QUANTITATIVE MEASURE OF "NOT-BEING" CATEGORY AND INNER PSYCHOPHYSICS

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Abstract - The present work is devoted to constructing the quantitative description of the categorization classes arising within the bisection of perceived stimuli according to some criterion, i.e., the description of the categories "being" and "notbeing" something. The main idea is to find such measures for this categorization that can be used in other situations to evaluate a current system state from various aspect.

Index Terms - Bisection, categorization classes, inner psychophysics, mental space, physics of mind, proximity measure

I. GOAL OF RESEARCH

Mental evaluation of current situation and the choice of appropriate actions are the heart of many processes where human role is crucial. Human intermittent control—a novel paradigm of human actions in balancing various unstable mechanical systems—illustrates them as a characteristic example of active human behavior with properties emphasizing the basic features of perception [1]. In particular, the theory of human intermittent control accepts the corresponding human actions to form a complex pattern including the sequence of alternate phases of active and passive behavior. Within the active phase a person affects directly the system dynamics.

Within the passive phase the person halts his control until the system deviates rather far from the desired state and the necessity of its correction becomes evident. This type behavior is actually the direct consequence of the bounded capacity of human cognition hampering the recognition of small perturbations in system dynamics.

The control activation in the transitions from passive to active phases is usually described within the threshold approximation, e.g., [2], [3]. However recently we put forward a new concept of noise-induced activation [4], [5] turning to the uncertainty in event recognition near the perception threshold (fuzzy threshold). The gist of this concept is that the control activation occurs at the moment when the person recognizes the current state of system under control no longer may be regarded as acceptable. The given example illustrates many phenomena where the categorization of current situation as "not-being something" induces special human actions. The purpose of the present work is to propose a general approach to quantifying such classification of current situation. The main idea of the proposed approach is to attach this problem turning to the ideas of inner psychophysics.

II. INNER PSYCHOPHYSICS & PSYCHOLOGICAL SPACE

Already the founder of psychophysics as quantitative psychology, Gustav Fechner (1801–1887), drew a distinction between outer and inner psychophysics [6]. As noted by Fechner, sensation is implemented as a sequence of two transformations with own properties:

- *stimulus* $\rightarrow$ *neural signal* by sense organs,
- *neural signal* $\rightarrow$ *sensation* via the brain.

Outer psychophysics deals with the two transformations as a whole and studies the stimulus-sensation relationship based on direct observation and experimentation. Inner psychophysics focuses on the latter transformation and has to draw inferences from the findings of outer psychophysics. Experimentation and quantification are rather problematic in inner psychophysics; for this reason inner psychophysics was rejected for a long time starting from Fechner's contemporaries. Actually G¨osta Ekman (1920–1971) returned inner psychophysics into the family of hard sciences admitting quantitative analysis. Instead of dealing with the intensity of physical stimuli he proposed to analyze the basic laws of human sensation in terms of sensory magnitudes based on the known laws of outer psychophysic. In particular, he demonstrated that the relationship between the variation of magnitude ΔM matching the just-noticeable difference of physical stimulus and the sensory magnitude obeys the law [7]

$$\Delta M = \kappa_e M \qquad (1)$$

similar to Weber's law. Relationship (1) is often referred to as Ekman's law and below the coefficient _e will be called Ekman fraction. The next step in founding inner psychophysics may be related to a number of hypotheses about the universality of inner psychophysics put forward by Robert Teghtsoonian (1932–2017) [8]–[10], I presented their summary in [11]. These hypotheses enabled Teghtsoonian [8], [9], [12] to posit a single central macrolevel mechanism responsible for all judgments of sensory magnitude.

Within this mechanisms the role of various receptor systems is reduced to performing the necessary expansions or compressions required to map the widely varying dynamic ranges of physical stimuli into the constant range of subjective magnitudes. The similarity between human perception of physical stimuli based on our sense organs and the mental evaluation of abstract objects. like numbers [13]–[17] is an additional argument for this mechanism.

The data accumulated in fMRI, EEG, and neuropsychological investigations also argue for this central mechanism. In particular Walsh and Bueti [18], [19] put forward a new paradigm about human judgment and evaluation of external stimuli called ATOM ("A Theory Of Magnitude"). The ATOM supposes that various dimensions of magnitude information, such as space, time, and quantity, are encoded by "common neural metrics" in the parietal cortex, which explains the emergence of common neurocognitive mechanisms governing human perception of various physical stimuli. Naturally, the situation can be more complex [20].

