DYNAMIC MODELING AND PARAMETRIC ANALYSIS ON NONLINEAR VERTICAL CUTTING PROCESS WITH VARIABLE STIFFNESS

[1]MAHMOOD REZA MEHRAN, [2]MEHDI MEHRAN

[1]Department of Mechanical Engineering, East Tehran Branch, Islamic, Azad University, Tehran, IRAN
[2]S. B. Manager of R&D in NIDC, Ahvaz, IRAN
E-mail: mahmood_mehran@yahoo.com, m_mehran57@yahoo.com

Abstract- In this investigation a nonlinear dynamic analysis was conducted on metal cutting process to evaluate system stability regarding to changing in the metal stiffness. For this propose a model with the forces due to coulomb friction and variable stiffness was solved. The manner is based on multiple time scales perturbation method that presents acceptable solution for non-resonance condition and stiffness with time variable. The important role of feed, depth and length of cutting and also effect of physical constants in system stability was carried out.

Keywords- Metal cutting Process, Perturbation Methods, Multiple Time Scales, Time Variable Stiffness

I. INTRODUCTION

TODAYS, there is no piecework in industry that no machining process was done on it. In some strategic industries such as power plants, airspace and so on, non-suitable operation even one part causes operations stop in massive projects. One of the important reasons of this can refer to the manner of production and machining accuracy. Products with complicate shapes need to high quality and this makes the role of machining be more colored than past. The promotion of machines and developments of mathematic models in these lately years states that there are good validations and harmony between output machining and result of math modeling for controlling of machine process specially in metal cutting.

The early dynamic models of the cutting process were more focused on the physical effect of interaction between machine mechanics and cutting process by solving of numerical equations of self-exited vibration models [1], [2]. Self-exited vibrations during cutting process are limitation factors in machining accuracy and lead to failure and break down the device quickly. Hence, this is undesired phenomena and many researches were dedicated to it [3]-[8]. Later another phenomenon in the form of stick-slip effects in chisel movement on work piece was entered in researches [9]-[10]. In more attempts researchers found chaos and turbulent in the machine response when chattering of machining tool were occurred [11]-[12]. Also, the effects of the tools and chisel arranging and their position on work piece with different dynamic variables were studied by them and found out that intermittency joint and disjoint that is chattering machine tool lead to instability behavior. With depth vision, there are many applied data for systems that causes instability and chaos. This means that control of metal cutting process is hard and the stability of cutting can't obligatory guaranty for all systems and also shows reduction efficiency in metal changing process. In practice, effects of stick-slip, chattering and intermittency may be due to work piece deviations and also chips replete due to change of chip thickness causes non-smoothness surface on work piece.

One of the important issue about the parts and work pieces is that should not only have sufficient strength but also hove logical justification for economic production that is the weight of parts could be reduced without strength decreasing which lead to stiffness reduction. Naturally the work piece with high softness and low stiffness can cause significant problems. Some outstanding researchers who studied on these problems are Warminski et al. [13] that conducted on the dynamic effects of the vertical metal cutting and dynamic chaos and they also worked on the dynamic analysis of chaos in a 2D model with constant coefficient. Vibration behavior of a cutting system based on random distribution was carried out by Lipski and the others [14].

The presence investigation has dedicated to cutting analysis of work piece with variable stiffness when chisel is perpendicular on work piece. In lately years, analytically approximation solutions were presented for modeling of nonlinear dynamic behavior of vertical cutting process and also studies on analysis of nonlinear dynamic cutting forces and their effects on cutting system using multiple time scales were presented by Hwang [15] and Nayfeh et al. [16]. In presence research also effects of cutting depth, cutting length and particular physical constants on stability were considered. For this propose, dynamic models were presented by Wiercigroch [17] and Rusinek [18] which refer to variable stiffness an coulomb friction is

regarded. Wiercigroch [17] propose the revision model that first time had been presented by Grabec [19], [20]. These Grabec [19] works has done base on Juilian [21] and point to existence inherent discontinuity in cutting process and also imply on a feedback between chisel and real cutting process.

