OPTIMAL LOT SIZE UNDER UNCERTAIN SUPPLY, STOCHASTIC DEMAND AND CYCLE SERVICE LEVEL

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Abstract- We consider a retailer who is facing uncertain supply as well as demand and who has to make a stocking decision for a given selling period. If demand exceeds supply, the unmet demand is lost and if the opposite happens, any units left can be salvaged at a reduce value. When demand is unmet, there is lost goodwill as well as the risk of loss of future business. In literature this problem is considered a variant of newsvendor problem. Retailers prefer using service level rather than the shortage penalty to determine the tradeoff between revenue and the loss of goodwill or loss of future business. In this paper, we develop a procedure for finding the optimal policy for the retailer assuming general distributions for the yield as well as demand. The goal is to maximize expected revenue given the cycle service level. The results have important implications for a retailer when a portion of the inventory may be lost due to variety of factors including failure to meet quality standards, theft or perishability.

Keywords- Data Mining, Intelligent Systems, Financial Risk, Early Warning.

I. INTRODUCTION LITERATURE REVIEW

Retailers are exposed to a variety of risks, one of which is having too much or too little stock to sell. This risk is usually due to uncertainty in consumer demand and is amplified when the supply is uncertain. The uncertainty in supply can be due to defects, theft, damage in transit or perishability. If demand outstrips the supply then a mid-season recourse may not be possible, resulting in costly shortages. Supply may be uncertain in many different environments such as electronic fabrication, chemical manufacturing, discrete parts manufacturing, food and agricultural products and pharmaceutical products. For the most recent industry specific studies that have looked at the issue of yield uncertainty, we refer the readers to [Yano and Lee (1995)].

When demand is uncertain, the newsvendor needs to make an implicit tradeoff between profit and the loss of goodwill and future profit associated with stock outs. Using a shortage penalty to represent the economic consequence of a stock out situation simplifies the problem to a single objective of maximizing the implicit profit or minimizing implicit cost. However, the shortage penalty is not easy to estimate and it needs to be estimated separately for each product.

Practitioners see the service level and revenue (or cost) as conflicting objectives and determining the tradeoff between the two objectives as a bi-criterion problem. It is not surprising that when the service level approach is feasible, practitioners prefer using it. In this paper we use a cycle service level to determine the optimal lot size for the newsvendor when both supply and demand are uncertain. We assume the yield to be stochastically proportional, i.e. the yield does not depend on the order size [Henig and Gerchak (1990)]. In section 2, we formulate the model taking into account the cycle service level. We analyze the model and provide a solution procedure in section 3. Finally we provide our conclusions as well as identify avenues for future research in section 4.

There is vast literature on the newsvendor problem. For a review of the newsvendor problem and its extensions, see [Khouja (1999)]. A comprehensive analysis of models of random supply is carried out by [Rekik et. al (2007)]. Gerchak et al. (1988) formulated a periodic review model assuming uncertain supply as well as demand where they place a single order for the entire planning horizon. Henig and Gerchak (1990) provide a comprehensive analysis of a periodic review inventory system with random yields. The above papers, similar to most of the literature on random yield problems, use the shortage penalty to model the economic consequence of a stock out situation.

II. MODEL FORMULATION

2.1. Notations and Assumptions

We define the notation used throughout this chapter.

c	unit cost, $c \geq 0$.
Q	lot size, a decision variable such that $Q > 0$.
h	salvage price such that $0 \leq h \leq c$.
r	random variable representing the supply yield defined over the range [0,1], with mean μ_r and standard deviation σ_r. Its probability density and cumulative functions are denoted by f and F, respectively.
u	realized value of yield r, $u \in [w,1]$ where w is the lowest yield that can occur.
$D(\varepsilon)$	demand function such that $D(\varepsilon) = \mu + \varepsilon$ where μ is the deterministic component of demand.
ε	random variable representing the uncertainty in demand defined over range $[A, B]$ with mean μ_ε and standard deviation σ_ε. Its probability density and cumulative functions are denoted by g and G, respectively.
v	realized value of the demand error ε.

Expected Profit
The revenue function is given as

$$\Pi(Q)=\begin{cases} p[\mu+v]+h[uQ-\mu-v], & \text{if } v\le uQ-\mu, \\ puQ & \text{if } v>uQ-\mu. \end{cases} \tag{1}$$

The expected profit function is given as

$$E[\Pi(Q)]=\int_{w}^{1}\int_{A}^{uQ-\mu}\{p[\mu+v]+h[uQ-\mu-v]\}g(v)f(u)dvdu + \int_{w}^{1}\int_{uQ-\mu}^{B}\{puQ\}g(v)f(u)dvdu - cQ. \tag{2}$$

Note that the limit of the integration may assume values such that $uQ-\mu\le A$ or $uQ-\mu\ge B$. However, since $v\in[A,B]$, $g(v)=0,\forall v\notin[A,B]$. Upon simplification we get

$$E\left[\Pi(Q)\right]=(p-h)\left[\mu+\mu_{\varepsilon}\right]+h\mu_{r}Q + (p-h)\int_{w}^{1}\int_{uQ-\mu}^{B}[uQ-\mu-v]g(v)dvf(u)du - cQ \tag{3}$$

The cycle service level constraint is

$$\Phi(Q)=\int_{0}^{1}\int_{A}^{uQ-\mu}g(v)dvf(u)du\ge CL \tag{4}$$

The problem of maximizing the expected profit is thus formulated as

$$(P)\quad \max_{Q}\ E[\Pi(p,Q)] \tag{5.1}$$

$$\text{s.t.}\quad \int_{w}^{1}\int_{A}^{uQ-\mu}g(v)dvf(u)du\ge CL \tag{5.2}$$

The optimization problem () P is highly non-linear. In the next section, we study its convexity properties.

