand forecast. Rajagopalan [11] indicated that inventory costs are slightly higher for the MTS strategy than for the MTO strategy, particularly for one-off and batch production.

In summary, the MTS strategy relies heavily on the accuracy of product demand forecasting. Because of accurate forecasting, the advantages of the MTS production strategy, including short delivery time, manageable long-term manufacturing schedule, and stable product prices, can be realized. In addition, accurate forecasting can optimize inventory levels; therefore, companies applying the MTS strategy can effectively control inventory costs. Some researchers have explored the inventory policies and material control mechanisms in MTO production [12]. The forecasting method proposed in this study provides a relatively accurate basis for forecasting random customer orders (demand) for MTS production.

B. Forecasting Methodology

Two main types of forecasting methodology exist: (1) statistical methods; and (2) data mining and machine learning [13]. Both types of forecasting methodology are aimed at identifying the relationship between influential factors (independent variables) and research variables (dependent variables), and identifying the effects of the influential factors on research variables [7]. These two methodologies involve distinct approaches to interpreting analysis models. The statistical methodology is based on the data derived from a specific mathematical model as well as unobservable errors. The machine-learning methodology avoids fitting data to a specific model and develops algorithms that are suitable for various types of data. These two methodologies differ in their strengths and characteristics [13]. The statistical methodology uses the probability distribution of errors to infer the significance of the influential factors in a model. The reliability of inferences correlates positively with the mathematical model. The machine learning methodology uses the size of forecast errors as a basis for selecting the optimal forecasting model.

Several typical forecasting methods are introduced as follows, the characteristics of which are shown in Table 1. The exponential smoothing method was proposed by Holt [14] and the statistical theoretical foundation for this method was established by Muth [15]. This method involves using a demand observation and predictive value in the current period to determine the predictive value for the subsequent period by using weighted mean. To date, the exponential smoothing method has been widely applied to forecast demand under the bullwhip effect [16] and to plan inventory control strategies [17]. Moreover, the methodology for exponential smoothing has been developed in recent years into one that incorporates the effect of influential factors on the accuracy of demand forecasts [7, 18, 19]. Wang [19] used a model selection method where crucial influential factors were included in the selected model, and nonsignificant factors were removed to avoid over-fitting the model.

Time-series model was first developed in the nineteenth century, and past studies related to such model were then systematically compiled by Box and Jenkins [20] into a book. A time-series autoregressive integrated moving average (ARIMA) model integrates an autoregressive process and moving average process after obtaining a finite difference from time-series data. The ARIMA model is used to estimate the correlations parameter between the time points of observed values, and the estimated parameter

Table 1. CHARACTERISTICS OF F ORECASTING M ETHODS . (○: YES ; △: YES FOLLOWING MODIFICATION BY OTHER STUDIES

Forecasting method	Can handle temporal data	Can include influential factors	Analyzing the importance of influential factors (e.g., *p* value)
Linear mixed-effect model	○	○	○
Exponential smoothing method	○	△	△
ARMA	○	△	△
Linear model	△	○	○

values can then be used for forecasting. Subsequently, Box and Tiao [21] added other time-series influential factor to the ARIMA model. Pankratz [22] called this model the dynamic regression model.

Linear regression models are a type of linear model that are most frequently mentioned in statistical analyses. Linear models assume that research variables and influential factors are linearly related, and thus can be used to explore the effect of influential factors on research variables. Furthermore, linear models assume that observation values are mutually independent; thus, this model is applicable for analyzing data containing mutually independent observation values. If linear models are used to analyze time-correlated data, i.e., the observation values being correlated over time, then unbiased but invalid model coefficient estimators can be obtained. Consequently, the standard errors of the model coefficient estimators would be incorrect, and problems regarding statistical testing within the models arise, such as whether the model coefficients are significantly greater than 0, whether the models exhibit explanatory power, and whether the predictive intervals are reliable in forecast analysis [23, 24].

Linear mixed-effect models can be considered as an extension of linear models. The linear mixed-effect models add random effects to linear models with fixed effects. Hence, a model that has both fixed and random effects is called a linear mixed-effect model. Linear mixed-effect models are typically used to describe the relationship between research variables and categorical factors with correlated observation values. A characteristic of the mixed-effect models is that observation values at the same categorical level

have identical random effect values for dependent variables; observation values at different levels have distinct values of random effect. This characteristic explains the correlation between observation values at an identical level. Therefore, linear mixed-effect models differ considerably from linear models. The mixed-effect model can be applied to data where observation values are correlated (e.g., longitudinal data, repeated measures data, and multilevel data). However, linear models can be applied only to data where the observation values are mutually independent. In industrial operations, the pattern of data observations is often time-correlated. For example, when forecasting monthly product demand or monthly inventory levels, the observation values are correlated over time. Under such circumstances, the linear mixed-effect model is more accurate than linear models for identifying statistically significant factors.

In the past 2 years, the linear mixed-effect model has been broadly applied in various fields, such as the timber industry [25], medicine [26, 27], and ecology [28], to identify crucial influential factors. In addition, numerous studies have established models for forecasting [29, 30]. However, in industrial engineering and management science [24, 31, 32, 33], no study has used the linear mixed-effect model to make predictions by using time-correlated data or to identify key influential factors. Therefore, in this study, a linear mixed-effect model was applied to business operations to analyze the importance of influential factors, and to forecast product demand; in addition, the performance of the linear mixed-effect model was compared with that of other methods, which are the research contributions of this study.

