ENTANGLEMENT BETWEEN BEC BOGOLIUBOV MODES AND OUTPUT OPTICAL FIELD OF A CAVITY

[1]MOHAMMAD EGHBALI-ARANI, [2]VAHID AMERI

[1]Department of Physics, University of Kashan, Kashan, Iran
[2]Department of Physics, University of Hormozgan, Hormozgan, Iran
E-mail: [1]m.eghbali@kashanu.ac.ir, [2]vahameri@gmail.com

Abstract— In this paper, we exploit the entanglement between output _field and the lowest bands (c_{10}; c_{01}) of a one dimensional BEC inside a driven optical cavity. We consider atomic collision on the behavior of the BEC in the weak photon-atom coupling, and use Bogoliubov approximation for the condensate. This entanglement is explained as being generated by a scattering process owing to which strong quantum correlations between the Bogoliubov modes and the optical Stokes sideband are created. We show that the Bogoliubov modes and the sideband modes form a fully bipartite and tripartite entangled system capable of providing a novel tool for generating and robust solutions for continuous-variable quantum-communication entanglement.

I. INTRODUCTION

It is acceptable that the quantum entanglement plays an essential role for the implementation of quantum information processing and continuous variables (CVs) are a powerful alternative approach to this purpose [1, 2]. In recent years, due to the vast application of quantum entanglement, several physical systems have been studied such as optomechanical systems [3], ultracold atomic ensemble trapped in optical lattice [4-6], trapped ions and atoms in optical cavities [7-10]. Interestingly, many of studies in cavity optomechanics have been emerged by combining the QED cavities with BEC modes [11-13]. It has been shown that an ensemble of ultracold atom inside a cavity can couple to the optical _field, where the excitation of the ultracold atoms plays the role of the vibrational mode of the mirror in an optomechanical system. Like a standard optomechanical system, nonlinear optics has been observed from the coherent motion of ultracold atoms, and the atomic collisions have changed the resonance frequency of the cavity [6].

In some of these opto-mechanical systems have been investigated schemes based on the free-space light modes scattered by a mirror, which could allow the implementation of continuous variables (CVs) entanglement swapping [14]. By using the traveling output modes, any quantum communications can be implemented rather than intracavity ones. So it is important to study how the entanglement generated within the cavity between the BEC modes and light is transferred to the output _field. It has been shown that quantized mechanical resonator can be described by CVs [15, 16], and recently by using it strong CV entanglement between different optical modes is generated [17].

This paper is organized as follows. In Sec. II we first provide a brief theoretical description of the system under consideration and the quantum Langevin equations (QLEs) are derived and linearized around the semiclassical steady state. In Sec. III, we study the quantum state transferring of output mode and BEC modes inside the one cavity and quantify the entanglement between the output optical modes by using the logarithmic negativity.

II. FORMULATION AND THEORETICAL DESCRIPTION OF THE PROBLEM

An alternative path to the studies of cavity optomechanics has been provided experimentally by systems consisting of ultracold atomic ensembles trapped in optical cavities [11, 12] where the excitation of a collective mode of the cold gas plays the role of the vibrational mode of the mirror. The standing electromagnetic wave inside the cavity forms a periodic potential, the so-called optical lattice, in which the cold atoms exhibit phenomena typical of solid state physics like the formation of energy bands and Bloch oscillations [18]. We study cigar-shaped gas of N ultracold bosonic two-level atoms with mass m and transition frequency !a in a double Fabry-Perot cavity as sketch in Fig.1.

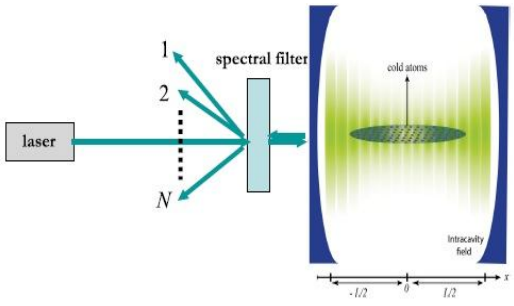


Fig1: Scheme of the cavity composed of cigar shaped BEC, which is driven by a laser. With appropriate filters one can select N independent modes from the cavity output _field.

