SIMILARITY OF OPTIMAL SOLUTION BETWEEN CLASSICAL AND ROBUST RANKING APPROACH IN FUZZY INTEGER PROGRAMMING PROBLEMS USING TRAPEZOIDAL MEMBERSHIP FUNCTIONS

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Abstract- Linear programming is intended to take care of crisp parameters, and however data regarding reality world systems is usually offered with in the style of imprecise descriptions. Fuzzy strategies are designed to handle imprecise terms, and are most suited to finding optimum solutions to issues with imprecise parameters. Throughout this article, solving procedure absolutely think about Fuzzy Integer Programming Problem with the support of trapezoidal membership functions and its arithmetic operations. Solving procedure has been applied from the approach of fuzzy Linear Programming problem and then extends the problem into two cases, first case discussed about Gomary's cutting plane in terms of fuzziness and also the second case discussed about the problem with Robust ranking approach. The fuzzified version of the problem has been mentioned with the assistance of a numerical example associated it is proving that the classical and Robust ranking approach offer an efficient tool for handling the Integer programming problem. Finally the comparison analysis of optimal solution was given with in the nature of fuzzy membership functions.

Keywords— Trapezoidal Fuzzy Numbers and its arithmetic operations, Fuzzy Linear Programming Problem, Fuzzy Integer Programming Problem, Robust Ranking approach. MSC 2010: 90C05, 90C30, 90C70, 90C90

I. INTRODUCTION

Integer programming problems are a special class of linear programming problems where all or some of the variables in the optimal solution are restricted to non-negative integer values. It may be recalled that the linear programming problems are based on the assumption that the decision variables are continuous, with the result that they may take fractional values in the optimal solution. However, many situations in real life may not allow for this and warrant for the decision variables to take only integer values. For example, in production manufacturing is frequently scheduled in terms of lots, batches or production runs. In allocation of goods a shipment of goods involves a discrete number of trucks, wagons or aircraft, and fractional values are meaningless. Similarly, situations like sequencing, scheduling and routing decisions necessitate the use of integer programming models. Hence the need for restricting variables restricted to assume only integer values and integer programming. The linear programming problems with variables restricted to non-negative integers are called pure, all – integer, or mixed – integer problems depending, respectively, on whether all or some of the variables are restricted to integer values. Also there is zero – one integer programming problems, which constitute an important class of such problems.
It may be appear that it would suffice to obtain a non-integer solution to the linear programming problems using the regular graphic or simplex methods and then rounding off the fractional values occurring in the optimal solution. Unfortunately, however, this rarely works. Usually the deviations from actual optimal integer values may become large enough to cause the solution infeasible. Hence there is a need to develop a systematic procedure for obtaining optimal solution to such problems, meeting the integer requirements. The first of the systematic approaches to solve integer programming problems were suggested by RE Gomory in 1956. While initially only the pure integer programming problems could be handled, the method was extended later to deal with the mixed integer programming problems. The method uses cutting plane algorithm and ensure the optimal integer solution in a finite number of iterations. Another significant approach to solving such problems is that of branch and bound method. This is also used to solve the zero – one integer programming problems.

In most of the real world issues it is unacceptable to possess all constraints and resources in exact from rather they are in expected or imprecise form. This finally ends up with in the thought of fuzzy logic that allows us to emulate the human reasoning process and make decisions based on vague or imprecise data, and fuzzy programming offers the methodology of solving the issues in fuzzy environment. Fuzzy set theory has received great attention in recent years. The concept of fuzzy linear programming (FLP) on general level was first proposed by Tanaka et al. Afterwards, many authors consider various types of the FLP problem and proposed several approaches for solving these problem[3, 7, 8, 13]. In particular, the most convenient methods are based on the concept of comparison of fuzzy numbers by use of ranking functions [6, 9, 11]. In 2004, Makani Das and

Hemanta developed the solving procedure for linear programming problem within the approach of triangular fuzzy membership functions [8].

Throughout this chapter, there is a tendency to investigate a lot of realistic issues, specifically the integer programming problem with fuzzy parameters [1,7,10,12]. Since the objectives are to maximize the total profit, subject to some crisp constraints, the objective function is taken into account additionally as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy numbers to search out the simplest different. The idea is to rework a problem with fuzzy parameters to a crisp version in the LPP form. Dominance of fuzzy numbers may be explained by several ranking methods. In these, Robust ranking approach [9] that satisfies the properties of compensation, linearity and additives. Here the attention of the entire problem with in the support of trapezoidal fuzzy membership functions of the integer programming problem, for the same investigated in to two cases. For the first case was presented about Gomary's cutting plane method in terms of fuzziness and also the second case discussed for the same with Robust ranking approach. The fuzzified version of the problem for both cases has been presented with the assistance of a numerical example associated it is proving that the classical and Robust ranking approach offer an efficient tool for handling the Integer programming problem. Finally, for justification the comparison analysis of optimal solution was given with in the nature of fuzzy membership functions. These kind of approaches and its justification shows that the novelty of our investigation in the area of optimization research in terms of fuzziness.