III. LINEAR MIXED-EFFECT MODEL

According to parameter attributes, two types of effect exist in a linear mixed-effect model: fixed and random effects [34, 35]. In a linear model, the parameters are all fixed values and therefore its corresponding covariates are referred to as fixed-effect parameters. The fixed effect describes the true value of the coefficient for an entire population, or the true value of the coefficient for a factor that can be repeatedly tested under identical conditions. If a factor in a model exhibits a random effect, then the factor is sampled from an entire population. The random effect is a coefficient of the factor; moreover, the coefficient is a random variable and not a fixed value. The following section introduces the linear mixed-effect model developed by Laird and Ware [36] and the estimation of model parameters, and describes how the research variables are forecasted.

A. Linear Mixed-Effect Model

In contrast to a multilevel model, a single-level linear mixed-effect model [36] was employed in this study. The multilevel model differs from the single-level model in terms of the covariance matrix of the observation values. The single-level model involves only one level, whereas the multilevel model involves at least two levels. The covariance matrix of the multilevel model is more complex than that of the single-level model. In practice, whether using a single-level or multilevel model is more appropriate depends on the data structure of the observation values. Although the covariance matrices of the two models differ, the observation values of the various groups at a fixed level are independent of each other, and the within-group observation values are intercorrelated. In the multilevel model, a group at one hierarchy level becomes the next level of the hierarchy.

The single-level linear mixed-effect model developed by Laird and Ware [36] is expressed as follows:

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i + \boldsymbol{\varepsilon}_i, \quad i = 1, \ldots, M \qquad (1)$$

$$\mathbf{b}_i \sim N(\mathbf{0}, \boldsymbol{\Psi}), \boldsymbol{\varepsilon}_i \sim N(\mathbf{0}, \boldsymbol{\Lambda}_i), \qquad (2)$$

where $\mathbf{b}_i$ is a matrix that is independent of $\boldsymbol{\varepsilon}_i$ (index i denotes the ith group at a single level), $\mathbf{y}_i$ contains n_i observation values for the ith group, M denotes the number of groups, $\boldsymbol{\beta}$ denotes a p-dimensional vector for the fixed effect, $\mathbf{b}_i$ denotes a q-dimensional vector for the random effect, $\mathbf{X}_i$ denotes an $n_i \times p$ covariance matrix for the fixed effect, $\mathbf{Z}_i$ is an $n_i \times q$ covariance matrix for the random effect, and $\boldsymbol{\varepsilon}_i$ denotes an n_i-dimensional within-group random error term. The variable $\boldsymbol{\varepsilon}_i$ obeys a multivariate normal distribution with an expected value of 0 and a covariance matrix of $\boldsymbol{\Lambda}_i$, and $\mathbf{b}_i$ obeys a multivariate normal distribution with an expected value of 0 and a covariance matrix of $\boldsymbol{\Psi}$. The model assumes that $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\varepsilon}_j$ are mutually independent ($i \neq j$); in addition, $\boldsymbol{\varepsilon}_i$ and $\mathbf{b}_i$ are mutually independent. Therefore, considering Models (1) and (2), the covariance matrix of the within-group observation values $\mathbf{y}_i$ is expressed as follows:

$$\mathbf{V}_i \equiv \mathrm{Var}(\mathbf{y}_i) = \mathrm{Var}(\mathbf{Z}_i\mathbf{b}_i) + \mathrm{Var}(\boldsymbol{\varepsilon}_i) = \mathbf{Z}_i\boldsymbol{\Psi}\mathbf{Z}_i^T + \boldsymbol{\Lambda}_i \qquad (1)$$

where the nondiagonal elements of $\mathbf{V}_i$ are not required to be 0. Therefore, according to (3), Models (1) and (2) allow the existence of the correlation between observation values within a group. This is a major difference that the two models have with the linear model.

B. Estimation of the Model Parameters

This section introduces estimation methods that adopt the linear mixed-effect model: the maximum likelihood (ML) and restricted ML (REML) estimation methods. Regarding the ML method, the

estimates of ML estimators are those that reach the maximum value of ML functions. By comparison, the REML method is aimed at identifying the estimators that exhibit unbiased characteristics. Therefore, estimators obtained using the REML method are unbiased, whereas those derived using the ML method could feature either biased or unbiased property. Therefore, most researchers prefer the REML method [34, 35]. We introduce the estimation procedures for both of these estimation methods, although only the REML method was used in this study.