The revision model of Wiercigroch [17] was based on adding friction term regarding to Hasting et al. [22]. The result of their work states that there is a mathematic relation between cutting forces and they show this relation formed form effects of discontinuity friction. So, based on above works and also regarding to variable stiffness that presented by Rusinek [18], a developed model as a logical indicator of cutting process was considered as in Fig. 1.

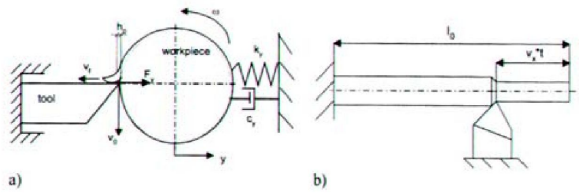

Fig. 1 physical model of vertical cutting process [18]

II. METAL CUTTING SYSTEM EQUATIONS

First the stiffness parameter introduced and discussed and then related dynamic equations regarding to variable stiffness is presented:

A. Evaluation of Stiffness Parameter

Based on test and laboratory results, the amount of experimental stiffness in the cutting process is less than the theoretical one [18].

$$k = \frac{3EI}{l_0^{\,3}} \tag{1}$$

It is clear that the equation (1) should correct with an extra coefficient called (α) which depends on work piece length (l_0) and real cutting length (l).

$$k_y = \frac{3EI\alpha}{l_0^{\,3}} \tag{2}$$

$$\alpha = \alpha_1 \frac{l}{l_0} \tag{3}$$

Therefore (α_1) is regarded as a particular coefficient for work piece which is hold by vice versa jaws.

Fig. 2 shows both theoretical and experimental stiffness curves. To fitting perfectly of these two curves on each other and convergence of the answers, correction of theoretical stiffness equation was done by adding (α) coefficient.

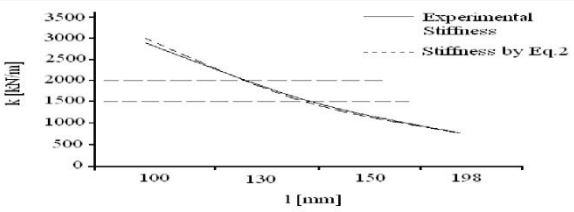

Fig. 2 theoretical and experimental stiffness of cutting process [18]

B. Dynamic Equations of Metal Cutting System

As mentioned before the model of cutting process based on developed model of Wiercigroch [17] and Rusinek [18] has been shown in Fig. 1. This model is one degree of freedom in Y direction. (f_y) is the force component related to friction force which heavy side and sign functions were appeared in it. These parameters are needed to show discontinuity of joint between chisel and work piece. The related velocity parameter (v_f) that shows feed rate of chisel in Y direction also should be presented. The developed dynamic equations of mentioned model based on Warminski [23] are:

$$m\ddot{y} + k_y(t)y + c_y\dot{y} = f_y \tag{4}$$

Introducing the ($k_z(t)$) as a monotonic time function and substitution in related equation, so:

$$k_z(t) = \frac{3EI\alpha_1}{l_0(l_0 - v_x t)^2} = \frac{3EI\alpha_1}{l_0^{\,3}\left(\frac{l_0 - v_x t}{l_0}\right)^2} \tag{5}$$

$$if \quad \beta = \frac{\alpha_1}{\left(\frac{l_0 - v_x t}{l_0}\right)^2} \;,\; a = \alpha_1 \tag{6}$$

$$\ddot{y} + 2\xi_y\sqrt{\beta}\dot{y} + \beta y = [C_2(|v_f| - 1)^2 + 1][C_3(h-1)^2 + 1]H(f)\mathrm{sgn}(v_f)q_0 h[C_1(|v_0| - 1)^2 + 1]$$
$$H(h)\left(H(v_0)\frac{1}{1+\mu_0} + \mathrm{sgn}(v_0)\frac{\mu_0}{1+\mu_0}\right) = f_y \tag{7}$$

