BIFURCATION DIAGRAMS OF LOGISTIC FUNCTION FOR DIFFERENT PARAMETER VALUES

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Abstract - Dynamical systems concept is a branch of mathematics that attempts to understand processes in motion. For instance, the motions of the stars and galaxies in the heaven are a dynamical system. The world's weather is another system that changes in time as is the stock market. The logistic map is a polynomial mapping of degree 2, often cited as archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. Bifurcation means a deflect, a rending aside. How and when physical, chemical and biological systems sustain sudden barters of the behavior is the appearance of bifurcation. Let us consider $F_r(x) = rx(1-x)$ denote the logistic map, Where r is considered as parameter. Here we observe the situations for different parameter values of r. In fact, we will observe attracting and repelling fixed points together with phase portrait. Of course, bifurcation diagram to be sketched in each situation. In the case of logistic bifurcation, we are considering the limits or end behaviors of logistic systems. To comprehend bifurcation behavior, it is always conductive to look at the bifurcation diagram. Saddle-node bifurcation and period-doubling bifurcation are speculated. Animation of a point "rolling" along the logistic curve, bifurcation diagram on logistic map for several iterations are analyzed. We have conferred aggregate espials for different parameter values. Finally, we have exhibited that the schemes are chaotic and non-chaotic for detached parameter appraises.

I. SADDLE-NODE (FOLD) BIFURCATION

A saddle-node is a conflict and invisibility of two positions in dynamical systems. In disembodied dynamical systems, the system mount bifurcation is continually substitute called a convolution bifurcation. Another name is blue skies bifurcation in allusion to the snappy designing of two fixed points. A saddle-node bifurcation also befalls if the trace of the bifurcation is tumbled. Revolved acme may endure a saddle-node bifurcation. If the cycle respite is one-dimensional, one of the equivalence mites hectic (the saddle), while the other is durable (the node).Saddle-node bifurcations may be affiliated with hysteresis noose and plague. Bifurcations narrate alteration in the durability or fixity of fixed points as a seizure parameter in the scheme innovates. As an excellently naïve affirmation of a bifurcation in a dynamical system, suspect a thing coherent on turret of a vertical beam. The mass of the thing can be envisaged of as the seizure parameter. As the mass of the thing increases, the beam's drift from vertical, which is x, the dynamic variable, resides comparatively consolidated. The mass proximity a certain point – the bifurcation point – the beam will abruptly ouch. Switches in the control parameter in the end switched the characteristic dealing of the system.

Consider the equation of the form $\frac{dx}{dt} = -x^2 + r$ shown in figure 1.1

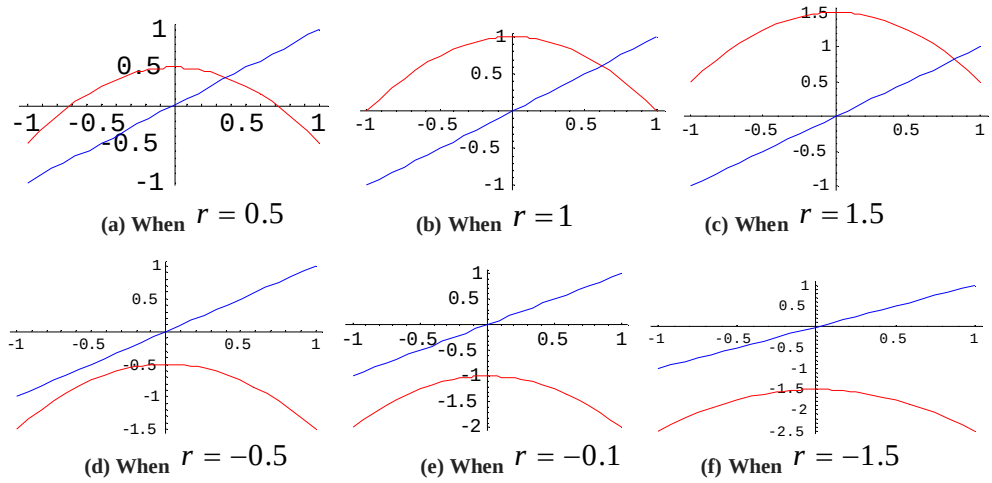


Figure1.1 Saddle-node bifurcation.