First, the model $\boldsymbol{\beta}$ coefficient and covariance matrix of observation values $\mathbf{V}_i$ are estimated as follows. In Models (1) and (2), the expected values of $\mathbf{b}_i$ and $\boldsymbol{\varepsilon}_i$ are assumed to be 0; thus, the expected value of y_i is $X_i\beta$ (i.e., $E(\mathbf{y}_i)=\mathbf{X}_i\boldsymbol{\beta}$). Because the covariance matrix of y_i is V_i (i.e., $Var(\mathbf{y}_i)=\mathbf{V}_i$) and because $\mathbf{b}_i$ and $\boldsymbol{\varepsilon}_i$ obey an independent multivariate normal distribution, the marginal distribution of y_i is a multivariate normal distribution expressed as follows:

$$\mathbf{y}_i \sim N(\mathbf{X}_i\boldsymbol{\beta},\ \mathbf{V}_i)$$

The ML function is expressed as follows:

$$L(\boldsymbol{\beta},\boldsymbol{\theta})=\prod_{i=1}^{M}(2\pi)^{\frac{-n_i}{2}}\det\left(\mathbf{V}_i\right)^{\frac{-1}{2}} \times\exp\left\{-\frac{1}{2}(\mathbf{y}_i-\mathbf{X}_i\boldsymbol{\beta})^T\mathbf{V}_i^{-1}(\mathbf{y}_i-\mathbf{X}_i\boldsymbol{\beta})\right\}$$

where $\boldsymbol{\theta}$ denotes the set of $\mathbf{V}_1,...,\mathbf{V}_M$. To facilitate differentiation, the natural logarithm of the ML function is used instead of the ML function to evaluate the ML and REML estimators, and define $l(\boldsymbol{\beta},\boldsymbol{\theta})=\ln L(\boldsymbol{\beta},\boldsymbol{\theta})$.

ML estimation method The ML estimates of β and θ are the values that maximize $l(\boldsymbol{\beta},\boldsymbol{\theta})$ and thus are also the values that maximize L(β,θ). Calculating the maximum value of $l(\boldsymbol{\beta},\boldsymbol{\theta})$ is challenging. Typically, let $\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}$, and evaluate the value of β such that it maximizes $l_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}(\boldsymbol{\beta},\boldsymbol{\theta})$. Subsequently, let $\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}$, and calculate the value of θ such that it maximizes the value of $l_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}(\boldsymbol{\beta},\boldsymbol{\theta})$. This process is iterated until the change in $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ is within a tolerance error (i.e., the $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ values converge).

Specifically, we first let $\boldsymbol{\theta}$ be $\hat{\boldsymbol{\theta}}$ (equivalent to letting $\mathbf{V}_i$ be $\hat{\mathbf{V}}_i$, $i=1,...,M$). Under these conditions, $\mathbf{y}_i$ obeys $N(\mathbf{X}_i\boldsymbol{\beta},\hat{\mathbf{V}}_i)$. An analytical solution for $\boldsymbol{\beta}$ can be obtained by using the generalized least squares method.

$$\hat{\boldsymbol{\beta}}=(\sum_i\mathbf{X}_i^T\hat{\mathbf{V}}_i^{-1}\mathbf{X}_i)^{-1}\sum_i\mathbf{X}_i^T\hat{\mathbf{V}}_i^{-1}\mathbf{y}_i \qquad (4)$$

Accordingly, $l_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}(\hat{\boldsymbol{\beta}},\boldsymbol{\theta})$ is the maximum value. Next, fix $\boldsymbol{\beta}$ in $l(\boldsymbol{\beta},\boldsymbol{\theta})$ as $\hat{\boldsymbol{\beta}}$, denoted by $l_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}(\boldsymbol{\beta},\boldsymbol{\theta})$, to obtain a $\boldsymbol{\theta}$ that maximizes the value of $l_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}(\boldsymbol{\beta},\boldsymbol{\theta})$, where

$$l_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}(\boldsymbol{\beta},\boldsymbol{\theta})=-\frac{1}{2}\left(\sum_i n_i\times\ln(2\pi)+\sum_i\ln(\det(\mathbf{V}_i)) +\sum_i(\mathbf{y}_i-\mathbf{X}_i\hat{\boldsymbol{\beta}})^T\mathbf{V}_i(\mathbf{y}_i-\mathbf{X}_i\hat{\boldsymbol{\beta}})\right) \qquad (5)$$

where $\mathbf{V}_1,...,\mathbf{V}_M$ are functions of $\boldsymbol{\theta}$. Typically, $l_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}(\boldsymbol{\beta},\boldsymbol{\theta})$ is not a linear function for $\boldsymbol{\theta}$. Consequently, no analytical solution for $\boldsymbol{\theta}$ exists, and an algorithm must therefore be used to obtain a numerical solution for $\boldsymbol{\theta}$. Commonly used algorithms include the expectation-maximization (EM) algorithm, Newton's method, and Fisher's scoring algorithm. Previous studies have described these algorithms in detail [36, 37, 38], including a comparison of their strengths and weaknesses [35]. An algorithm can be used to obtain a numerical solution for $\boldsymbol{\theta}$ (i.e., $\hat{\boldsymbol{\theta}}$), the result of which can be converted to $\hat{\mathbf{V}}_i$. Subsequently, the calculation is performed iteratively by using Equations (4) and (5) until the values of $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ converge.