II. PRELIMINARIES

L.A. Zadeh advanced the fuzzy theory in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and issues that have several potential solutions. The conception of fuzzy mathematical programming on a general level was initially projected by Tanaka et al 1974 within the frame work of fuzzy decision of Bellman and Zadeh [2]. Now it tends to present some necessary definitions are

2.1 Basic Definitions

1. Definition

A real fuzzy number A is a fuzzy subset of the real number R with membership function μ_A satisfying the subsequent conditions.

(i). μ_A is continuous from R to the closed interval [0, 1]

(ii). μ_A is strictly increasing and continuous on $[a_1, a_2]$

(iii). μ_A is strictly decreasing and continuous on $[a_3, a_4]$ wherever a_1, a_2, a_3 & a_4 are real numbers, and the fuzzy number denoted by A (=$\tilde{A}$) (=) $[a_1, a_2, a_3, a_4]$ is named as a trapezoidal fuzzy number.

2. Definition

A trapezoidal fuzzy number A can be expressed as A (=$\tilde{A}$) (=) $[a_1, a_2, a_3, a_4]$ and its membership function is outlined as:

$$\mu_A(x)(=)\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2\\ 1 \qquad , a_2 \le x \le a_3\\ \dfrac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4\end{cases}$$

The Pictorial representation of trapezoidal membership function μ_A is given below within the figure.

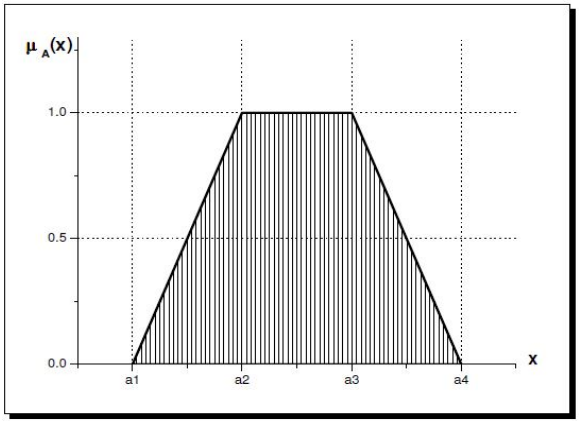


Figure: 1 Trapezoidal Membership function μ_A

2.2. Robust Ranking Technique:

Robust ranking technique that satisfy compensation, linearity, and additively properties and provides results which are consist human intuition. If A could be a fuzzy number then the Robust Ranking is outlined by

$$\mathrm{R(A)}\ (=)\ \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\, d\alpha$$

where

$(A_\alpha^L, A_\alpha^U)(=)[(a_2-a_1)\alpha + a_1, a_4 + (a_3-a_4)\alpha]$ is the α level cut of the fuzzy number A.

Here the Robust ranking index R(A) offers the representative value of fuzzy number A.

III. FUZZIFIED FORM OF INTEGER PROGRAMMING PROBLEM

The pure integer programming problem is to obtain the decision variables,

$$\left([x_1^{(1)}, x_1^{(2)}, x_1^{(3)}, x_1^{(4)}], [x_2^{(1)}, x_2^{(2)}, x_2^{(3)}, x_2^{(4)}], [x_3^{(1)}, x_3^{(2)}, x_3^{(3)}, x_3^{(4)}], \dots, [x_j^{(1)}, x_j^{(2)}, x_j^{(3)}, x_j^{(4)}]\right)$$

which maximizes the objective function $[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}](=)[(c_1x_1)^{(1)}, (c_1x_1)^{(2)}, (c_1x_1)^{(3)}, (c_1x_1)^{(4)}]$ (+)

