FREE VIBRATION ANALYSIS OF AXIALLY FUNCTIONALLY GRADED TAPERED MICRO BEAM, USING MODIFIED STRAIN GRADIENT ELASTICITY

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Abstract- In this paper, free Vibration analysis of axially functionally graded tapered micro beam are investigated in conjunction with strain gradient elasticity theory. It is assumed that cross section and material properties of the non-uniform and non-homogenous micro beam vary continuously in the longitudinal direction. Natural frequencies of the fixed-free micro beam are obtained by means of Rayleigh-Ritz method. The results show the effects of material properties and taper ratios on natural frequencies of axially functionally graded tapered micro beams and illustrate the effects of considering length scales parameters by comparing the results with those determined by classical theories. To validate and demonstrate accuracy of the present analysis, some results are compared with the available results from the existing literature.

Keywords- Vibration, Functionally Graded, Tapered Micro Beam, Strain Gradient Theory.

I. INTRODUCTION

Functionally graded materials (FGMs) are a new category of inhomogeneous composite materials which are produced by varying the material of microstructure from one to another, with a specified gradient, hence material properties vary continuously along one or more direction. This enables the new material to have the best of both materials without undesirable stress concentration to occur that may cause in laminated composites. Due to these mentioned features, FGMs are more advantageous than the conventional laminated materials and have been widely used in different engineering applications such as electronics, computer circuit industry, optics, biomedicine, aerospace engineering, military, semiconductor industry, and general structural elements in thermal environments.

Recently, understanding static and dynamic behavior of FGM becomes increasing important. Reviewing the literature on beams that are made of FG materials are indicating that most of the research works on FG beams with varying material properties throughout thickness [1] and very few studies exist on axially functionally graded (AFG) beams that variations of material are considered along beam axis. Since the governing Equations of AFG beams have variable coefficients, solving these equations becomes complicated.

Elements of structures are not always fabricated with uniform cross-section. Usually, non-uniform with variable cross-section type of them are used in different engineering applications, owing to these elements are make lighter, economical and optimal. Also they are utilized in special purpose, i.e., micro- and nano-size sensor and actuators [2], non-uniform composite beam-like structures [3], carbon nanotubes [4], turbines and helicopter blades, and etc. Recently, many studies have been carried out on the vibration behavior of non-uniform beams [5]. Shahba et al [6] and Alshorbagy et al [7] studied vibration characteristics of AFG tapered beams by employing a finite element method. Shahba and Rajasekaran [8] investigated free vibration and stability of AFG tapered Euler-Bernouli beams using numerical method. Free vibration behavior of AFG beams with non-uniform cross section are presented by Huang et al [9]. With the advance of science and technology, micro and nano structures in the order of microns and sub-microns have found applications in biosensors, atomic force microscopes, micro actuators, nano probes, shape memory alloys as thin films and also in micro- and nano-electromechanical systems (MEMS and NEMS). Recently, micro scale structures made of FGM materials developed in a wide range applications [10].

The torsion and bending behaviors of the micro-scale structures are experimentally investigated [11], and seen that the classical elasticity theory is not adequately capable to explain and estimate the size dependent behavior of these structures. In contrast, non-classical elasticity theories (higher-order continuum theories) such as non-local theory [12], couple stress theory [13] and strain gradient theory [14] becomes acceptable to prediction and explanation of size-dependent behaviors. During past years, Yang et al [15] made some development on couple stress theory and considered only one additional material length scale parameter in addition to classical material constants, and introduce the modified couple stress theory. Lam et al [16] proposed a modified strain gradient theory by considering three additional material length scale parameters. Many studies on size-dependent vibration behavior correlate with non-classical elasticity theories have been carried out during the past years [17].

The researches dedicated to vibration analysis of non-uniform cross section of small-scale structures have been limited. Vibration analysis of a non-uniform nano cantilever is studied by Murmu et al [2]. Rafiei et al [4] and Mustapha et al [18] investigated vibration behaviors of non-uniform carbon nano tubes.