REML estimation method The REML method is another approach for estimating $\boldsymbol{\theta}$. The REML estimate of $\boldsymbol{\theta}$ is obtained by applying an iterative method to a restricted natural-logarithm ML function.

$$l_{REML}(\boldsymbol{\theta})=-\frac{1}{2}\left((\sum_i n_i-p)\times\ln(2\pi)+\sum_i\ln(\det(\mathbf{V}_i)) +\sum_i(\mathbf{y}_i-\mathbf{X}_i\hat{\boldsymbol{\beta}})^T\mathbf{V}_i(\mathbf{y}_i-\mathbf{X}_i\hat{\boldsymbol{\beta}}) +\sum_i\ln(\det(\mathbf{X}_i^T\mathbf{V}_i\mathbf{X}_i))\right) \qquad (6)$$

Regarding the difference between the restricted natural-logarithm ML function (6) and Equation (5), Equation (6) accounts for the loss in degrees of freedom. Therefore, the estimator of $\boldsymbol{\theta}$ obtained using the REML is an unbiased estimator. The REML method involves applying Equation (4) to obtain the estimator of $\boldsymbol{\beta}$. For the REML, Equations (4) and (6) are iteratively used until the values of $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\theta}}$ converge. Equation (4) is used in both the ML and REML estimation methods to estimate $\boldsymbol{\beta}$. However, the functions employed to estimate θ (i.e., the ML and REML methods use Functions (4) and (6) to estimate $\boldsymbol{\theta}$, respectively) differ between these methods, and they thus yield different values for $\hat{\boldsymbol{\theta}}$. In addition,

because $\hat{\mathbf{V}}_i$ is a function of $\hat{\boldsymbol{\theta}}$, different values are obtained for $\hat{\mathbf{V}}_i$; consequently, different $\hat{\boldsymbol{\beta}}$ values are obtained through using these two methods.

Estimating random effect parameters Given $\mathbf{b}_i$, the following equation can be derived from (1):

$$\mathbf{y}_i \mid \mathbf{b}_i \overset{d}{=} N(\mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{b}_i, \boldsymbol{\Lambda}_i)$$

where "$\overset{d}{=}$" represents "distribution equals" and $\boldsymbol{\Lambda}_i$ is given by (2). Therefore, the generalized least squares method can be applied to estimate $\mathbf{b}_i$, which is equal to $(\sum_i \mathbf{Z}_i^T\boldsymbol{\Lambda}_i^{-1}\mathbf{Z}_i)^{-1}\sum_i \mathbf{Z}_i^T\boldsymbol{\Lambda}_i^{-1}(\mathbf{y}_i - \mathbf{X}_i\boldsymbol{\beta})$. In the equation, $\boldsymbol{\Lambda}_i$ (a function of $\boldsymbol{\theta}$) and $\boldsymbol{\beta}$ are true values. Therefore, by substituting the ML or REML estimates (i.e., $\hat{\boldsymbol{\beta}}$ or $\hat{\boldsymbol{\Lambda}}_i$), we can obtain the estimator of $\mathbf{b}_i$ as follows:

$$\hat{\mathbf{b}}_i = (\sum_i \mathbf{Z}_i^T\hat{\boldsymbol{\Lambda}}_i^{-1}\mathbf{Z}_i)^{-1}\sum_i \mathbf{Z}_i^T\hat{\boldsymbol{\Lambda}}_i^{-1}(\mathbf{y}_i - \mathbf{X}_i\hat{\boldsymbol{\beta}}).$$

C. Forecasting Research Variables

After the explanatory variables $\mathbf{X}_i^{new}$ and $\mathbf{Z}_i^{new}$ have been obtained, the estimates of $\boldsymbol{\beta}$ and $\mathbf{b}_i$ (i.e., $\hat{\boldsymbol{\beta}}$ and $\hat{\mathbf{b}}_i$) described in the previous section can be used to forecast the research variable $\mathbf{y}_i$. The predictive value is as follows:

$$\hat{\mathbf{y}}_i = \mathbf{X}_i^{pred}\hat{\boldsymbol{\beta}} + \mathbf{Z}_i^{pred}\hat{\mathbf{b}}_i. \quad (7)$$

IV. A CASE STUDY

This study adopted a single-level linear mixed-effect model to forecast product demand. In the case study, the sample was a leading professional industrial LCD/OLED display manufacturer. This manufacturer produces products that are critical components of various devices used in daily life and are applied in various industries. Moreover, the company has an international customer base. Table 2 shows the number of orders, total product demand, average product demand per order, and quantity of finished goods from 2009 to 2013. Before 2013, the manufacturer produced more than 5,000 product types, and the average quantity of products required in an order was approximately 400. Thus, the manufacturer is considered to be a suitable example of a business that produces a diverse combination of high-mix products.

A characteristic of high-mix low-volume manufacturers is that they typically commence production only after receiving a customer order. This production pattern is typical of the MTO production pattern, which is mainly adopted to serve customers in niche markets. In recent years, the manufacturer's profits have decreased despite an increasing revenue and market share. Therefore, the manufacturer aimed at changing its production strategy by adopting the MTS production strategy for some product types in order to increase its batch production capacity, reduce its production costs, and improve its production efficiency. In addition, the manufacturer believed that adopting the MTS production strategy would enhance customer satisfaction by ensuring the rapid delivery of customer orders, thereby providing a competitive advantage. Thus, being able to accurately forecast product demand was crucial. Following evaluation, to test the implementation of the MTS production strategy, this study selected the top 20 standard finished products that were most frequently ordered between 2011 and 2013 by customers of the sample manufacturer. As shown in Figure 1, these 20 standard products accounted for 20% of the manufacturer turnover for standard products in 2013, with 86 orders placed in the same year. After implementing the MTS production strategy, the manufacturer planned to run production of each product type once per month per year. Accordingly, the production frequency, cost of handling orders, and frequency of changing production lines was reduced. Thus, its long-term production capacity plans can be implemented to maximize the benefits of producing a high volume of products with fewer runs.