The cavity is compose of N ultracold bosonic two level atoms with mass M, and transition frequency ω_a. The optical cavities with length *L*, are driving at rates *E*, and the wave number $K = 2\pi / \lambda = \omega_p / c$. In the dissipative regime where the laser pump is far detuned from the atomic resonance ($\Delta_a = \omega_p - \omega_a$) and ignoring spontaneous emission, we can adiabatically eliminated the excited electronic state of the atoms [19].

The many-body Hamiltonian of system in the frame rotating at pump frequency reads

$$H = -\hbar\Delta_c a^\dagger a + i\hbar E(a^\dagger - a) + \int \psi^\dagger(x) H_0 \psi(x) dx + H_{aa}, \quad (1)$$

where $\Delta_a = \omega_p - \omega_c$ is the cavity-pump detuning, and a is the annihilation operator for the cavity photons of each cavity. H_{aa} and H_0 are the atom-atom interaction, and the single-particle Hamiltonian of an atom inside the optical lattice of each cavity respectively.

$$H_{aa} = \frac{1}{2} U_s \int_{-L/2}^{L/2} dx \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x),$$
$$H_0 = \frac{p^2}{2M} + \hbar U_0 \cos^2(Kx) a^\dagger a. \quad (2)$$

The parameter $U_0 = g_0^2 / \Delta_a$ is the optical lattice barrier height per photon and represent the backaction on the field, g_0 is the vacuum Rabi frequency, $4\pi\hbar^2 a_s / M$ as M, and as is the two-body s-wave scattering length [20, 21].

It is well-known that the eigenfunction of a particle inside a periodic potential are Bloch functions, so by using Bloch theorem, Born-Van Kaman periodic boundary condition, parity symmetry and retaining the lowest Bogoliubov modes, the Hamiltonian (1) reduces to the following forms [6]:

$$H = -\hbar\Delta_c a^\dagger a + i\hbar E(a^\dagger - a) + \hbar(4\omega_R + \omega_{sw}) c_{10}^\dagger c_{10}$$
$$+\hbar\left(\frac{4}{l^2}\omega_R + \omega_{sw}\right) c_{01}^\dagger c_{01} + H_{ac} + H_{aa}$$
$$H_{ac} = \frac{\sqrt{2N}}{4} h U_0 a^\dagger a (c_{10}^\dagger + c_{10}) + \frac{1}{2} h U_0 a^\dagger a (c_{10}^\dagger c_{10} + c_{01}^\dagger c_{10}),$$
$$H_{ac} = \frac{1}{4} h\omega_{sw} (c_{10}^2 + c_{10}^{\dagger 2} + c_{01}^2 + c_{01}^{\dagger 2}) \quad (3)$$

Where $c_{10}(c_{10}^\dagger)$, and $c_{01}(c_{01}^\dagger)$ is the lowest atomic field operator that annihilates (creates) a particle in Bogoliubov modes. $\tilde{\Delta}_c = \Delta_c - \frac{NU_0}{2}$ and $\omega_R = \frac{\hbar K^2}{2M}$ are the effective Stark shift detuning and the recoil frequency respectively. $\omega_{sw} = \frac{8\pi\hbar N}{ML\omega^2}$ is the s-wave scattering frequency and ω is the waist of the optical potential. Also we treat classically with zero mode i.e $c_{00} = \sqrt{N}$, and by substituting a=α+δa, $c_{10} = \sqrt{N}\beta_1 + \delta c_{10}$, $c_{01} = \sqrt{N}\beta_0 + \delta c_{01}$ into Eq.(**Error! Reference source not found.**), one can obtain a set of nonlinear algebric equation for the steady state value:

$$\alpha = \frac{E}{\sqrt{\Delta_d^2 + \kappa^2}},$$
$$\beta_1 = \frac{\sqrt{2N}}{4} \frac{U_0 \alpha}{\sqrt{\Omega_{10}^{(+)^2} + \gamma_i^2}},$$
$$\beta_0 = 0. \quad (4)$$