In this study, free vibrations of an axially functionally graded tapered Euler-Bernoulli micro-beam have been investigated based on the modified strain gradient theory. The cross section and materials properties are supposed varied along the beam axis direction and approximate natural frequencies of free transverse vibration of cantilever tapered AFG micro-beam are obtained by Rayleigh-Ritz method. The results are investigated the effects of material properties and tapered ratio on natural frequencies and illustrated considering of length scale parameters had a major impact on results rather than classical and modified couple stress theories. For validate and demonstrate accuracy of present study, some results are compared with some previous literature and shows a good in agreement between them.

II. FORMULATION

According to modified strain gradient theory [16], stored strain gradient energy of a linear elastic material that occupy a region V, and have infinitesimal deformations, can be written as

$$U = \int_V \bar{u}\, dv = \frac{1}{2}\int_V (\sigma_{ij}\,\varepsilon_{ij} + p_i\,\gamma_i + \tau^{(1)}_{ijk}\,\eta^{(1)}_{ijk} + m^s_{ij}\,\chi^s_{ij})\, dV \quad (1)$$

In which σ_{ij}, ε_{ij}, γ_i, $\eta^{(1)}_{ijk}$, χ^s_{ij} and θ_i, denote the components of Cauchy stress tensor σ, the classical strain tensor ε, the dilatation gradient vector γ, the deviatoric stretch gradient tensor $\eta^{(1)}$, the symmetric part of the rotation gradient tensor χ^s, and the infinitesimal rotation vector θ, respectively, and can be introduced for small deformation as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (2)$$

$$\gamma_i = \varepsilon_{mm,i} \quad (3)$$

$$\eta^{(1)}_{ijk} = \frac{1}{3}(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}) - \frac{1}{15}\delta_{ij}(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}) - \frac{1}{15}[\delta_{jk}(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}) + \delta_{ki}(\varepsilon_{mm,j} + 2\varepsilon_{mj,m})], \quad (4)$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \quad (5)$$

$$\theta_i = \frac{1}{2}(curl(u))_i = \frac{1}{2}e_{ijk}u_{k,j} \quad (6)$$

Where u_i are the Cartesian components of the displacement vector, and following a comma denote the partial derivatives with respect to the space coordinates. According to the constitutive equations for a linear isotropic elastic material, the classical stress parameter σ, and higher-order stresses parameters p, $\tau^{(1)}$, and m^s can be obtained by differentiating the strain energy density $\bar{u}$with respect to kinematics parameters ε, γ, $\eta^{(1)}$ and χ^s, respectively, as follows [33]

$$\sigma_{ij} = \lambda tr(\varepsilon)\delta_{ij} + 2\mu\varepsilon_{ij} \quad (7)$$

$$p_i = 2\mu l_0^2\gamma_i \quad (8)$$

$$\tau^{(1)}_{ijk} = 2\mu l_1^2\eta_{ijk} \quad (9)$$

$$m^s_{ij} = 2\mu l_2^2\chi^s_{ij} \quad (10)$$

In which,λ and μ are lame constants, called the bulk and shear modules, respectively, and can be represented in terms of the Young modulus E and Poisson's ratio νas

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)},\ \mu = \frac{E}{2(1+\nu)} \quad (11)$$

Also, l_0, l_1 and l_2 are additional independent material length scale parameters, related to dilatation gradients, deviatoric stretch gradients, and rotation gradients, respectively. In order to determine these parameters for a specific material, some typical experiments such as micro-bend test, micro torsion test and specially micro/nano indentation test can be carried out [20-22].For specific case, when ($l_0 = l_1 = 0$), the strain energy U which introduced in Eq. (1), will be reduced to the strain energy that are defined in the modified couple stress theory and also the strain energy U conjugate to the classical theory can be obtained by vanishing all three length scale parameters ($l_0 = l_1 = l_2 = 0$) in Eq. (1).

III. GOVERNING EQUATION AND VIBRATION ANALYSIS

In this section, governing equations of an AFG tapered Euler-Bernoulli micro beam have been developed and then vibration behavior is investigated. The components of displacement field$u$ of an Euler-Bernoulli beam can be expressed as

$$u_1 = -z\frac{\partial w(x,t)}{\partial x} \quad (12)$$

$$u_2 = 0 \quad (13)$$

$$u_3 = w(x,t) \quad (14)$$

Whereu_1, u_2 and u_3 indicate displacement along x, $y$and z direction, respectively. By substituting Eqs. (12)-(14) into Eq. (2), the components of strain tensor ε can be written as

$$\varepsilon_{11} = -z\frac{\partial^2 w(x,t)}{\partial x^2} \quad (15)$$

$$\varepsilon_{22} = \varepsilon_{33} = \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0 \quad (16)$$