Table 2. NUMBER OF ORDERS AND PRODUCT DEMAND

Year	Number of orders	Total product demand	Average product demand for an order	Finished goods quantity
2009	12,929	3,603,141	278.69	2,727
2010	17,968	8,343,884	464.37	3,518
2011	20,169	6,721,194	333.24	4,546
2012	22,589	8,062,890	356.94	5,822
2013	22,361	9,045,056	404.50	5,468

A. Data Structure

The data structure comprised 20 types of standard finished products. The monthly product demand data were collected from January 2007 to December 2013 for each product type (see S1 Table). The historical data before 2012 were used to estimate model parameters, and the model was used to forecast the product demand for 2013 (January–December). Not all 20 products were manufactured from 2007. The historical data used to estimate model parameters comprised 1295 observation values (64 observation values on average for each product type). The product lifecycle varied by year, and the product demand varied by month. Therefore, year and month were crucial predictors. For each type of product, the monthly product demands in each month were related. In this study, the explanatory variables (year and month) were added to the linear mixed-effect model

to analyze the monthly product demand data. Regarding product sales, the product demand varied by product type. Accordingly, product type was regarded as a crucial categorical variable because of its influence in forecasting the product demand. In this study, according to the characteristics of the mixed-effect model, we used product type as a random-effect term and included the demand for each product type in a universal model to

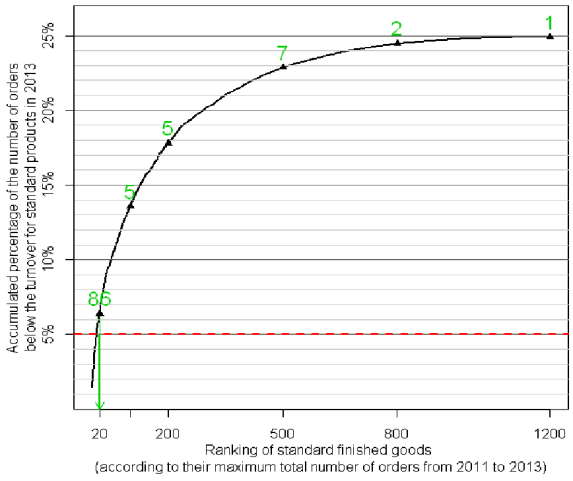


Figure 1. Maximum total number of orders (2011–2013). The plot shows that the accumulated percentage of the maximum total number of orders from 2011 to 2013 is less than the turnover of standard products in 2013. The first 20 products accounted for approximately 20% of the turnover for of standard products. The numbers in green denote the number of orders for standard products in 2013 corresponding to the horizontal axis. forecast the demand for type separately. Subsequently, we compared other commonly used forecasting methods. Unlike the mixed-effect model, other methods did not have a universal model to account for 20 unique product types. Therefore, for the other forecasting methods, the data are required to be divided into multiple data sets according to product type, and the partitioned data are then applied to the forecasting methods depending on the product type for analysis and forecasting. This approach substantially reduces the sample size, reducing the accuracy of the forecast.

B. Model Development

Product demand differed by product type, and thus we assumed the demand for each type of product to be mutually independent. In Model (1), which is the single-level model, random effect was set to be product type, thus yielding various random-effect coefficient for each product type. The model is expressed as follows:

$$\begin{aligned}\mathbf{y}_i &= \beta_0 + \beta_1 \times (\text{year-2007}) \\ &+ \beta_2 \times (\text{year-2007})^2 + \text{month} \times \boldsymbol{\beta}_3 \\ &+ b_{i0} + b_{i1} \times (\text{year-2007})^2 + \boldsymbol{\varepsilon}_i\end{aligned} \quad (8)$$