Where (c_y, k_y, f_y) are stiffness, damping and force of cutting respectively. (q_0) is magnitude of cutting force, (h_0) is initially cutting depth, (h) is real cutting depth, (μ_0) is static friction coefficient and (v_f) is related velocity of chisel in Y direction. Also, ($C_{1\text{-}4}$, v_0, R, R_0) are constants of process, absolute velocity of work piece, coefficient of cutting plastic deformation and the constant of cutting plastic deformation respectively, that define as follow:

$$v_f = v_0 - R\dot{y} \quad , \quad h = h_0 - y \tag{8}$$

$$\ddot{y} + 2\xi_y\sqrt{\beta}\dot{y} + \beta y = Q_3(h_0 - y)[C_1\sigma_2 + 1][C_2\delta_1^2 - 2C_2\delta_1\delta_4\dot{y} + C_2\delta_4^2\dot{y}^2 + 1] * [C_3\sigma_1 + C_3y^2 + 2C_3y - 2C_3h_0y + 1] \tag{9}$$

where

$$\begin{aligned} Q_3 &= H(f)\,\mathrm{sgn}(v_f)Q_1, \\ \sigma_1 &= h_0^2 - 2h_0 + 1, \\ \sigma_2 &= v_0^2 - 2v_0 + 1, \\ \delta_1 &= v_0 - 1, \\ \delta_2 &= R_0C_4, \quad \delta_3 = 2R_0C_4(v_0 - 1), \\ \delta_4 &= R_0(C_4v_0^2 - 2C_4v_0 + C_4 + 1), \\ Q_1 &= q_0\left[H(v_0)\frac{1}{1+\mu_0} + \mathrm{sgn}(v_0)\frac{\mu_0}{1+\mu_0}\right]H(h) \end{aligned} \tag{10}$$

$$C_1 = \varepsilon\overline{C}_1 \ , \ C_2 = \varepsilon\overline{C}_2 \ , \ C_3 = \varepsilon\overline{C}_3 \ , \ \xi_y = \varepsilon v_y \tag{11}$$

$$\begin{aligned} P_1 &= \varepsilon\overline{P}_1 = [-2Q_3C_3h_0(1-h_0) + Q_3C_1\sigma_2 + Q_3C_3\sigma_1 + Q_3C_2v_0^2 - 2Q_3C_2v_0 + Q_3C_2], \\ P_2 &= \varepsilon\overline{P}_2 = [Q_3C_1h_0\sigma_2 + Q_3C_3h_0\sigma_1 + Q_3C_2h_0v_0^2 - 2Q_3C_2h_0v_0 + Q_3C_2h_0] \end{aligned} \tag{12}$$

$$\begin{aligned} &\overline{\lambda}_1 = 2\overline{C}_2Q_3h_0R_0(1-v_0), \quad \overline{\lambda}_2 = \overline{C}_2Q_3h_0R_0^2, \\ &\overline{\lambda}_3 = \overline{C}_3Q_3, \quad \overline{\lambda}_4 = 2\overline{C}_2Q_3R_0(1-v_0), \\ &\overline{\lambda}_5 = \overline{C}_2Q_3R_0^2, \\ &\overline{\lambda}_6 = \overline{C}_3Q_3(3h_0 - 2), \quad \overline{\lambda}_7 = Q_3h_0 \end{aligned} \tag{13}$$

$$\ddot{y} + (\beta + Q_3)y = \varepsilon[-2v_y\sqrt{\beta} + \overline{\lambda}_1]\dot{y} - \varepsilon\overline{P}_1y + \varepsilon\overline{\lambda}_2\dot{y}^2 - \varepsilon\overline{\lambda}_3y^3 - \varepsilon\overline{\lambda}_4y\dot{y} - \varepsilon\overline{\lambda}_5y\dot{y}^2 + \varepsilon\overline{\lambda}_6y^2 + \varepsilon\overline{P}_2 + \varepsilon^0\overline{\lambda}_7 \tag{14}$$