III. ANALYSIS OF THE MODEL

We first establish the following lemma:

Lemma1: $E[\Pi(Q)]$ is a strictly concave function of Q.

Proof is given in Appendix A.

Lemma2: There is a unique Q, say $\tilde{Q}$, satisfying constraint (5.2) when imposed as the equality.

Lemma 3: Let $\bar{Q}$ represent the global maximum of $E[\Pi(Q)]$. Given Lemma 2, $\bar{Q}$ is unique. Then the optimal solution to problem (P) is $Q^{*}=\max(\bar{Q},\tilde{Q})$.

Given the above results, the solution procedure for solving problem (P) will entail determining both $\bar{Q}$ and $\tilde{Q}$ and then choosing the larger of the two values. Given the complexity of $E[\Pi(Q)]$ as well as $\Phi(Q)$ the solution can be obtained only numerically barring the case when both yield and demand are uniformally distributed. Although the yield can be uniformally distributed, demand will not be. In most situations, demand can be modeled using the truncated Normal distribution.

4.1 Example

Suppose

$\mu=2000,\mu_{\varepsilon}=0,\sigma_{\varepsilon}=300,A=-300,B=300,h=\$2,c=\$6;p=\$18,w=0.6$ and $CL=0.9$.

Note given the cycle service level, probability of stock out at the end of the selling is 10%.

$$g(v)=\frac{\frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon}}e^{-\frac{1}{2}\left(\frac{v-\mu_{\varepsilon}}{\sigma_{\varepsilon}}\right)^{2}}}{GB1-GA1}\quad\text{where } GB1=\int_{-\infty}^{B}g(v)dv=.9986 \text{ and}$$

Also, GA1= $\int_{-\infty}^{A} g(v)dv = 0.0013$. Similarly let r be distributed according to uniform .

distribution or let $f(u) = \frac{1}{1-w}$.

In this case we find that $\bar{Q} = 2628$ and $E[PI(\bar{Q})] = \$18998.69$. Similarly $\tilde{Q} = 3131$ and $E[PI(\tilde{Q})] = 18062.75$. Thus, the optimal lot size is equal to 3131 units. The retailer has to sacrifice expected profit of \$935.94 to provide cycle service level of 90%.

CONCLUSIONS

We have provided a procedure for determining optimal lot size for the single period problem when both supply and demand are uncertain. Our approach uses cycle service level instead of shortage penalty to model the economic consequence of a stock out situation. Practitioners find specifying cycle service level easier and moreover, cycle service level can be specified collectively for a broad category of items in one step.

Given the uncertain yield, cycle service levels such as 0.95 or 0.99, which are routine in non-perishables goods or reliable supply situations, will be high when yield is highly uncertain. In fact not always, the optimal the cycle service level constraint may be binding at the optimal point and *Q can be equal to the unconstrained optimum Q. Finally, we have assumed in this paper that the demand function is additive. Similar analysis can be performed when the demand function is multiplicative; that is when $D(\varepsilon) = \mu\varepsilon$.

Appendix A

Proof of Lemma 1: Using Leibniz rule, $\frac{dE[\Pi(Q)]}{dQ} = h\mu_r + (p-h)\int_w^1 \int_{uQ-\mu}^{B} ug(v)dvf(u)du - c$

Similarly, $\frac{dE^2[\Pi(Q)]}{dQ^2} = -(p-h)\int_w^1 u^2 g(uQ-\mu)f(u)du < 0$ given that

$g(uQ-\mu) > 0$ for $A < uQ - \mu < B$.

This completes the proof.

Proof of Lemma 2:

Again uisng Leibniz rule, $\frac{d\Phi(Q)}{dQ} = \int_w^1 ug(uQ-\mu)f(u)du > 0$ given that $g(uQ-\mu) > 0$ for $A < uQ - \mu < B$. Thus $\Phi(Q)$ is an increasing function of Q.

Proof of Lemma 3:

Let □ be the Lagrange multiplier associated with constraint (5.2). The Kuhn-Tucker conditions for problem (P) are:

$$h\mu_r + (p-h)\int_w^1 \int_{uQ-\mu}^{B} ug(v)dvf(u)du - c + \lambda\int_w^1 ug(uQ-\mu)f(u)du = 0 \quad \text{A(1)}$$

$$\lambda[\Phi(Q) - CL] = 0 \quad \text{A(2)}$$

$$\lambda \geq 0 \quad \text{A(3)}$$

Case 1: Suppose $\tilde{Q} < \bar{Q}$. It is easy to see that in this case $\Phi(\bar{Q}) > CL$. Thus $\bar{Q}$ satisfies constraint (5.2); i.e. $Q^* = \bar{Q}$.

Case 2: $\tilde{Q} \geq \bar{Q}$. From Lemma 1, in (A1), $h\mu_r + (p-h)\int_w^1 \int_{u\tilde{Q}-\mu}^{B} ug(v)dvf(u)du - c < 0$.

Thus (A1) can be satisfied only if $\lambda = 0$. That is constraint (5.2) is binding at the optimal point; or $Q^* = \tilde{Q}$.

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