where $\mathbf{y}_i$ is a vector that denotes the monthly product demand (the vector length is equal to the data quantity for product i); β_0, β_1, β_2, and β_3 denote the intercept, year, year-squared, and month for the fixed-effect term; and b_{i0} and b_{i1} denote the intercept and year-squared for the random-effect term. In Model (8), year was considered as a continuous variable with 2007 used as the baseline. Month was a categorical variable; therefore, the month term in Model (8) was a dummy variable. The dummy variable for month had 11 indicator variables with a value of 0 or 1, and the total product demand in January was used as the baseline. Expressing Equation (1) as Model (8), the fixed-effect explanatory variable $\mathbf{X}_i$ is a matrix comprising a column of 1's vector for the intercept, year, year-squared, and month covariates. Thus, the expression $\boldsymbol{\beta}=[\beta_0\ \beta_1\ \beta_2\ \boldsymbol{\beta}_3^T]^T$ is a 14 × 1 vector, where $\boldsymbol{\beta}_3$ is the coefficient of the dummy variable for the month covariate and has 11 elements. To account for the various product types, we chose the intercept and year-squared covariate as the random-effect explanatory variable, where the intercept was used to account for the average difference of demands between product types, and the year-squared covariate was used to consider the difference between product demands decreased or increased over time. The explanatory variable $\mathbf{Z}_i$ in the random-effect explanatory variable comprised the intercept and year-squared covariate, of which the coefficients are a 2 × 1 vector expressed as $\mathbf{b}_i=[b_{i0}\ b_{i1}]^T$. In Model (8), the year-squared covariate in the random-effect explanatory variable was also a part of the fixed-effect explanatory variable, and was used to account for the fact that the expectation of $\mathbf{b}_i$ was probably unequal to 0; thus, the assumption that $\mathbf{b}_i$ in (2) was equal to 0 was reasonable. The year-squared covariate was included to prevent the annual growth trend from being linear, which enabled the model to more accurately reflect the current situation. The year-squared covariate is crucial to practical operations. The year and year-squared covariates added into the fixed-effect explanatory variable facilitated establishing a grand model for the 20 product types. The year and year-squared covariates for the fixed effect indicated the average growth trend for the 20 product types, whereas the random effect reflected the specific annual growth trends for each product type. To forecast the monthly product demand for 2013, 2013 was used as the value for the year and year-squared covariates. Both covariates and the target month were input into the explanatory variable to form $\mathbf{X}_i^{new}$ and $\mathbf{Z}_i^{new}$. Subsequently, $\hat{\boldsymbol{\beta}}$ and $\hat{\mathbf{b}}_i$ in (7) were used to obtain the forecasted value $\hat{\mathbf{y}}_i$.

C. Other Forecasting Methods

Comparing forecasting methods is crucial in methodological studies [39 , 40 , 41 , 42 , 43]. The model proposed in this study was compared with commonly used statistical forecasting methods, beginning with the following linear model:

$$Y_j = \alpha_0 + \alpha_1 \times (\text{year}_j\text{-}2007) + \alpha_2 \times (\text{year}_j\text{-}2007)^2 + \alpha_3 \times \text{month}_j + \delta_j \quad (9)$$

Table 3. Linear Mixed-Effect Model Versus the Linear Model.

Explanatory variable	Linear mixed-effect model			Linear model		
	Coefficient	Standard error	P value	Coefficient	Standard error	P value
The intercept term	39.46	320.14	.9019	174.54	294.07	.5529
(Year-2007)	800.55	133.52	.0000 ***	746.35	153.69	.0000 ***
$(\text{Year-2007})^2$	-99.97	25.82	.0001 ***	-93.23	27.62	.0008 ***
February	206.41	283.03	.4660	171.14	327.19	.6010
March	736.88	281.57	.0090 **	716.23	325.51	.0280 *
April	753.30	281.56	.0076 **	762.56	325.50	.0193 *
May	536.51	280.89	.0564 -	504.53	324.70	.1205
June	253.56	281.62	.3681	218.06	325.53	.5031
July	591.73	271.46	.0295 *	556.53	313.77	.0764 -
August	91.35	271.48	.7366	56.21	313.77	.8579
September	711.75	271.46	.0088 **	664.40	313.77	.0344 *
October	297.69	271.05	.2723	255.00	313.28	.4158
November	473.91	272.52	.0823 -	432.47	314.94	.1699
December	360.30	270.62	.1833	308.34	312.71	.3243

" - ": p < .1; " * ": p < .05; " ** ": p < .01; " * ": p < .001.**

where α_0, α_1, α_2, and α_3 are regression coefficients and α_3 denotes the coefficient of the dummy variable for the month covariate, and δ_j is the error term. Model (9) (i.e., the linear model) includes only the fixed-effect term in Model (8) (i.e., the mixed-effect model); therefore, Model (9) was compared with Model (8) to examine the differences when the random-effect term is present or absent in the model. A total of 1295 observations of monthly product demand (Y_j, $j = 1,...,1295$) were used to estimate the coefficients in Model (9) and the significance of the coefficients with P values. In the Results section, Models (8) and (9) are compared regarding forecast accuracy and the P values.

Next, the model proposed in this study was compared with the exponential smoothing method, in which the product demand observation values Y_t's and its predictive values F_t's were used to obtain the predictive values for the subsequent period by calculating a weighted mean. The forecast formula is as follows:

$$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$$

where α is the weighted coefficient. To accurately forecast the monthly product demand in this case, we adjusted the exponential smoothing method to account for two influential factors (i.e., month and product type). The data were divided into 20 data sets according to each product type, and each data set was divided into 12 subsets (one for each month). For each product type, no more than six observations from each month in the historical data were used. The pre-2012 monthly product demand data were used to forecast the product demand for the corresponding months in 2013. The weighed coefficient was $\alpha = \frac{1}{2(N+1)}$, where N is the number of observations for a month ($N \leq 6$).