The components of dilatation gradient vector γ, and non-zero components of the deviatoric stretch gradient tensor $\eta^{(1)}$ are obtained by substitution Eqs. (15) and (16) into (3) and (4) respectively.

$$\gamma_1 = -z\frac{\partial^3 w(x,t)}{\partial x^3}, \quad \gamma_2 = 0, \quad \gamma_3 = -\frac{\partial^2 w(x,t)}{\partial x^2} \quad (17)$$

$$\eta^{(1)}_{111} = -\frac{2}{5}z\frac{\partial^3 w(x,t)}{\partial x^3}$$
$$\eta^{(1)}_{122} = \eta^{(1)}_{331} = \eta^{(1)}_{313} = \eta^{(1)}_{133} = \eta^{(1)}_{221} = \eta^{(1)}_{212} = \frac{1}{5}z\frac{\partial^3 w(x,t)}{\partial x^3}$$
$$\eta^{(1)}_{232} = \eta^{(1)}_{223} = \eta^{(1)}_{322} = \frac{1}{15}\frac{\partial^2 w(x,t)}{\partial x^2}$$
$$\eta^{(1)}_{113} = \eta^{(1)}_{131} = \eta^{(1)}_{311} = -\frac{4}{15}\frac{\partial^2 w(x,t)}{\partial x^2}$$
$$\eta^{(1)}_{333} = \frac{1}{5}\frac{\partial^2 w(x,t)}{\partial x^2} \quad (18)$$

Moreover, by considering Eq. (6), the components of rotation vector θ are obtained as

$$\theta_1 = 0, \quad \theta_2 = -\frac{\partial(w(x,t))}{\partial x}, \quad \theta_3 = 0 \quad (19)$$

According to following expression and Eq. (5), the non-zero components of the symmetric part of the rotation gradient tensor χ^s are determined as:

$$\chi^s_{12} = \chi^s_{21} = -\frac{1}{2}\frac{\partial^2 w(x,t)}{\partial x^2} \quad (20)$$

For slender beam with a large aspect ratio, the Poisson effect is minor, and may be neglected to simplify the governing equations.

Therefore, using the obtained components of ε, $\eta^{(1)}$ and m^s,one can obtain the non-zero components of stress tensor$\sigma$and higher-order stress tensors, $\tau^{(1)}$ and m^s introduced in Eq. (7), (9) and (10) as

$$\sigma_{11} = -zE(x)\frac{\partial^2 w(x,t)}{\partial x^2} \quad (21)$$

$$p_1 = -2\mu(x)l_0^2(x)z\frac{\partial^3 w(x,t)}{\partial x^3}$$
$$p_2 = 0, \quad p_3 = -2\mu(x)l_0^2(x)\frac{\partial^2 w(x,t)}{\partial x^2} \quad (22)$$

$$\tau^{(1)}_{111} = -\frac{4}{5}\mu(x)l_1^2(x)z\frac{\partial^3 w(x,t)}{\partial x^3}$$
$$\tau^{(1)}_{122} = \tau^{(1)}_{331} = \tau^{(1)}_{313} = \tau^{(1)}_{133} = \tau^{(1)}_{221} = \tau^{(1)}_{212} = \frac{2}{5}\mu(x)l_1^2(x)z\frac{\partial^3 w(x,t)}{\partial x^3}$$
$$\tau^{(1)}_{232} = \tau^{(1)}_{223} = \tau^{(1)}_{322} = \frac{2}{15}\mu(x)l_1^2(x)\frac{\partial^2 w(x,t)}{\partial x^2}$$
$$\tau^{(1)}_{113} = \tau^{(1)}_{131} = \tau^{(1)}_{311} = -\frac{8}{15}\mu(x)l_1^2(x)\frac{\partial^2 w(x,t)}{\partial x^2}$$
$$\tau^{(1)}_{333} = \frac{2}{5}\mu(x)l_1^2(x)\frac{\partial^2 w(x,t)}{\partial x^2} \quad (23)$$