The concept of psychological space takes the central place in inner psychophysics. This concept is rooted in the works of Louis Leon Thurstone (1887–1955) formulated the law of comparative judgment converted then into the law of categorical judgment by Warren S. Torgerson (1924–1999). However the psychological space as an object admitting sophisticated mathematical description appears in the works by Robert Duncan Luce (1925–2012) (e.g., [21]–[24], also [25] for a general discussion). The gist of these constructions is the use of the probabilistic properties of human decision-making in categorizing various external stimuli for constructing the topology of the inner psychological space, whose points are various categories. Exactly the introduction of psychological space with own properties and governing the final step in stimulus evaluation explains the universality of psychophysical laws for all the sensory modalities, e.g., [26]–[30].

Among the recent publications this universality has been also discussed, e.g., in [31], [32] and [33]–[35]. In this connection I want to note the Bayesian approach to describing the interaction between the perceptual and central subsystems as well as their individual operations turning to the formalism of conditional probabilities, for review see [36], [37]. The psychological space can be also introduced axiomatically, e.g., [38], [39]. As far as psychological approach is concerned, Kurt Lewin (1890—1947) put forward the concept of psychological force field describing psychological processes in topological spaces [40]–[43]. Then George Kelly (1905–1967) formulated personal construct psychology, as complete theory of cognition, action, learning and intention, using geometry of psychological spaces as alternative to logic [44]. For a review of advances in this direction see [45].

Developing the ideas of physics of mind [46]–[49] in [11] I have proposed the notion of mental space where human perception of external stimuli is treated as the emergence of the corresponding images in the mental space. On one hand, these images are caused by external stimuli and their magnitude bears the information about the intensity of the corresponding stimuli. On the other hand, the individual structure of such images as well as their persistence after emergence is determined only by the properties of mental space on its own. In this approach I turned to the two-component description of human behavior [49]. As demonstrated, the developed theory of stimulus perception is able to explain from a common standpoint the basic regularities found in outer psychophysics.

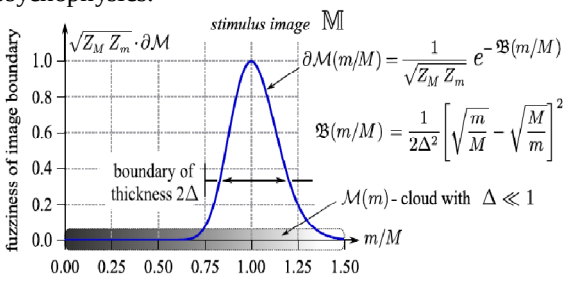


Fig. 1. The mental image M with magnitude M as the M(m)-cloud—a fuzzy segment with blurred boundary on the m-axis. In this illustration the parameter _ = 1=8 corresponding to the boundary thickness of 25% was used.

III. MENTAL SPACE AND IMAGE COMPARISON

The concept of mental space whose elements are images of physical stimuli takes the central place in my further constructions. Exactly the relations between these images enable me to introduce new objects, the categorization classes A and A describing the fuzzy bisection of images into the class of images meeting a certain criterion A and the opposite class. For this reason I start this section with a partial description of the mental space introduced in [11].

A. Mental Space of Stimulus Images

Each image M emerging in the mental space after processing a certain neural signal is assumed to have the following properties:

P1: The image M possesses a certain property called the magnitude M representing in some way the amplitude of the corresponding neural signal.

P2: After the emergence induced by the neural signal, the image M no longer needs this signal to be present. So the mental images of physical stimuli are considered to be individual objects existing on their own in the mental space, which implies the memory-based nature of the mental space.

P3: Within a given prothetic perceptual continuum (e.g., loudness) stimulus images admit direct

comparison with one another based only on their magnitudes.

P4: The image M possesses a certain universal structure represented in Fig. 1. On the axis of magnitudes (further the m-axis) the image M is represented as some fuzzy region—the M(m)-cloud—with a blurred boundary of thickness _ centered at a point M. The distance between the m-axis origin and the latter point is the magnitude M of the image M.