Finally, the model proposed in this study was compared with a seasonal time-series model; specifically, the autoregressive moving average model $(\text{ARMA}(2,2)_{12})$, which was considered to be a suitable model because the data were not nonstationary time-series data. The mathematical model for ARMA $(p,q)_s$ is expressed as follows:

$$(1 - \sum_{i=1}^{p} \phi_i B^{s \times i}) Y_t = (1 + \sum_{i=1}^{q} \theta_i B^{s \times i}) \xi_t$$

where ϕ_i is the ith order autoregressive process coefficient, B is a backward shift operator, θ_i is the ith order moving-average process coefficient, ξ_t is a normally distributed confounding term, and s is a seasonal parameter. Longitudinal data were collected for each of the 20 product types. A time-series model was established for each of the 20 product types. In this case, the month was regarded as a crucial influential factor for forecasting and thus the seasonal parameter s was set to 12, which indicates the existence of correlations in the data for every 12 month. The samples were categorized by product type, yielding an average of 64 samples for each type of product. The parameters p and q were determined based on the characteristics of an autocorrelation function, a partial autocorrelation function, and an extended autocorrelation function (p = 2 and q = 2). Finally, the $\text{ARMA}(2,2)_{12}$ model was used to forecast the product demand for each product type.

Table 4. Error Indicators for the Four Forecasting Methods.

	MAE		MAPE		RMSE	
	M	*SD*	*M*	*SD*	*M*	*SD*
Linear mixed-effect model	1,412.71	1,500.04	1.52%	1.50%	1,849.42	1,919.86
Linear model	1,828.96	2,091.93	3.77%	6.00%	2,259.99	2,712.69
$\text{ARMA}(2,2)_{12}$	1,509.22	1,938.23	1.92%	2.04%	1,942.48	2,533.25
Exponential smoothing method	1,565.54	1,547.88	2.01%	1.77%	2,003.87	2,193.16

D. Results

In this study, mean of absolute error (MAE), mean of absolute percent error (MAPE), and root-mean-square

error (RMSE) were used as error indicators. The definitions for these error indicators are provided as follows:

$$MAE = n^{-1}\sum_{t=1}^{n} |F_t - Y_t|$$

$$MAPE = 100n^{-1}\sum_{t=1}^{n} \left|\frac{F_t - Y_t}{Y_t}\right|$$

$$RMSE = \left(n^{-1}\sum_{t=1}^{n} (F_t - Y_t)^2\right)^{0.5}$$

where n denotes the number of months to be forecasted (n = 12 in this case), Y_t represents the true product demand for month t of 2013, and F_t is the forecasted product demand for month t. The fixed-effect term in the linear model was compared with that in the linear mixed-effect model. As shown in Table 3, the absolute values of the coefficients for the explanatory variables in the linear mixed-effect model containing the random-effect term are greater (i.e., further from 0) than all of those in the linear model except for April. In addition, the standard errors and P values for all of the explanatory variables in the linear mixed-effect model are smaller than those in the linear model. Regarding the linear fixed-effect model, compared with January in a given year, the product demand was significantly greater in May and November (P value < 0.1), in July (P value < 0.05), and in March, April, and September (P value < 0.01). Compared with the linear fixed-effect model, the linear model yielded less significant results. The linear model is suitable for data containing mutually independent observation values. In this case, the observation values for product demand were correlated over time, thereby violating the assumption of the linear model. Therefore, the standard errors and P values for the linear model (Table 3) are not valid estimates, whereas those for the linear mixed-effect model are more reliable. Table 4 shows the error indicators for the four forecasting methods. Because this case involved three error indicators for each of the 20 product types, Table 4 presents the mean and standard deviation of the three error indicators. As shown in Table 4, the means and standard deviations of MAE, MAPE, and RMSE for the linear mixed-effect model are lower than those for the linear, ARMA, and exponential smoothing models, indicating that, in this case, the linear mixed-effect model is superior to the other three models. Regarding the model comparison (Table 5), the predictive values obtained through using the linear model to process the correlated data are unbiased [23]. However, the linear mixed-effect model (8) contains the random-effect term, whereas the linear model (9) does not. Therefore, in Model (8), the intercept and year-squared terms differ according to the product type, and thus the corresponding intercept values and coefficients differ based on the product type. In Model (9), the covariate of product type is not included in the explanatory variables, which generates identical predictive values for various product types in the same years and months. Thus, this model cannot predict the product demand for the individual product types, rendering its forecasting effectiveness inferior to that of Model (8). Regarding the exponential smoothing method, we considered product type and month as crucial influential factors, which were used as the basis for dividing the data into 240 data sets. For each product type, the pre-2012 monthly data were used to forecast the monthly product demand for 2013. In this manner, the exponential smoothing method was applied 12 times for each of the 20 product types. In addition, less than six observations from the historical data were used in the exponential smoothing method (for a given month, there were at most 6 sets of data from 2007 to 2012); consequently, the risk of inferential error was high because only a few observations were involved in the prediction. Regarding the seasonal time-series model $ARMA(2,2)_{12}$, we considered product type as a crucial influential factor and divided the data into 20 data sets according to product type. For each product type, 64 observations were used on average. The $ARMA(2,2)_{12}$ model was used to forecast the product demand for each product type by considering the correlation between the data for every 12 month. For both the exponential smoothing method and the $ARMA(2,2)_{12}$ model, the data were divided into subsets according to the product type and then used to estimate the monthly effect of each product type. Accordingly, although such procedure could consider the various monthly effects for various product types and the interaction between product type and month, it reduces the number of data observations involved in the prediction. In the linear mixed-effect model, 1295 data observations were used to estimate the random effect for each product type. The number of data observations used in the linear mixed-effect model was considerably more than that used in the exponential smoothing and time-series models, which could explain

Table 5. Comparison of the Four Models.