$[(c_2x_2)^{(1)},(c_2x_2)^{(2)},(c_2x_2)^{(3)},(c_2x_2)^{(4)}](+)\ldots(+)\ [(c_nx_n)^{(1)},(c_nx_n)^{(2)},(c_nx_n)^{(3)},(c_nx_n)^{(4)}]$

subject to the linear constraints

$[(a_{i1}x_1)^{(1)},(a_{i1}x_1)^{(2)},(a_{i1}x_1)^{(3)},(a_{i1}x_1)^{(4)}](+)[(a_{i2}x_2)^{(1)},(a_{i2}x_2)^{(2)},(a_{i2}x_2)^{(3)},(a_{i2}x_2)^{(4)}]$

$(+)\ldots(+)\left[(a_{ij}x_j)^{(1)},(a_{ij}x_j)^{(2)},(a_{ij}x_j)^{(3)},(a_{ij}x_j)^{(4)}\right](=)[b_i^{(1)},b_i^{(2)},b_i^{(3)},b_i^{(4)}]$

$[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]\geq 0$ & are all integers, for $i=1,2,3,\ldots,m$ & $j=1,2,3,\ldots,n$

3.1 Gomary's Cutting Plane Method in terms of Fuzziness

In this method we generate new constraints so as to ensure integral solution of the given linear programming problem in terms of fuzziness. These additional constraints do not cut off that portion of the original feasible region which contains a feasible integer point. Also it cuts off the current non integer solution of the linear programming problem. First solve the linear programming problem, ignoring the integer requirement, by the usual simplex algorithm in terms of fuzziness [13]. If the optimal solution contains integer values for the variables then it is itself the solution of the integer programming problem. If some of the variables have fractional values then choose the variable having the largest fractional part, say $[x_r^{(1)},x_r^{(2)},x_r^{(3)},x_r^{(4)}]$. The corresponding r^{th} constraint can be written as,

$[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}](=)[x_r^{(1)},x_r^{(2)},x_r^{(3)},x_r^{(4)}](+)$

$\sum_{j=1}^{n}[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}](*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}];\ j\neq r$

If $\left[[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]\right]$ denotes the integral part of $[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]$ and $\left[[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]\right]$ denotes the integral part of $[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]$, then we can write

$[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}](=)\left[[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]\right](+)f_r,\ 0<f_r<1$ and

$[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}](=)\left[[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]\right](+)f_{rj},\ 0<f_{rj}<1$

Therefore we get, $\left[[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]\right](+)f_r(=)[x_r^{(1)},x_r^{(2)},x_r^{(3)},x_r^{(4)}]$

$(+)\sum_{j=1}^{n}\left(\left[[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]\right](+)f_{rj}\right)(*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}];\ j\neq r$

$$f_r(+)\left(\left[[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]\right](-)[x_r^{(1)},x_r^{(2)},x_r^{(3)},x_r^{(4)}](-)\sum\begin{bmatrix}\left[[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]\right](*)\\ [x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]\end{bmatrix}\right)$$

$(=)\sum f_{rj}(*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]\ ;\ j\neq r$

$$\text{Since}\left\{\left[[b_r^{(1)},b_r^{(2)},b_r^{(3)},b_r^{(4)}]\right](-)[x_r^{(1)},x_r^{(2)},x_r^{(3)},x_r^{(4)}](-)\sum\begin{bmatrix}\left[[a_{rj}^{(1)},a_{rj}^{(2)},a_{rj}^{(3)},a_{rj}^{(4)}]\right]\\ (*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]\end{bmatrix}\right\}\geq 0.$$

We have,

$$f_r\leq\sum f_{rj}(*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]$$

Introducing slack variable, we get

$$f_r+s=\sum f_{rj}(*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]$$

$$\text{i.e. } s-\sum f_{rj}(*)[x_j^{(1)},x_j^{(2)},x_j^{(3)},x_j^{(4)}]=-f_r \qquad (1)$$

This is a new constraint called Gomary's constraint in terms of fuzziness which represents a cutting plane. Add this new constraint at the bottom of the simplex table and also find a new optimal solution using dual simplex method. If the new solution gives integer values to the variables it is the required optimal solution. Otherwise continue this process is repeated until all the variables assume integer values.

3.2. Numerical Example

Let us consider a numerical example in two cases which is able to make simpler the methods for the solution of fuzzy integer programming problem with the assistance of trapezoidal membership functions and its arithmetic operations [9, 10, 13], fuzzy Gomary's constraint and its method follows case 1. For case 2, the same problem investigated with the help robust ranking approach.