P5: Attributing magnitudes to these images enables us to introduce the following two general operations implemented consciously in the mind:

a) A qualitative comparison of two images M1 and M2 with respect to which one has a smaller or larger magnitude, or whether their magnitudes are identical, i.e.,

$$\mathbb{M}_1 \prec \mathbb{M}_2\,, \quad \mathbb{M}_1 \succ \mathbb{M}_2\,, \quad \mathbb{M}_1 \simeq \mathbb{M}_2\,. \tag{2}$$

b) A certain scaling of the image amplitudes to be called the image fusion determined by the structure of these

images

$$n \otimes \mathbb{M}_1 \mapsto \mathbb{M}_n\,, \tag{3}$$

where n is an integer meeting the condition $n \leq n_c$. Here n_c is a certain critical value ruling out the possibility of fusing many images.

B. Image Mental Comparison: Ekman's Law

The mental comparison of two physical stimuli is reduced to the comparison of their images. I want to emphasize that the accuracy of comparison depends on various conditions. The limit cases are represented by the situation when the analyzed stimuli affect the perceptual subsystem simultaneously and when these stimuli are separated by some time gap and their comparison requires memory to be involved. In the former case the Ekman fraction k_e and the value n_c are related as $\kappa_e \sim 1/n_c^2$ c and using the experimental data analyzed by

Teghtsoonian and the estimate $\kappa_e \approx 0.03$ the value n_c may be estimated as $n_c \sim 4$–6 [11].

The comparison of stimuli based on memory can be quantified as the overlapping of the corresponding M(m)-clouds

$$\langle \mathbb{M}_1 | \mathbb{M}_2 \rangle = \exp\left\{ \quad \mathfrak{B}(M_2/M_1) \right\}. \tag{4}$$

Here generalizing the results of [11] the proximity potential (proximity measure) is written as

$$\mathfrak{B}\left(\frac{M_2}{M_1}\right) = \frac{n_c^2}{2p_w^2}\left[\left(\frac{M_1}{M_2}\right)^{p_w/2} - \left(\frac{M_2}{M_1}\right)^{p_w/2}\right]^2, \tag{5}$$

where the exponential $p_w > 0$ is a certain value about unity. It is necessary to emphasize that here the proximity potential $\mathfrak{B}_{12}$ is assumed not to exceed unity substantially, otherwise the categorization of compared images as equal in magnitude does not have much sense.

Expression (5) gives rise to Ekman's law directly. Indeed, let us estimate the just-noticeable difference in image magnitudes $\Delta M = |M_2 - M_1|$ via the criterion of image equality

$$\langle \mathbb{M}_1 | \mathbb{M}_2 \rangle = \exp\left\{ \quad \mathfrak{B}(M_2/M_1) \right\} < e^{-1/2}. \tag{6}$$

Whence it immediately follows that

$$\Delta M = \frac{1}{n_c}\sqrt{M_1 M_2}\,, \tag{7}$$

which is Ekman's law for the image comparison involving memory with the proportionality coefficient $\kappa_e - 1/n_c$. It should be noted that Exp. (1) is Ekman's law for the image comparison when the corresponding stimuli affect sensory system simultaneously.

IV. IMAGE COMPARISON VS IMAGE CATEGORIZATION

The image comparison with respect to equivalence in magnitude and the image categorization with respect to meeting some criterion A have some common "core." Within the former procedure two images with amplitudes M1 and M2 are compared to figure out when their difference jM1 □M2j becomes recognizable. Within the latter procedure, first, we fix the image Ma that best meets the criterion A, then the proximity of a given image M to the standard image Ma is analyzed. The image categorization admits also the examination of image magnitude variations _M with respect to their noticeability. It may be done via perception of how the proximity between two images M and Ma changes as the magnitude M varies. In this case the images M and Ma should not be in close proximity to each other. Therefore, generally speaking, the two procedures do not necessarily give the same estimate of the just-noticeable difference _M, which, turning to psychological reasons, should not be the case. To find a description reconciling the two procedures I need to introduce the notion of natal probability. In the given case this notion stands for the probability of identifying two images M1 and M2 in magnitude on their own, i.e., when other factors and other images are outside the scope of analysis. In this aspect, the natal probability is just a conventional probability. However, it does not describe a similar probability when other images are involved in the comparison. Under such conditions the natal probability is closer to the membership function of fuzzy sets in property. I posit that the natal probability P12 and the measure B12 of two image proximity should be related by an expression similar to the ansatz

$$\mathfrak{P}_{12} = \left[1 + \frac{2p_w}{n_c}\mathfrak{B}_{12}\right]^{-n_c/2p_w} \tag{8}$$

to lead to Ekman's law (7) in the limits $|M_1 - M_2| \ll M_1, M_2$ and $|M_1 - M_2| \gtrsim M_1$ or M_2. For $|M_1 - M_2| \sim M_1 \sim M_2$ in the image comparison three images of similar magnitudes are involved and some deviation from relationship (7) is quite expectable.