C. Perturbation Analysis

For solving the nonlinear equation the multiple time scales was applied. The principle of this manner is that dependent variables have been expanded in the form of infinitive series by two or more terms of independent variables which called as scales. The first term of these expansions is a linear term and gradually order of these terms would be increased. Also, time derivations of dependent variables define similarity. The independent variable of multiple time (T_j) in terms of real time (t) is defined as $T_j = \varepsilon^j t, j = 0,1,2,...$

So, time derivations are defined as:

$$\begin{aligned} \frac{d}{dt} &= D_0 + \varepsilon D_1 + ... \\ \frac{d^2}{dt^2} &= {D_0}^2 + 2\varepsilon D_0D_1 + ... \end{aligned} \tag{15}$$

Where ($D_j = \frac{\partial}{\partial T_j}$) and one can assume the solution form of y as:

$$y = y_0 + \varepsilon y_1 \tag{16}$$

Ordering and separation the terms lead to perturbation equation of system for y, lead to:

$$O(\varepsilon^0): \qquad {D_0}^2y_0 + (\beta + q)y_0 = \overline{\lambda_7} \tag{17}$$

$$O(\varepsilon^1): \qquad {D_0}^2y_1 + (\beta + q)y_1 = -2D_0D_1y_0 + [-2v_y\sqrt{\beta} + \overline{\lambda}_1]D_0y_0 - \overline{P}_1y_0 + \overline{\lambda}_2(D_0y_0)^2 - \overline{\lambda}_3{y_0}^3 - \overline{\lambda}_4(D_0y_0)y_0 - \overline{\lambda}_5(D_0y_0)^2y_0 + \overline{\lambda}_6{y_0}^2 + \overline{P}_2 \tag{18}$$

III. EVALUATION OF EFFECTIVE PARAMETERS IN STABILITY OF CUTTING PROCESS WITH VARIABLE STIFFNESS

Based on the solution of perturbation method used before, effect of parameters such as depth, length and velocity of cutting and also particular physical coefficient on stability of system with variable stiffness was considered. Numeric data used in all curves include: l=1m, h_0=0.5, Q_3=1, C_1=.3, C_2=0.7, C_3=1.5, C_4=1.2, $R_0 = 2.2, \mu_0 = 0.1, \xi_x = \xi_y = 0.1$ with boundary conditions of $y(0) = 0, \dot{y}(0) = 0$. Note that it was attempted to magnify significant parts of each figures below itself.

In Fig. 3, the (a) parameter is defined as one of particular physical coefficient. As seen in the start cutting tool movement the group of the curves is all the same that shows this parameter has no effect on start of movement. As time is passing, and increasing this parameter the system becomes stable. It is notable that when the (a) parameter approaches to zero the nonlinear equation convert to linear one.

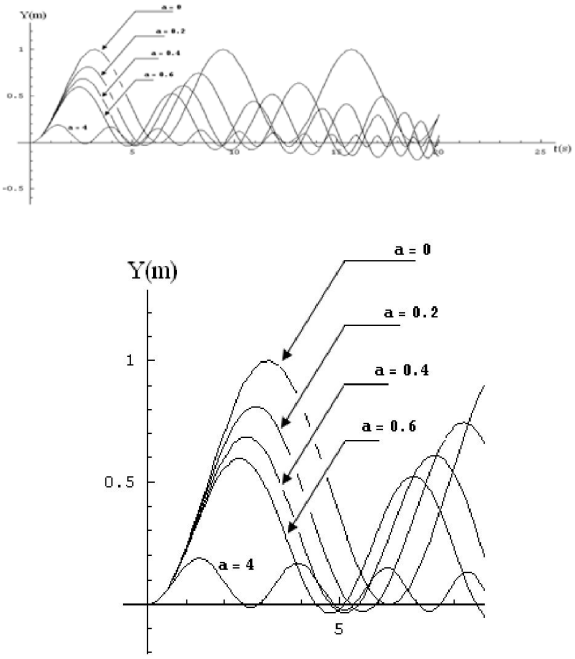

Fig. 3 time response due to particular physical coefficient changes of (a) on vibration amplitude