The integer programming problem in terms of fuzziness is as follows:

Maximize $[Z^{(1)},Z^{(2)},Z^{(3)},Z^{(4)}](=)[x_1^{(1)},x_1^{(2)},x_1^{(3)},x_1^{(4)}](+)[x_2^{(1)},x_2^{(2)},x_2^{(3)},x_2^{(4)}]$

Subject to the constraints,

$$[0,2,4,6](*)[x_1^{(1)},x_1^{(2)},x_1^{(3)},x_1^{(4)}](+)[-2,2,3,5](*)[x_2^{(1)},x_2^{(2)},x_2^{(3)},x_2^{(4)}]\leq[2,4,6,8]$$

$$[-2,0,2,4](*)[x_2^{(1)},x_2^{(2)},x_2^{(3)},x_2^{(4)}]\leq[-2,2,3,5]$$

$$[x_1^{(1)},x_1^{(2)},x_1^{(3)},x_1^{(4)}],[x_2^{(1)},x_2^{(2)},x_2^{(3)},x_2^{(4)}]\geq 0 \text{ and are integers.}$$

3.2.1. Case (i): Integer programming problem in terms of fuzzy membership functions

Let us solve the above integer programming problem by means of the fuzzy simplex procedure [8, 13]: The standard form of the starting table is tabulated below:

		$\widetilde{x_1}$	$\widetilde{x_2}$	S_1	S_2	$\widetilde{x_B}$
$\widetilde{C_B}$	B	[-2,0,2,4]	[-2,0,2,4]	[-2,1,1,2]	[-2,1,1,2]	
[-2,-1,1,2]	S_1	[0,2,4,6]	[-2,2,3,5]	[-2,0,2,4]	[-2,1,1,2]	[2,4,6,8]
[-2,-1,1,2]	S_2	[-2,-1,1,2]	[-2,0,2,4]	[-2,1,1,2]	[-2,0,2,4]	[-2,2,3,5]
	$Z_1 - C_1$	[-4,-3,1,2]	[-4,-3,1,2]	0	0	[-14,-7,7,14]

Iteration 1

[-2,0,2,4]	$\widetilde{x_1}$	[0,2/3,4/3,2]	[-2/3,2/3, 1,5/3]	[-2/3,0, 2/3,4/3]	[-2/3,-1/3, 1/3,2/3]	[2/3,4/3,6/3,8/3]
[-2,-1,1,2]	S_2	[-2,-1,1,2]	[-2,0,2,4]	[-2,-1,1,2]	[-2,0,2,4]	[-2,2,3,5]
	$Z_2 - C_2$	0	[-4/3,-1, 1/3,2/3]	[-4/3,-1, 1/3,2/3]	0	[-22/3,-2, 16/3,32/3]

Iteration 2

[-2,0,2,4]	$\widetilde{x_1}$	[0,2/3,4/3,2]	0	[-2/3,0, 2/3,4/3]	[0,-1, -2/3,-1]	[6/3,0,0,-2/3]
[-2,0,2,4]	$\widetilde{x_2}$	[-2,-1,1,2]	[-2,0,2,4]	[-2,-1,1,2]	[-2,0,2,4]	[-2,2,3,5]
	$Z_3 - C_3$	0	0	[4/3,1, -1/3,-2/3]	[4/3,1, -1/3,-2/3]	[-14/3,0, 14/3,28/3]

In the optimal solution obtained $\widetilde{x_2}$ assumes integer value. But x_1 has a fractional value. Hence we generate a gomary's cut. The constraint corresponding to $\widetilde{x_1}$ is given below.

$[0,2/3,4/3,2] (*) \widetilde{x_1} (+)[-2/3,0,2/3,4/3] (*)S_1 -[0,-1,-2/3,-1] (*)S_2 (=)[6/3,0,0,-2/3]$

Using (1), we get

$[6/3,0,0,-2/3] < [-2/3,0,2/3,4/3] (*)S_1 (+)[-2/3,0,2/3,4/3] (*)S_2$

Introducing a slack variable S, we get

$S(-)[-2/3,0,2/3,4/3] (*)S_1 (-)[-2/3,0,2/3,4/3] (*)S_2 (=)[2/3,0,-2/3,-4/3]$

Include the above gomary's constraint in to the last simplex iteration table, which are tabulated below :