In terms of natal probabilities the just-noticeable difference ΔM is introduced via the transition $M_2 \to M_2 + \Delta M$ meeting a condition similar to equality (6), namely,

$$\mathfrak{P}_{12}\big|_{M_2+\Delta M} = e^{-1/2} \cdot \mathfrak{P}_{12}\big|_{M_2}. \tag{9}$$

Figure 2 depicts the obtained results. As seen, the exponent $p_w = 2$ may be accepted as a reasonable estimate. Indeed for $p_w = 2$, on one hand, the shown variations in Ekman's fraction are located inside a rather narrow region. On the other hand, the corresponding exponent $n_c/2p_w \gtrsim 2$ for $n_c \sim 4$–6 and, thereby, there is a certain finite crossover between the ansätz (6) and (8), which reconciles the two procedures in specifying the just-noticeable difference.

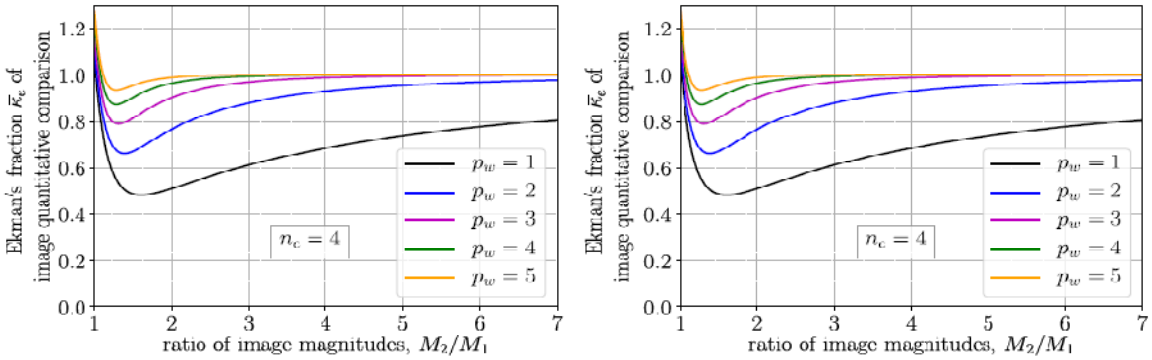


Fig. 2. Ekman's fraction for two image comparison based on criterion (9) and ansatz (5). Here Ekman's fraction _e is given in units of n_c, i.e., $\bar{\kappa}_e = n_c\kappa_e$.