Where $\Delta_d = \tilde{\Delta}_c - \frac{1}{2} N U_0 \beta_1 (\beta_1 + \sqrt{2})$,

$\Omega_{10}^{(\pm)} = \hbar\omega_{10} \pm \frac{1}{2}\omega_{sw}$, $\tilde{\omega}_{10} = \omega_{10} + \frac{1}{2} U_0 \alpha^2$. Finally after linearized QLEs for the fluctuating operator, we derive 6×6 drift **A**, and diffusion **D** matrices for each cavity:

$$\mathbf{A} = \begin{pmatrix} -\gamma_i & \Omega_{10}^{(-)} & 0 & 0 & 0 & 0 \\ -\Omega_{10}^{(+)} & -\gamma & 0 & 0 & -G & 0 \\ 0 & 0 & -\gamma & \Omega_{01}^{(-)} & 0 & 0 \\ 0 & 0 & \Omega_{01}^{(+)} & -\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & -\kappa & \Delta_d \\ -G & 0 & 0 & 0 & \Delta_d & -\kappa \end{pmatrix}, \quad (5)$$

$$\mathbf{D} = diag[\gamma(2n_B + 1), \gamma(2n_B + 1), \gamma(2n_B + 1), \gamma(2n_B + 1), 2\kappa, 2\kappa] \quad (6)$$

where $G = U_0 \sqrt{N}\alpha(\beta_1 + \frac{\sqrt{2}}{2})$ is effective atom-photon coupling parameter for each cavity.

Due to the linearized dynamics of the fluctuations and since all noises are Gaussian the steady state is a zero-mean Gaussian state which is fully characterized by the stationary correlation matrix (CM) **V**. For entanglement between two BECs, we are interested an expression for output optical fields, So we derive the following stationary output correlation matrix V^{out} for each cavity

$$V^{out} = \int d\omega \tilde{\mathbf{T}}_i(\omega)(\tilde{\mathbf{M}}^{ext}(\omega) + \mathbf{P}^{out})\mathbf{D}(\tilde{\mathbf{M}}^{\dagger ext}(\omega) + \mathbf{P}^{out})\tilde{\mathbf{T}}^\dagger(\omega), \quad (7)$$

where $\tilde{\mathbf{M}}^{ext}(\omega) = (i\omega + \mathbf{A})^{-1}$, and $\mathbf{P}^{out} = diag[0,0,0,0,1/2\kappa, 1/2\kappa]$. $\tilde{T}(\omega)$ is the Fourier transform of

$$\tilde{\mathbf{T}}(\omega) = \begin{pmatrix} \delta(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & R & -I \\ 0 & 0 & 0 & 0 & -I & R \end{pmatrix}, \quad (8)$$

where $R = \sqrt{2\kappa} Re[g(t)]$ and $I = \sqrt{2\kappa} Im[g(t)]$ are determined by the causal filter functions[22].g(t) with bandwidth $1/\tau$ and central frequency Ω_1.

III. ENTANGLEMENT BETWEEN OUTPUT OPTICAL FIELD AND BOGOLIUBOV MODES

However like an optomechanical system that the motion of mechanical resonator generates Stokes and anti-Stokes motional sidebands, the BEC modes generates Stokes and anti-Stokes sidebands, consequently modifying the cavity output spectrum. Therefore it may be nontrivial to determine which is the optimal frequency bandwidth of the output field that carries most of the optomechanical entanglement generated within the cavity. The cavity output spectrum associated with the photon number fluctuations $S(\omega)=\langle\delta a(\omega)^{\dagger}\delta a(\omega)\rangle$ is shown in Fig.**Error! Reference source not found.**, where we have considered a parameter regime with recoil frequency $\omega_R/2\pi=3.57\times10^3$, cavity detuning $\Delta_c=28000\omega_R$, the s-wave scattering frequency $\omega_{sw}=\omega_R$, the number of atoms $N=6\times10^4$, the optical lattice barrier height per photon $U_0=0.96\omega_R$, the cavity damping rate $\kappa=363.9\omega_R$, dissipation of collective density excitations of the BEC $\gamma=0.001\kappa$, laser input power $E=80.06\omega_R$, and BEC temperature $T=10^{-7}$.