The effect of cutting depth on the stability was shown in Fig. 4. There are no phases difference with different cutting depths and equality of frequencies for all cutting depths in every semi cycle are visible.

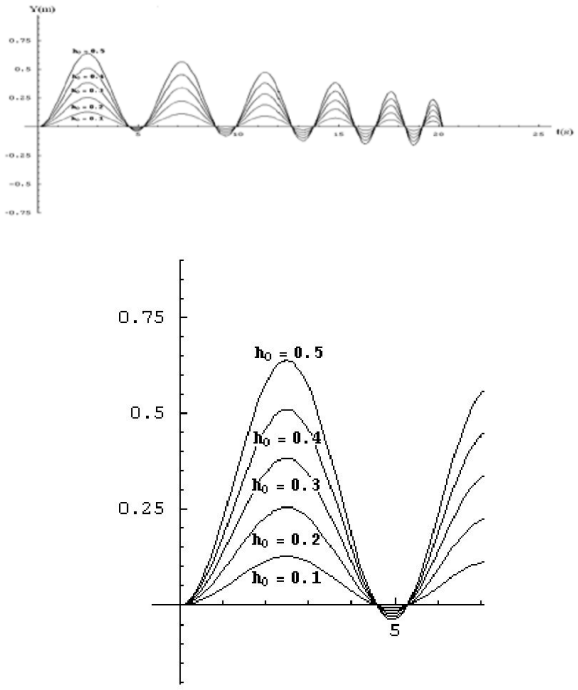

Fig. 4 time response due to cutting depth (h_0) on vibration amplitude

Q_3 is also another particular physical coefficient and effect of this parameter on system stability with great amount of it, is observed in Fig. 5.

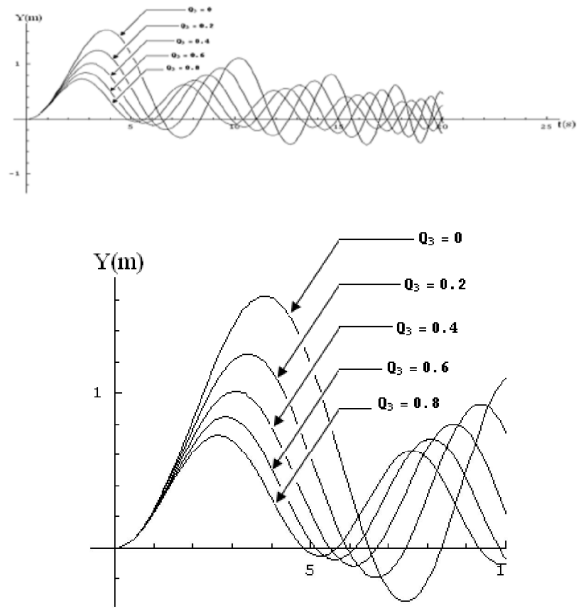

Fig. 5 time response due to particular physical coefficient changes of (Q_3) on vibration amplitude

FIG. 6, shows when the length is changing several amplitudes and frequencies are appeared which due to inherent properties of a continuous system.