This equations are derived by assumption that both material and geometrical properties and also the length scale parameters l_0, l_1 and l_2 vary along x direction and constant through thickness. Variations are linear through longitudinal direction and can be defined typically as

$$F(x) = F_L\left(1 - f\frac{x}{L}\right) \quad (24)$$

In which

$$f = 1 - \frac{F_L}{F_R} \quad (25)$$

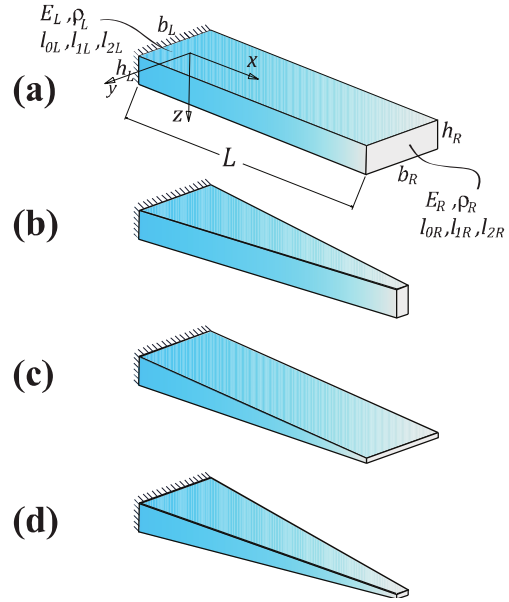


Figure 1- Different taper ratios of an AFG micro beam (a) $f_h = f_b = 0$, (b) $f_h = 0$, $f_b \neq 0$, (c) $f_h \neq 0$, $f_b = 0$ and (d) $f_h \neq 0$, $f_b \neq 0$.

Following to Fig. 1, the subscripts R and Lin equation (24) indicates the right and the left ends of the micro beam.

Therefore, the effective material properties (Young's modulus (E), shear modulus (μ), mass density (ρ) and material length scale parameters (l_0, l_1 and l_2))and also the variation of geometric properties (height (h) and weight (b)of cross-section of microbeam)can be expressed as

$$E(x) = E_L(1 - f_E\frac{x}{L})$$
$$\mu(x) = \mu_L((1 - f_\mu\frac{x}{L})$$
$$\rho(x) = \rho_L(1 - f_\rho\frac{x}{L})$$
$$h(x) = h_L(1 - f_h\frac{x}{L})$$
$$b(x) = b_L(1 - f_b\frac{x}{L})$$
$$l_0(x) = l_{0L}(1 - f_{l_0}\frac{x}{L})$$
$$l_1(x) = l_{1L}(1 - f_{l_1}\frac{x}{L})$$
$$l_2(x) = l_{2L}(1 - f_{l_2}\frac{x}{L})$$

where

$$f_E = 1 - (E_R/E_L)$$
$$f_\mu = 1 - (\mu_R/\mu_L)$$
$$f_\rho = 1 - (\rho_R/\rho_L)$$
$$f_h = 1 - (h_R/h_L)$$
$$f_b = 1 - (b_R/b_L)$$
$$f_{l_0} = 1 - (l_{0R}/l_{0L})$$
$$f_{l_1} = 1 - (l_{1R}/l_{1L})$$
$$f_{l_2} = 1 - (l_{2R}/l_{2L}) \quad (28)$$

The cross sectional area A and the second moment of cross-sectional area I of AFG microbeam are written as

$$A(x) = A_L(1 - f_b\frac{x}{L})(1 - f_h\frac{x}{L})$$
$$I(x) = I_L\left(1 - f_b\frac{x}{L}\right)\left(1 - f_h\frac{x}{L}\right)^3 \quad (29)$$

The strain energy U which expressed in Eq. (1) for the linearly tapered AFG microbeam can be rewritten as following

$$U = \frac{1}{2}\int_0^L \left\{(EI)_{eq}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 + 2(a_0' I)_{eq}\left(\frac{\partial^3 w(x,t)}{\partial x^3}\right)^2 + 2(a_0' A)_{eq}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 + \frac{4}{5}(a_1' I)_{eq}\left(\frac{\partial^3 w(x,t)}{\partial x^3}\right)^2 + \frac{8}{15}(a_1' A)_{eq}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 + (a_2' A)_{eq}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2\right\} dx \quad (30)$$

where

$$(EI)_{eq} = E(x)I(x)$$
$$(a_0' I)_{eq} = l_0^2(x)\mu(x)I(x) \quad (31)$$
$$(a_0' A)_{eq} = l_0^2(x)\mu(x)A(x)$$
$$(a_1' I)_{eq} = l_1^2(x)\mu(x)I(x)$$
$$(a_1' A)_{eq} = l_1^2(x)\mu(x)A(x)$$
$$(a_2' A)_{eq} = l_2^2(x)\mu(x)A(x)$$