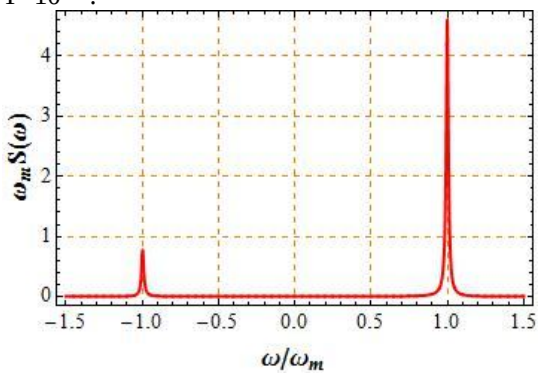


Fig 2: Cavity output spectrum in the case of an oscillator with $\omega_R=2\pi\times3.57\times10^3$, $\Delta_c=28000\omega_R$, $\omega_{sw}=\omega_R$, $N=6\times10^4$, $U_0=0.96\omega_R$, $\kappa=363.9\omega_R$, $\gamma=0.001\kappa$, $E=80.06\omega_R$, and $T=10^{-7}$. In this regime photons are scattered only at the two first motional sidebands, at $\omega_0\pm\omega_m$.

In order to determine the output optical mode that is best entangled with the Bogoliubov modes, we study the logarithmic negativity E_N associated with the output CM V^{out}. As a function of the central frequency of the mode Ω_1 and its bandwidth τ^{-1}, at the considered operating point. The results are shown in Fig. **Error! Reference source not found.**, where bipartite entanglement E_N between c_{10}, c_{01}, and output optical field is plotted versus Ω_1/ω_m. This figure show that the maximum entanglement is between c_{10} mode and output optical field (c_{10},f). Fig. **Error! Reference source not found.** shows entanglement between two Bogoliubov modes (c_{01},c_{10}), and entanglement between c_{01} and output optical filed (c_{01},f), while Dalafi et. al [6]. only showed that only exist entanglement between c_{10} and intracavity optical field.

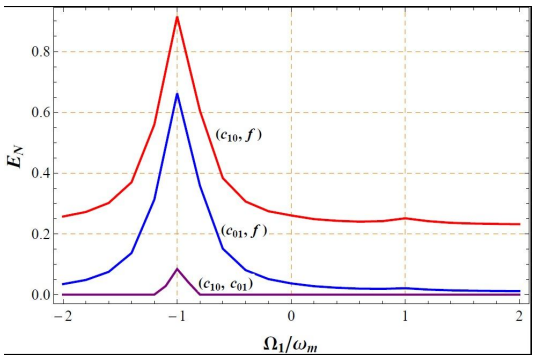


Fig 3: Three CV bipartite logarithmic negativity E_N of BEC modes and a single cavity output mode (c_{10},f),(c_{01},f), and (c_{10},c_{01}) versus the central frequency of the detected output mode Ω_1/ω_m. The other parameters are the same as in Fig **Error! Reference source not found.**.

In Fig. **Error! Reference source not found.** the entanglement between c_{10} mode and output optical field E_N is plotted versus Ω_1/ω_m at five different values of $\varepsilon=\omega_m\tau$. If $\varepsilon\lesssim1$, i.e., the bandwidth of the detected mode is larger than ω_m, the detector does not resolve the motional sidebands, and E_N has a value (roughly equal to that of the intracavity case) which does not essentially depend upon the central frequency. For smaller bandwidths (largerε), the sidebands are resolved by the detection, and the role of the central frequency becomes important. In particular, E_N becomes highly peaked around the Stokes sideband $\Omega_1=-\omega_m$, showing that the optomechanical entanglement generated within the cavity is mostly carried by this lower-frequency sideband. What is relevant is that the entanglement of the output mode is significantly larger than its intracavity counterpart and achieves its maximum value at the optimal value $\varepsilon\simeq8$ i.e., a detection bandwidth $\tau^{-1}=\omega_m/8$. This means that, in practice, by appropriately filtering the output light, one realizes an effective entanglement distillation because the selected output mode is more entangled than the intracavity mode Dalafi et. al [6]. with the Bogoliubov mode. It is finally important to