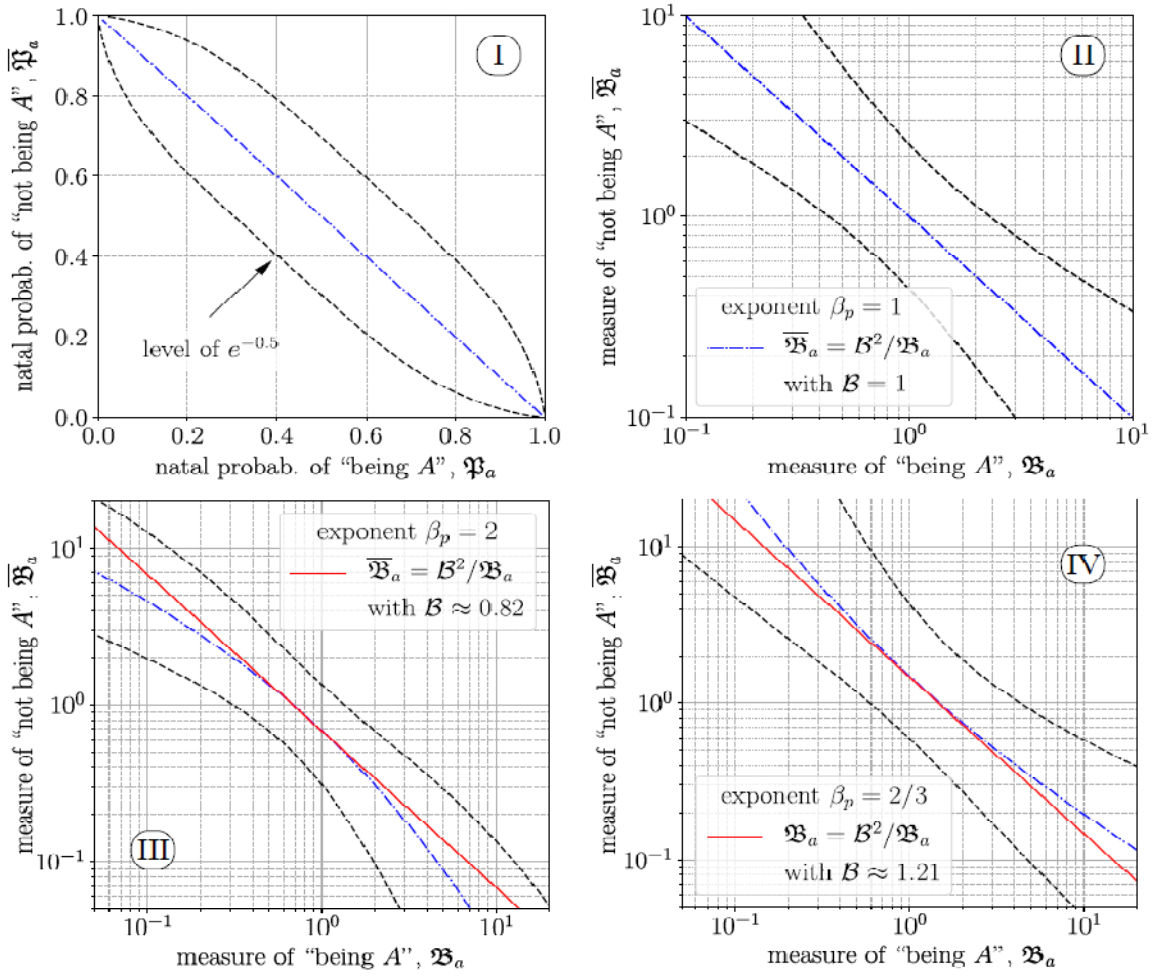


Fig. 3. Fussy region on the $\{\mathfrak{P}_a, \mathfrak{P}_a\}$-plane and the corresponding regions on the $\{\mathfrak{P}_a, \mathfrak{P}_a\}$-plane representing the image of relationship (11) in the mind.

V. "BEING ..." AND "NOT-BEING ..." CATEGORIES

The introduced notion of natal probability enables me to ascribe certain individual measures to the categorization classes $\mathcal{A}$ and $\overline{\mathcal{A}}$ in the following way. As noted above, first, for the class $\mathcal{A}$ there can be find an image $\mathbb{M}_a$ best meeting the condition A. Then, in the bisection of all the images into the categories $\mathcal{A}$ and $\overline{\mathcal{A}}$ the membership function—the natal probability of classification—is constructed as it follows from Exp. (8) where $\mathfrak{B}_{12} = \mathfrak{B}(M/M_a) := \mathfrak{B}_a(M)$ and M, M_a are the magnitudes of a trial images $\mathbb{M}$ and the class representative $\mathbb{M}_a$, respectively. I want to emphasize that the natal probability may be used as a probability only within the given bisection. Contrarily, the measure $\mathfrak{B}_a(M)$ admits a much wider use, e.g., it may be employed in comparing different categorical classes $\mathcal{A}$ and $\mathcal{N}$. In this sense the measure $\mathfrak{B}_a(M)$ may be treated as a individual property of the categorization class $\mathcal{A}$ which quantifies the relation between the given class and the images $\{\mathbb{M}\}$—the "points" of mental space.