Iteration 3

$\widetilde{C_B}$	B	$\widetilde{x_1}$	$\widetilde{x_2}$	S_1	S_2	S	$\widetilde{x_B}$
[-2,0,2,4]	$\widetilde{x_1}$	[0,2/3,4/3,2]	0	[-2/3,0, 2/3,4/3]	[0,-1, -2/3,-1]	[-2,-1, 1,2]	[6/3,0, 0,-2/3]
[-2,0,2,4]	$\widetilde{x_2}$	[-2,-1,1,2]	[-2,0,2,4]	[-2,-1,1,2]	[-2,0, 2,4]	[-2,-1, 1,2]	[-2,2,3,5]
[-2,-1,1,2]	S	[-2,-1,1,2]	[-2,-1,1,2]	[2/3,0, -2/3,-4/3]	[2/3,0, -2/3,-4/3]	[-2,0, 2,4]	[2/3,0, -2/3,-4/3]
	$Z_4 - C_4$	0	0	[4/3,1, -1/3,-2/3]	[4/3,1, -1/3,-2/3]	[-2,-1, 1,2]	[-14/3,0, 14/3,28/3]

Iteration 4

[-2,0,2,4]	$\widetilde{x_1}$	[-2,-1/3, 7/3,4]	[-2,-1,1,2]	0	[2/3,-1, -4/3,-7/3]	[-4,-1, 3,6]	[-2,-1,1,2]
[-2,0,2,4]	$\widetilde{x_2}$	[-2,-1,1,2]	[-2,0,2,4]	[-2,-1,1,2]	[-2,0, 2,4]	[-2,-1, 1,2]	[-2,2,3,5]
[-2,-1,1,2]	S_1	0	0	[-2,0,2,4]	[-2,0,2,4]	[6,0, -6,-12]	[-2,0,2,4]
	$Z_5 - C_5$	0	0	0	0	[-2,0, 2,4]	[-2,2,3,5]

Here all $Z_5 - C_5 \geq 0$, hence the optimal solution of the stated integer programming problem is

$$\widetilde{x_1} (=)\left[x_1^{(1)}, x_1^{(2)}, x_1^{(3)}, x_1^{(4)}\right](=)[-2,-1,1,2], \widetilde{x_2}(=)\left[x_2^{(1)}, x_2^{(2)}, x_2^{(3)}, x_2^{(4)}\right](=)[-2,2,3,5] \&$$

$$\text{Max}\,\tilde{Z}(=)\left[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}\right](=)[-2,2,3,5]$$

3.2.2. Case (ii): Integer programming problem in terms of fuzzy membership functions with robust ranking approach

Let us solve the above integer programming problem by means of robust ranking approach [9].

From the above problem, the ranking index of R[0,2,4,6] by applying Robust ranking approach, the

membership function of above fuzzy number is defined as follows

$$\mu(x)(=)\begin{cases}\frac{x}{2} & ,0 \le x \le 2\\ 1 & ,2 \le x \le 4\\ \frac{x-6}{-2} & ,4 \le x \le 6\\ 0, otherwise\end{cases}$$

The interval of confidence for every level α & the trapezoidal shapes are going to be represented by functions of α within the following manner.

Here $\alpha = \frac{x_1^{(\alpha)}}{2}$ & $\alpha = \frac{(x_2^{(\alpha)}-6)}{-2}$

Therefore, $[x_1^{(\alpha)}, x_2^{(\alpha)}] = (A_\alpha^L, A_\alpha^U) = [(a_2-a_1)\alpha + a_1, a_4 + (a_3-a_4)\alpha]$

$= [2\alpha, -2\alpha + 6]$

$$R(A) = R[0,2,4,6] = \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\,d\alpha = \int_0^1 (0.5)(6)\,d\alpha = 3$$

Similarly, the ranking index of R[2,4,6,8] is given as follows

$$\mu(x)(=)\begin{cases}\frac{x-2}{2} & ,2 \le x \le 4\\ 1 & ,4 \le x \le 6\\ \frac{x-8}{-2} & ,6 \le x \le 8\\ 0, otherwise\end{cases}$$

The interval of confidence for every level α & the trapezoidal shapes are going to be represented by functions of α within the following manner.

Here $\alpha = \frac{x_1^{(\alpha)}-2}{2}$ & $\alpha = \frac{(x_2^{(\alpha)}-8)}{-2}$

Therefore, $[x_1^{(\alpha)}, x_2^{(\alpha)}] = (A_\alpha^L, A_\alpha^U) = [(a_2-a_1)\alpha + a_1, a_4 + (a_3-a_4)\alpha]$

$= [2\alpha + 2, -2\alpha + 8]$

$$R(A) = R[2,4,6,8] = \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\,d\alpha = \int_0^1 (0.5)(10)\,d\alpha = 5$$

Similarly apply the robust ranking index for all the other fuzzy numbers are computed as follows:

$$R(A) = R[-2,0,2,4] = \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\,d\alpha = \int_0^1 (0.5)(2)\,d\alpha = 1 \&$$

$$R(A) = R[-2,2,3,5] = \int_0^1 (0.5)(A_\alpha^L, A_\alpha^U)\,d\alpha = \int_0^1 (0.5)(4)\,d\alpha = 2$$

By means of the above approach the fuzzy integer programming problem reduced into standard crisp problem which are given as follows:

Maximize $Z = x_1 + x_2$,

Subject to the constraints, $3x_1 + 2x_2 \le 5; x_2 \le 2;\ x_1, x_2 \ge 0$ & are integers.