W{\displaystyle {\dot {x}}=-x^{2}+\varepsilon }WWwWhere r is the seizure parameter. As r is greater than 0, the scheme has one adjusted fixed point and one unadjusted fixed point. As r abates the fixed point's shakes simultaneously, shortly clash into a semi-adjusted fixed point at $r=0$, and to exist when $r<1.$

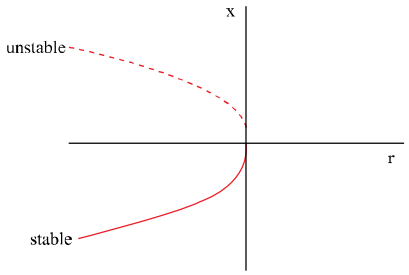


Figure 1.2 Bifurcation diagram for a saddle node bifurcation of the equation $\frac{dx}{dt}=-x^2+r$

The behavior of the scheme changes meaningful when the seizure parameter r is 0, 0 is a bifurcation point. This example bifurcation is called the saddle-node bifurcation.

II. PERIOD DOUBLING BIFURCATION

A period doubling bifurcation in a discrete dynamical system is a bifurcation in which a slight change in a parameter value in the system's equations leads to the system switching to a new behavior with twice the period of the original system. With the doubled period, it bears twice as many repetitions of a process as prior for the numerical values visited by the system to revolve them. A period doubling cataract is a succession of doublings and into the bargain doublings of the repeating feast, as the parameter is constant therewithal and therewithal. Period doubling bifurcations can also ensue in incessant dynamical systems, namely as newish tether arise from an existing tether cycle, and the epoch of the newish tether cycle is twice that of the old one. As the parameter innovates, a fixed point may innovate from attracting to repelling and, at the same, give birth to an attracting 2-cyle. Otherwise, the fixed point may innovative from repelling to attracting and, at the equivalent time, give birth to a repelling hoop of period 2.

III. RESULTS AND DISCUSSIONS

3. 1 Proposition

Let $F_r(x)=rx(1-x)$ be a logistic family of functions. Then we have the following propositions:

3.1 The values of r does F_r have an attracting fixed point at $x=0$.

3.2 The values of r does F_r have a nonzero attracting fixed point.

3.3 The bifurcation that occurs when $r=1.$

3.4 Phase portrait and bifurcation diagram near at $x=1$.

3.5 The bifurcation that occurs when $r=3.$

3.6 An explicit formula for the periodic points of period two for F_r.

3.2 Bifurcation diagram on logistic map for several iterations

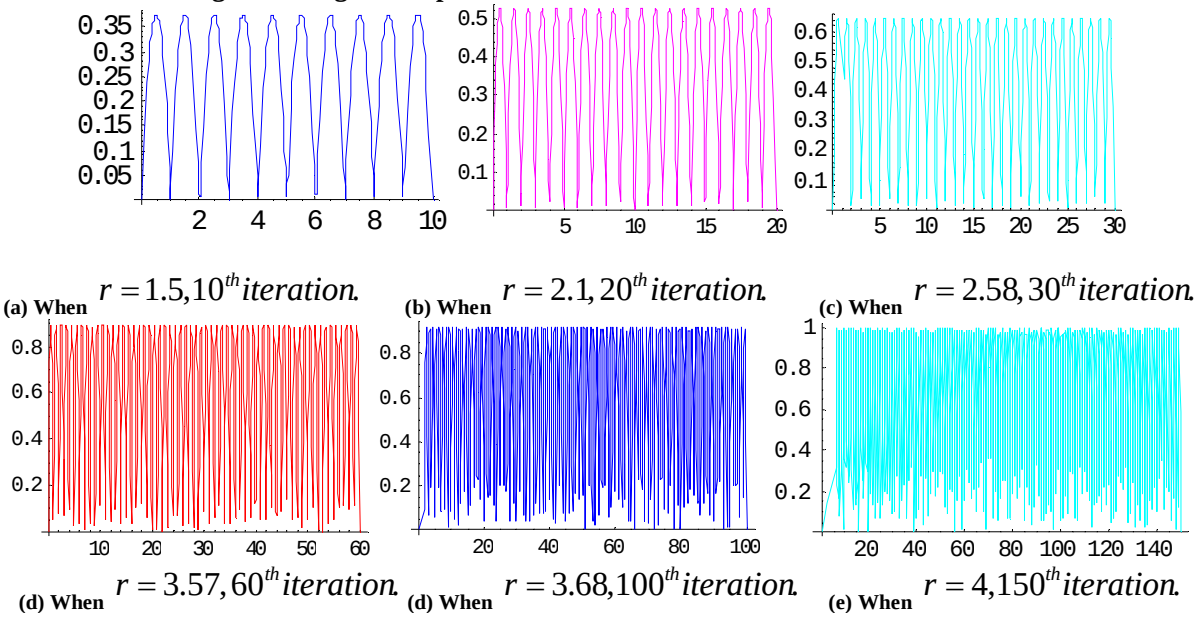
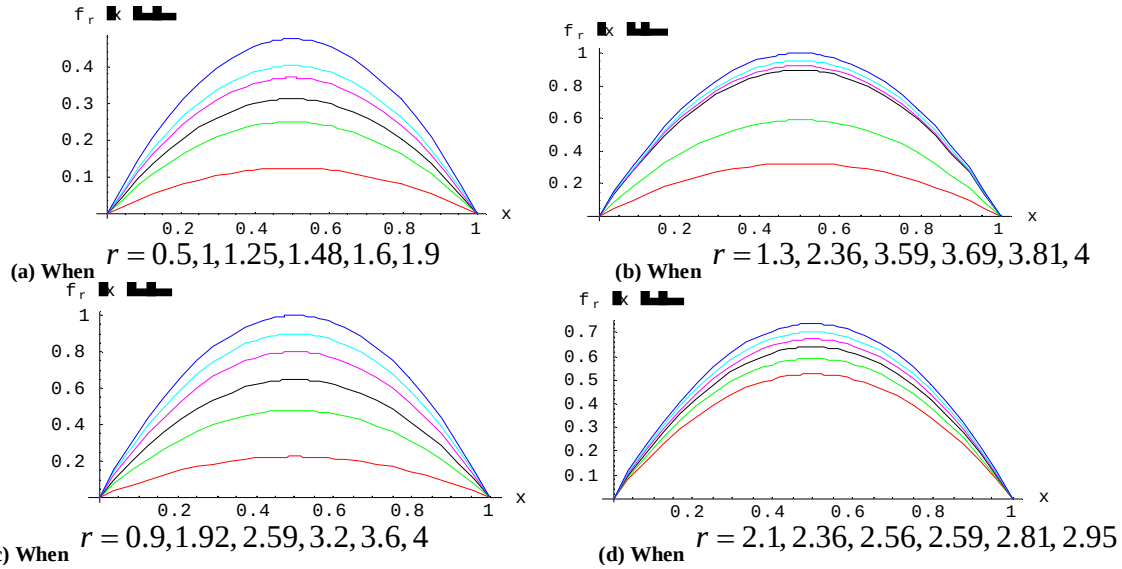


Figure 3.2.1 A bifurcation in the logistic family $f_r(x)=rx(1-x)$ **, for several parameter values of r and several iterations.**

We have accomplished that when $r = 1.5$ to 4 and $r = 3.57$ to 4 and the number of iterations taken 10 to 50 times, the illustrated figures behooving narrow to narrower.



(a) When $r = 0.5, 1, 1.25, 1.48, 1.6, 1.9$ **(b) When** $r = 1.3, 2.36, 3.59, 3.69, 3.81, 4$

(c) When $r = 0.9, 1.92, 2.59, 3.2, 3.6, 4$ **(d) When** $r = 2.1, 2.36, 2.56, 2.59, 2.81, 2.95$

Figure 3.2.2 A bifurcation in the logistic family $f_r(x) = rx(1-x)$ **for different values of** r.