	Number of models	Number of samples	Consideration for the effect of product type	Consideration for the effect of month	Consideration for the interaction effect of product type and month
Linear mixed-effect model	1	1,295	○	○	※1
Linear model	1	1,295		○	
ARIMA(2,2)	20	≤72	○	○	○
Exponential smoothing method	240	≤6	○	○	○

※1 This effect is nonsignificant

why the linear mixed-effect model produced lower forecast errors. In addition, in Model (8), the random effect of the interaction term for month and year-squared term was considered and the likelihood ratio test was employed to examine whether this term is significant to this model. The results showed that only

the random effects of the intercept and year-squared terms were significant, and the random effect of the month term did not significantly enhances its explanatory power for the data. Therefore, the random effect of the interaction term was not included in Model (8).

V. DISCUSSION

In summary, when applying the linear mixed-effect model, all of the historical data were used in one model to predict the monthly product demand for each product type, and to avoid problems resulting from dividing the data into smaller data sets. In this case study, using the linear mixed-effect model enables manufacturers who adopt the MTS production strategy to predict the amount of inventory they should stock. Furthermore, the model is more effective in forecasting product demand than is the time-series, exponential smoothing, and linear models.

Similar to the linear model, the linear mixed-effect model is typically used to examine the relationship between explanatory and research variables. Unlike the linear model, which assumes the observation values to be mutually independent, the linear mixed-effect model is suitable for examining correlated data. Because the data pertaining to business operations are generally correlated over time, the linear model is limited in applicability. By contrast, the linear mixed-effect model was initially developed to handle correlated data. Other methods such as the time-series and exponential smoothing methods formulate the correlation between observation values as parameters, and then estimate the parameters by data and forecast the observations by the estimates. When the time-series and exponential smoothing models were first developed, these methods were not aimed at analyzing the relationship between explanatory and dependent variables. Wang [19] proposed an exponential smoothing method that included explanatory variables and can be used to explore the association of research variable. Because this method is a relatively new development, most of statistical software packages have not yet incorporated related functions, and thus this method has not been widely used. By contrast, the linear mixed-effect model was developed more than 30 years ago, and related functions have been included in various statistical software packages.

Using linear mixed-effect, time-series, and linear models to forecast product demand can yield negative predictive values. This phenomenon occurs when the linear mixed-effect model is used because $\boldsymbol{\varepsilon}_i$ in (2) is assumed to be normally distributed and the link function is an identity function. Negative values are usually obtained from historical data where product demand is zero or very low. To prevent this, predictive value was truncated at 0 (i.e., $F_t = \max(\hat{Y}_t, 0)$, where $\hat{Y}_t$ denotes a predictive value derived from any method, and F_t denotes an actual predictive value obtained from any prediction method). In other words, if $\hat{Y}_t > 0$, then $F_t = \hat{Y}_t$; if $\hat{Y}_t \le 0$, then $F_t = 0$. Some link functions in generalized linear mixed-effect model can deal with the case where dependent variable is restricted to $\hat{Y}_t \ge 0$ [44]. However, the prediction intervals for the random-effects in linear mixed-effect model are well developed [45, 46, 47, 48, 49]. It is useful to apply the prediction intervals in business operations for knowing whether the random-effect exists.

Implementing an MTS production strategy can enhance the competitive advantages of a manufacturer, enabling the manufacturer to rapidly satisfy product demand, thereby reducing internal and external transaction costs for handling orders. Employing this strategy also enables high batch centralized production and thus can reduce production costs and assist manufacturers in negotiating with material suppliers about the cost of materials. Because this approach enables short delivery times, customer satisfaction can be improved, thus attracting potential customers who need products immediately. Consequently, market share can be increased. MTS production also enhances the usage rate of production equipment. Companies that adopt an MTS strategy require an accurate forecasting method to realize these advantages. This study proposed an accurate forecasting method for determining the stock levels a company should determine for adopting the MTS production strategy, a topic that has seldom been discussed in studies on MTS production.

Using an MTS production strategy involves the potential risk of increasing inventory costs. Therefore, future studies should adequately apply the strengths of the linear mixed-effect model (e.g., accurately forecasting demand for multiple product types in one go) when forecasting. Future studies should consider investigating whether the forecasting intervals of the linear mixed-effect model can be coupled with various inventory strategies to assist manufacturers with adopting the MTS production strategy in order to develop an optimal business operation model in terms of optimal inventory time points and minimal inventory costs. In addition, to remain competitive, companies should enhance their organizational capability for elevating the threshold that enables competitors to develop similar operating models. Future studies are also recommended to explore the benefits that the MTS production strategy involving a linear mixed-effect model brings to the various departments of an enterprise and the effects of such strategy on customer satisfaction and loyalty.

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