analyse the robustness of the present entanglement with respect to temperature.

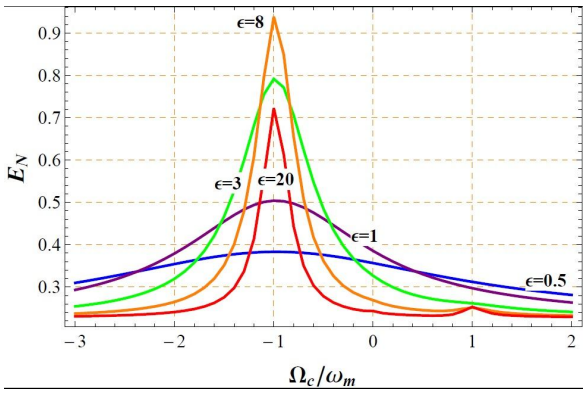


Fig 4: Logarithmic negativity E_N of the CV bipartite system formed by the BEC mode and a single cavity output mode versus the central frequency of the detected output mode Ω_1/ω_m at five different values of its inverse bandwidth $\varepsilon=\omega_m\tau$ The other parameters are the same as in Fig **Error! Reference source not found.**.

It is important to see if this robustness is retained also by the Bogoliubov mode and field mode entanglement of the output mode. This is shown by Fig. **Error! Reference source not found.**, where the entanglement E_N of the output mode centred at the Stokes sideband $\Omega_1=-\omega_m$ is plotted versus the temperature of the reservoir at two different values of the bandwidth, the optimal one $\varepsilon=8$, and at a larger bandwidth $\varepsilon=1$. We see the expected decay of E_N for increasing temperature, but above all that also this output optomechanical entanglement is robust against temperature because it persists even above liquid He temperatures, at least in the case of the optimal detection bandwidth $\varepsilon=8$.

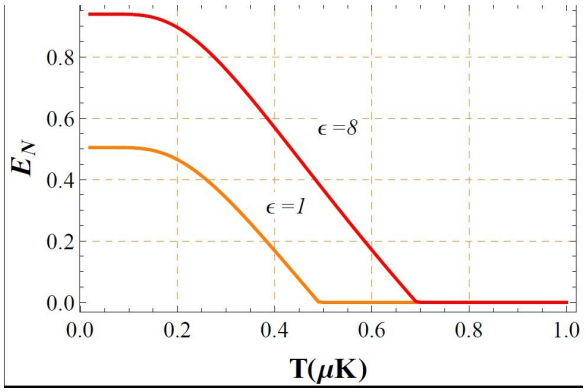


Fig 5: Logarithmic negativity E_N of the CV bipartite system formed by the Bogoliubov mode and the cavity output mode centered around the Stokes sideband $\Omega_1=-\omega_m$ versus temperature for two different values of its inverse bandwidth $\varepsilon=\omega_m\tau$. The other parameters are the same as in Fig. **Error! Reference source not found.**

SUMMARY AND CONCLUSIONS

In conclusion, we have investigated the entanglement between BEC modes and output optical field, which is more important from a practical point of view because any quantum-communication application involves the manipulation of traveling optical fields. We have developed a general theory showing how it is possible to define and evaluate the entanglement properties of the multipartite system formed by the BEC modes and N independent output modes of the cavity field. It has been shown that the Stokes output mode is strongly entangled with the BEC modes and shows robust entanglement when the bandwidth is appropriately chosen.

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