In a similar way I may ascribe a certain measure $\overline{\mathfrak{B}}_a(M)$ to the categorization class $\overline{\mathcal{A}}$ and introduce the corresponding natal probability

$$\overline{\mathfrak{P}}_a = \left[1 + \frac{2p_w}{n_c}\,\overline{\mathfrak{B}}_a\right]^{-n_c/2p_w}. \qquad (10)$$

There is only one difference in constructing the measures for classes $\mathcal{A}$ and $\overline{\mathcal{A}}$; it is the absence of representative element similar to M_a for $\overline{\mathcal{A}}$. To overcome this problem let us turn to the identity holding true for membership functions of fuzzy sets as well as conventional probabilities. Namely, within the analyzed bisection I may write

$$\mathfrak{P}_a + \overline{\mathfrak{P}}_a - 1\,. \qquad (11)$$

To finalize the construction of the measure for $\overline{\mathcal{A}}$ it is necessary to convert identity (11) into a relationship between the measures $\mathfrak{B}_a(M)$ and $\overline{\mathfrak{B}}_a(M)$.

Before doing it directly I want to attract attention to that in the mind these measures are not *strict numerical* quantities. In fact they are some fuzzy objects—some clouds—characterized by (*i*) the mean value—the given measures—and (*ii*) a certain uncertainty. Taking into account the construction of the just-noticeable difference this uncertainty in the natal probabilities may be represented by the level of $e^{\,1/2}$, see Exp. (9). Thereby instead of the rigor equality (11) I may regard all the points within the fuzzy region shown in Fig. 3 (I) as a layer bounded by dashed lines to meet equality (11). The corresponding images of this region on the $\{\mathfrak{B}_a, \overline{\mathfrak{B}}_a\}$-plane are depicted in Figs. 3 (II–IV) for different values of the exponent $\beta_p = n_c/2p_w$. The used values of the exponent β_p match, e.g., the pairs $\{n_c = 4, p_w = 1\}$, $\{n_c = 4, p_w = 2\}$, and $\{n_c = 4, p_w = 3\}$. As seen, within a region where the choice between the categories $\mathcal{A}$ and $\overline{\mathcal{A}}$ has some sense from the psychological standpoint the relationship between the measures $\mathfrak{B}_a(M)$ and $\overline{\mathfrak{B}}_a(M)$ admits the approximation

$$\mathfrak{B}_a \cdot \overline{\mathfrak{B}}_a - \mathcal{B}^2\,, \qquad (12)$$

where $\mathcal{B} \sim 1$ is some constant depending on the parameters n_c and p_w (see also Fig. 3 for the corresponding particular values of $\mathcal{B}$).

CONCLUSION

Previously [11], turning to the concepts of inner psychophysics and the two-component description of human actions [49] I constructed a certain mental space whose elements are the images of physical stimuli affecting our sensory system. It has been demonstrated that the properties ascribed to these images, in particular, the image proximity measure (potential) explain from a common standpoint the main laws found in outer psychophysics.

In the given paper, following the gist of the previous theory, I have generalized this mental space to quantify human evaluation of a current stimulus (or the state of system under control). Namely, I have considered the bisection of possible images into two categorization classes $\mathcal{A}$ and $\overline{\mathcal{A}}$ with respect to a certain criterion A. The fuzzy class $\mathcal{A}$ comprises images meeting the criterion A, the class $\overline{\mathcal{A}}$ represents the opposite situation. The relationship between the given classes and the images of mental space is specified by certain equivalence measures $\mathfrak{B}_a(M)$ and $\overline{\mathfrak{B}}_a(M)$ attributed to the two classes individually.

For the class $\mathcal{A}$ the measure $\mathfrak{B}_a(M)$ is constructed in two steps: (*i*) finding the image $\mathbb{M}_a$ best meeting the criterion A and (*ii*) identifying the desired measure $\mathfrak{B}_a(M)$ with the proximity potential $\mathfrak{B}(M/M_a)$ between the trial image $\mathbb{M}$ of intensity M and the class representative $\mathbb{M}_a$ of intensity M_a.

The class $\overline{\mathcal{A}}$ does not allow the introduction of representative image. To overcome this problem I have turned the relationship between the natal probabilities of classification—the membership functions of this bisection. The obtained expression (12) represents the main result of the given paper. It established the relationship between the measure $\mathfrak{B}_a(M)$ and the desired $\overline{\mathfrak{B}}_a(M)$. The latter value is ascribed to the categorization class $\overline{\mathcal{A}}$ and is treated as its own property. It is essential because the classes $\mathcal{A}$ and $\overline{\mathcal{A}}$ may be used together or separately in describing various phenomena, e.g., decision-making and its retrospective evaluation or a time sequence of events including their permanent evaluation.

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