It can be easily seen that strain energy U for modified couple stress theory which reported by Akgöz et al [39] are obtained by vanishing l_0 and l_1.

Neglecting rotating inertia, the kinetic energy for the tapered AFG micro beam is expressed as

$$T = \frac{1}{2}\int_0^L \rho(x)A(x)\left(\frac{\partial w(x,t)}{\partial t}\right)^2 dx \quad (32)$$

Assuming harmonic vibration, the auxiliary function $w(x,t)$ can be introduced as

$$w(x,t) = W(x)\cos\omega t \quad (33)$$

Where $W(x)$ and ω are the eigenfunction and circular frequency of free vibration, respectively. Substituting from Eq. (33) into Eqs. (30) and (32), the maximum potential and kinematic energy are given as

$$U_{max} = \frac{1}{2}\int_0^L \left\{\left[(EI)_{eq} + 2(a_0' A)_{eq} + \frac{8}{15}(a_1' A)_{eq} + (a_2' A)_{eq}\right]\left(\frac{d^2 W(x)}{dx^2}\right)^2 + \left[2(a_0' I)_{eq} + \frac{4}{5}(a_1' I)_{eq}\right]\left(\frac{d^3 W(x)}{dx^3}\right)^2\right\} dx \quad (34)$$

$$T_{max} = \frac{1}{2}\omega^2\int_0^l \rho(x)A(x)(W(x))^2 dx \quad (35)$$

The Rayleigh-Ritz Method, are used to determine an approximate solution. The expansion of the mode shape in terms of n linearly independent admissible functions $\phi_i(x)$, $i = 1, 2, \dots, n$ are considered in the form

$$W(x) = \sum_{i=1}^{n} c_i\phi_i(x) \quad (36)$$

Where c_i are unknown constant coefficients which must be chosen suitably to minimize the Rayleigh's quotient which are written as

$$R[W(x)] = \omega^2 = \frac{U_{max}}{T_{max}} \quad (37)$$

The geometric boundary conditions for a cantilever beam as shown in Fig. 1, are given as

$$w(0,t) = 0, \quad \frac{\partial w(x,t)}{\partial x}|_{x=0} = 0 \quad (38)$$

The admissible function ϕ_i is chosen depending on the above geometric boundary conditions which be constructed using polynomials function as following

$$\phi_i(x) = \left(\frac{x}{L}\right)^{i+1}, \quad i = 1,2,3,\dots,10 \quad (39)$$

The coefficients $c_1, c_2, \dots, c_{10}$, can be obtained by differentiating Eq. (37) with respect to the each of the coefficients to minimize Rayleigh's quotient.

$$\frac{\partial(R)}{\partial c_i} = 0, \qquad i = 1,2,3,\dots,10 \tag{40}$$

Eq. (40) denotes a system of ten algebraic linear homogenous equations with constant $c_1, c_2, \dots, c_{10}$ which the determinant of the coefficient must be equal to zero to have a nontrivial solution. This yields characteristic equation that gives ten natural frequencies.

IV. RESULT

In this section, effects of small-scale and also section and material properties ratios, on vibration response of a fixed-free AFG tapered micro beam, are illustrated by numerical results. To easily study, all three length scale parameters are considered equal ($l_0 = l_1 = l_2 = l$). The numerical results for different cases demonstrated in Fig. 1, which conclude linear variation of height and/or width of cross section and material properties including elasticity modulus, mass density and length scale parameters of micro beam are obtained. Length of micro beam is assumed 20 times larger than height of cross section at fixed end ($L = 20h_L$). The rate of width to height of cross section of fixed end is considered to be ($b_L/h_L = 2$).To simplify the notation, the abbreviations for the modified strain gradient, the modified couple stress theory and the classical theory are introduced by MSGT, MCST and CT, respectively.