Now apply the existing integer programming problem of the method with the assistance of Gomary's constraint and the optimal solution obtained for the above problem is $x_1 = 0, x_2 = 2$ & the maximum Z = 2.

IV. COMPARISON ANALYSIS OF OPTIMAL SOLUTION OBTAINED FROM EXISTING AND ROBUST RANKING APPROACHES.

To compare the existing and robust ranking approach, for the results of fuzzy integer programming problem chosen in the above numerical example obtained by using the existing and ranking approach are shown in the following table. It is obvious from the results shown in the table that irrespective of whether by existing or ranking approaches gives same results, which are obtained in the above numerical example. On the basis of above result, it can be recommended that it is ideal to utilize any of the given approaches instead of existing approach, namely the fuzzy membership function approach or robust ranking approach [9].

Existing methodology based on the standard approach [14]	The optimal solution obtained for the above problem is $x_1 = 0, x_2 = 2$ & Max Z = 2.
Existing methodology based on the standard approach in terms of fuzziness [13]	The fuzzy optimal solution is $Max\,\tilde{Z}(=)[-2,2,3,5]$ and its decision variables are $\tilde{x_1}(=)[-2,-1,1,2]$ & $\tilde{x_2}(=)[-2,2,3,5]$
Proposed methodology based on the Robust ranking approach	The fuzzy optimal solution is $Max\,\tilde{Z}(=)[-2,2,3,5](=)R[-2,2,3,5] = 2$ and its decision variables are $\tilde{x_1}(=)[-2,-1,1,2](=)R[-2,-1,1,2](=)0$ & $\tilde{x_2}(=)[-2,2,3,5](=)R[-2,2,3,5](=)2$

V. RESULTS AND DISCUSSION

Using the above approach shows that the fuzzy optimum solution is [-2, 2, 3, 5], which may be physically understood as follows:

The optimal fuzzy value is going to be continuously bigger than -2 and fewer than 5 and most possibility is that the value is going to be between 2 and 3. The variations in cost with relevancy possibility is shown within the below figure.

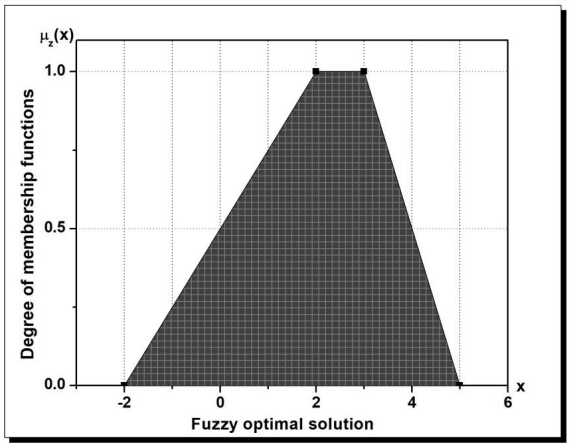


Figure: 5. Membership function of the fuzzy optimal solution

Let x represent the value of overall fuzzy optimal solution and then the proportion of the favourness in the decision maker for $x = \mu_z(x)$, where

$$\mu_z(x)(=)\begin{cases}\dfrac{x+2}{4}, -2 \le x \le 2\\ 1, 2 \le x \le 3\\ \dfrac{x-5}{-2}, 3 \le x \le 5\\ 0, \text{otherwise}\end{cases}$$

CONCLUSION

In this article, the integer programming problem fully considered as by trapezoidal fuzzy membership functions which are additional realistic and general in nature. Moreover, Numerical example shows the optimum solution obtained from the manner of fuzzy membership functions and also the same problem with the assistance of Robust ranking approach [9,13] offers an efficient tool for handling the Integer programming problem. For justification the comparison analysis of optimal solution was given with in the nature of fuzzy membership functions, which might be a brand new try in solving the integer programing problem in fuzzy environment. An equivalent approach for solving the fuzzy issues may be utilized in future studies of operational research analysis.

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