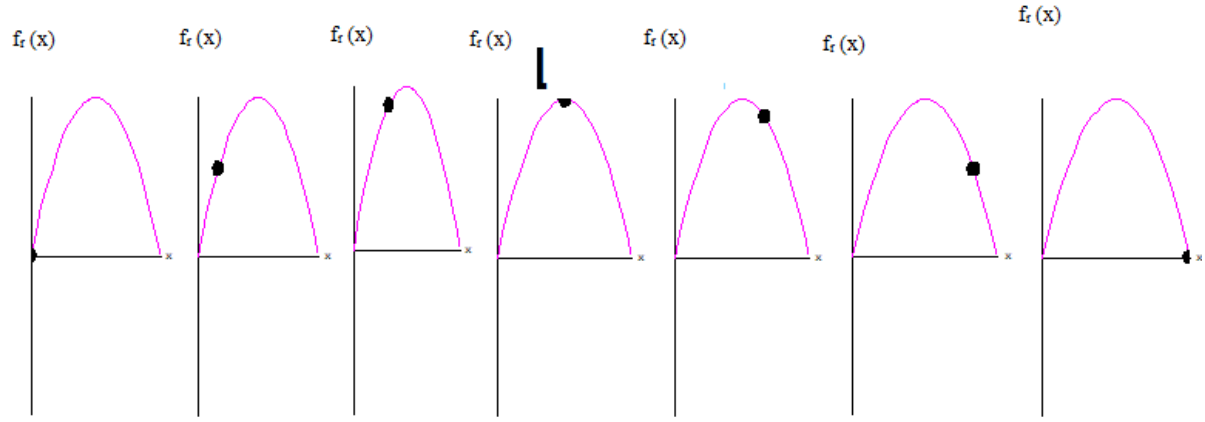


Figure 3. 2.3 Animation of a point "rolling" along the logistic curve $f_r(x) = rx(1-x)$ **from 0 to 1, when** $r = 4$.

3.3 Bifurcation Diagram of Logistic Function for Different Parameter Values

A period-doubling bifurcation is $f_r(x) = rx(1-x)$ for $r = 3$. This function is called Logistic function. The Logistic function $f_r(x) = rx(1-x)$, where r is a parameter. For fixed point $f_r(x) = x$

$\Rightarrow x = 0 \quad or, \; x = \dfrac{r-1}{r} = P_r$ (say).

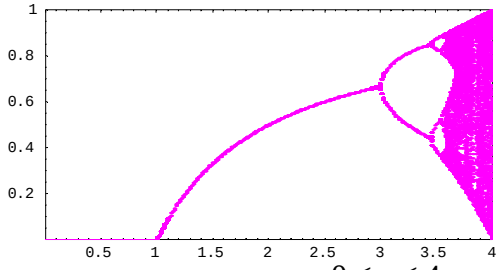
If $r > 1$, then f_r have two fixed points: one at $x = 0$ another at $x = P_r$.

If $r = 1$, then $x = 0$ is the only fixed point for f_r.

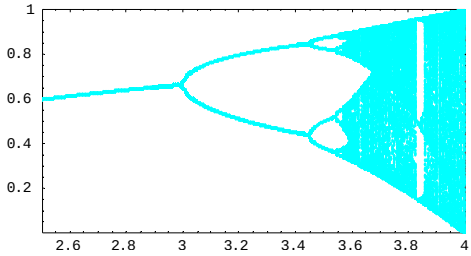
1. If $r = 1$, a transcritical bifurcation occurs at $P_1 = 0$.

2. If $r = 3$, a period-doubling bifurcation occurs at $P_1 = \dfrac{2}{3}$.

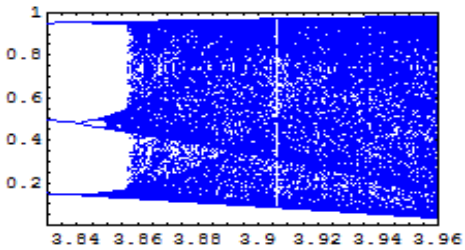
3. If $r = 3.45$, another period-doubling bifurcation occurs and the period two attracting orbit splits into a period four attracting orbit and a period two repelling orbit.



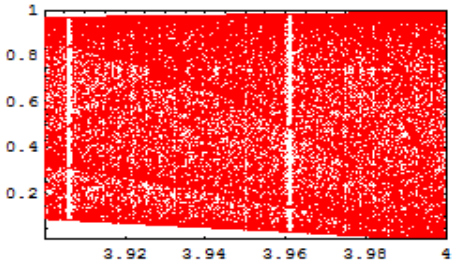
(a) Logistic bifurcation when $0 \le r \le 4$



(b) Logistic bifurcation when $2.5 \le r \le 4$



(c) Logistic bifurcation when $3.83 \le r \le 3.96$



(d) Logistic bifurcation when $3.86 \le r \le 3.98$

From the aloft sketch, tidy bifurcations are perceptible in the chaotic virtues of the diagram. Sometimes bifurcation is considered as a fractal. The same is true for all other non-chaotic points. We noticed that the windows of period three at $r = 3.83$, period five at $r = 3.74$, and period six at $r = 3.63$ in the above diagram. Sooth to say, chaotic nature ensues between the interim3.57 and 4.

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