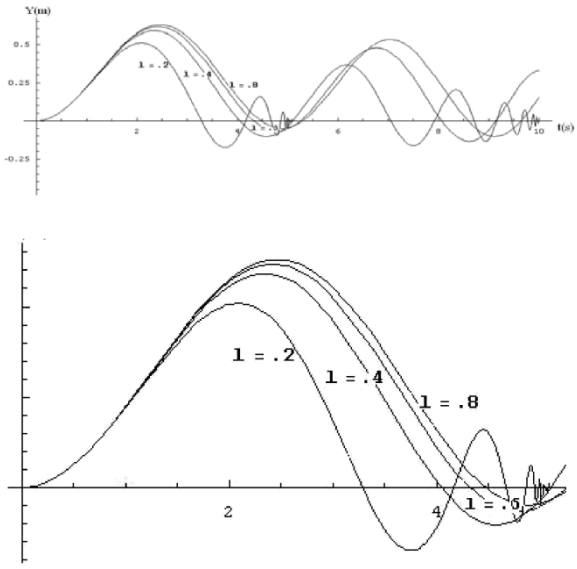

Fig. 6 time response due to cutting tool (chisel) along the work piece and its effect on vibration amplitude

Finally, velocity changes causes changes in vibration rate which shown in Fig. 7. It is also shows that chattering in low feed rate causes instability even with presence of friction and in high speed lead to stability in system.

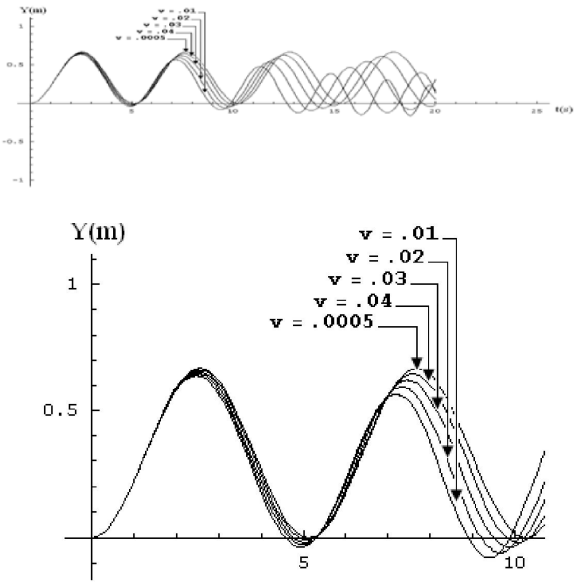

Fig. 7 time response due to cutting velocity changes (V) on vibration amplitude

CONCLUSION

In this research a numeric solution for vertical cutting process with self-exited vibration with time variable stiffness due to chattering was presented and solved by multiple time scales perturbation method. The

following conclusions are made:

1) During the cutting process, generate the sets of frequencies that due to inherent properties of continuous system and also as chisel approach to vice jaws the amplitude of vibrations was reduced.

2) It is observed chattering and vibration of chisel even with presence of friction causes instability in low speed rates.

3) This research shows that results have harmony with expectations and outputs provide suitable and logical responses.

REFERENCES

[1] Opitz H., Bernardi F., *"Investigation and calculation of the chatter behavior of lathes and milling machines "*, Annuals of the CIRP 18, 1970, pp.335-343.

[2] Tlusty J., *"Dynamics of high speed milling "*, ASME, J. Engineering for industry 108, 1986, pp. 59-67.

[3] Tlusty J., Ismail F., *"Basic nonlinearity in machining chatter "*, Annuals of the CIRP 30, 1981, pp.299.

[4] Shi H.M., and Tobias S.A., *"Theory of finite amplitude machine tool instability "*, Int. J. Machine and Tool Design Research, 24, 1984, pp.45.

[5] Lin J.S., Weng C.I., *"A nonlinear dynamic model of cutting "*, Int. J. Machine Tool Manufacture, 30, 1990, pp.53-64.

[6] Lin J.S., Weng C.I., *"Nonlinear dynamic of the cutting process "*, Int. J. Mechanical Sciences, Vol. 33, 1991, pp.645-657.

[7] Gradisk J., Govekar E., Grabec I., *"A chaotic cutting process and determining optimal cutting parameter using neural networks "*, Int. J. Machine Tool manufacture, 36, 1996, pp. 1161-1172.

[8] Nosyreva E.P. and Molinari A., *"Analysis of Nonlinear Vibrations in Metal Cutting "*, Int. J. Mechanical Sciences, Vol. 40, No.8, 1998, pp. 735-748.

[9] Bell R., Burdekin M., *"A study of stick-slip motion of machine tool drives "*, Proceeding of Institution of Mechanical Engineering, 184, 1970, pp. 543-557.

[10] Wiercigroch M., "Modeling of dynamic interaction between machine tool and cutting process (in polish), Ph.D. Thesis, Silesian technical university, Gliwice, Poland, 1989.

[11] Kalmar-Nagy T., Pratt J.R., *"Experimental and analytical investigation of the subcritical instability in metal cutting "*, Proceeding of *DETC* 1997, 17th biennial Conf. on Mechanical Vibrations and Noise, Las Vegas, USA, DETC99/VIB-8060.

[12] Wiercigroch M., Cheng A.H.D., *"Chaotic and stochastic dynamics of orthogonal metal cutting"*, Chaos, Solutions and fractals 8, 1997, pp.715-726.

[13] Warminski J., Litak G., Lipski J., Wiercigroch M., Cartmell M.P., *"Chaotic Vibrations in Regenerative Cutting Process."*, IUTAM / IFTOMM Symposium on Synthesis of Nonlinear Dynamical Systems. Kluwer Academic Publishers 2000.

[14] Lipski J., Litak G., Rusinek R., Szabelski K., Teter A., J.Warmiński J., K.Zaleski K., *"Modeling of nonlinear vibrations in a cutting process considering surface quality of work material"*, 3rd International Symposium "Investigation of Nonlinear Dynamics Effects in Production Systems" Cottbus 2000.

[15] Hwang C.C., Fung R.F., Lin J.S., *"Strong nonlinear dynamics of cutting process "*, J. Sound and Vibration, Vol.203, 1997, pp.363-372.

[16] Nayfeh A.H., Chin C.M., Pratt J., *"Perturbation methods in nonlinear dynamics applications to machining dynamics "*, J. Manufacturing Science and Engineering, Vol.195, 1997, pp.485-493.

[17] Wiercigroch M., *"Chaotic vibrations of a simple model of machine tool-cutting process system "*, Transaction of the ASME, Journal of Vibration and Acoustics, Vol. 119, 1997, pp. 468-475.

[18] Rusinek R., Warmiński J., Szabelski K., Teter A., Litak G., Lipski J., *" Dynamical model of cutting process with stiffness changeable in time. "*,W: Insperger S.G. (ed.) Proceedings of 2-nd Workshop on Dynamics and Control of Mechanical Processing, Budapest, 15.VI.2001. Publishing Company of Technical University of Budapest, Budapest, 2001, p. 47-51.

[19] Grabec I., *"Chaotic generated by the cutting process "*, Physics Letters A 117, 1986, pp. 384-386.

[20] Grabec I., *"Chaotic generated by the cutting process "*, International Journal of Machine Tool Manufacture V28, 1988, pp. 19-31.

[21] Juilian F., Zhejun Y. and Yingxue Y., *"A unified system model of cutting chatter and its transformation function "*, International Journal of Machine Tool Manufacture V29, 1989, pp. 601-609.

[22] Hasting W.F., Mathew P. and Oxley P.L.B., *"A machining theory for predicting chip geometry, cutting forces, etc. from material properties and cutting conditions "*, Proceedings of Royal Society of London, A 371, 1980, pp.343-354.

[23] Warminski J., Litak G., Cartmell M.P., Khanin R. and Wiercigroch M., *"Approximate analytical solutions for primary chatter in the nonlinear metal cutting model "*, J. Sound and Vibration, Vol. 259, No.4, 2003, pp. 917-933.

★★★