Fig. 2-3 show respectively the first three non-dimensional natural frequencies of AFG double tapered micro beam predicted by CT and MSGT for different material and tapered ratios.

Fig. 2 shows contrary effects on non-dimensional fundamental frequencies by tapered and material properties ratios for both CT and MSGT models.

As shown in Fig. 3, the second non-dimensional natural frequencies increase as material properties ratio increase for both CT and MSGT. Also, it is noted that an increment in taper ratio, the second non-dimensional natural frequency values decreases for CT, in contrast with MSGT that has an increasing effect on the second and third non-dimensional natural frequency values.

Fig. 5 shows the effects of the dimensionless length scale parameters (l/h_L) on the non-dimensional natural frequencies of AFG double tapered microbeam with different elasticity theories. It is evident that the first and second non-dimensional natural frequencies are predicted by MSGT are larger than those determined by MCST and CT. Also, it can be seen, a decrease in l/h_L leads to decrease of non-dimensional natural frequencies and it can be inferred that those predicted by MSGT, MCST and CT become closed to each other for small value of l/h_L as $l/h_L < 0.3$.

CONCLUSION

In this paper, free vibration analysis of cantilever tapered AFG micro beams based on modified strain gradient and Euler-Bernoulli beam theories for various tapered and material properties ratios was presented. Rayleigh-Ritz Method is utilized to show the effects of considering material length scale parameters, tapered ratio and material gradient on natural frequencies. The results of the case studies can be outlined as

- The dimensionless natural frequencies are obtained by the modified strain gradient theories (MSGT) are always larger than those determined by the modified couple stress theory (MCST) and the classical theory (CT). It can be concluded the MSGT predict the tapered AFG micro beams stiffer than does predicted by MCST and CT.

- The dimensionless natural frequencies are obtained by MSGT are closed to those determined by MCST and CT for small dimensionless length scale parameters.

Tapered and material properties ratios have considerable influence on the dimensionless natural frequencies of AFG micro beams.

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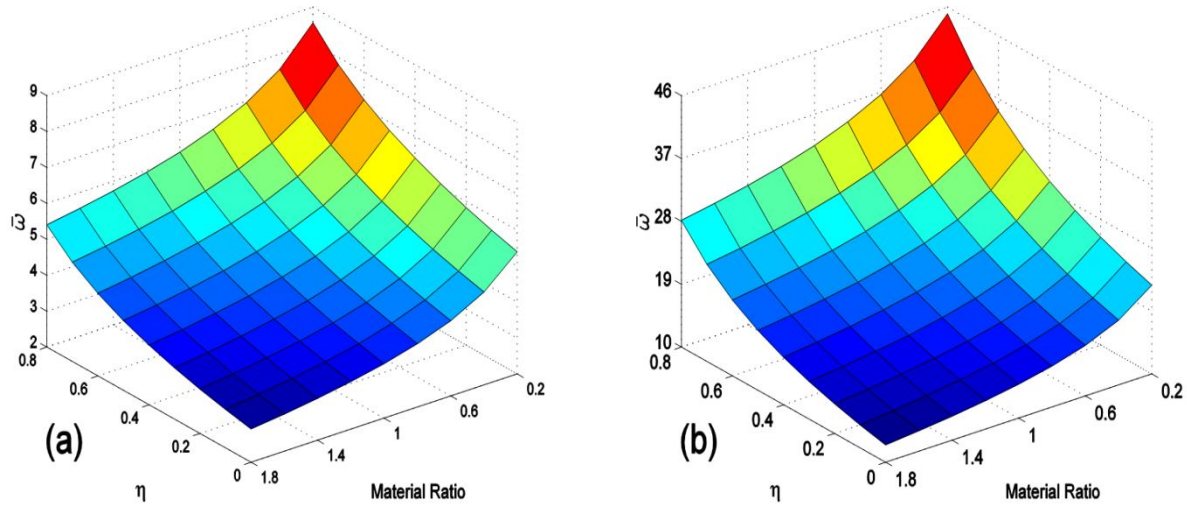


Fig 2. *Variation of non-dimesional fundamental frequencies of AFG double tapered micro beam for different values of material and taper ratio($f_h = f_b = \eta$), (a)CT, (b) MSGT*

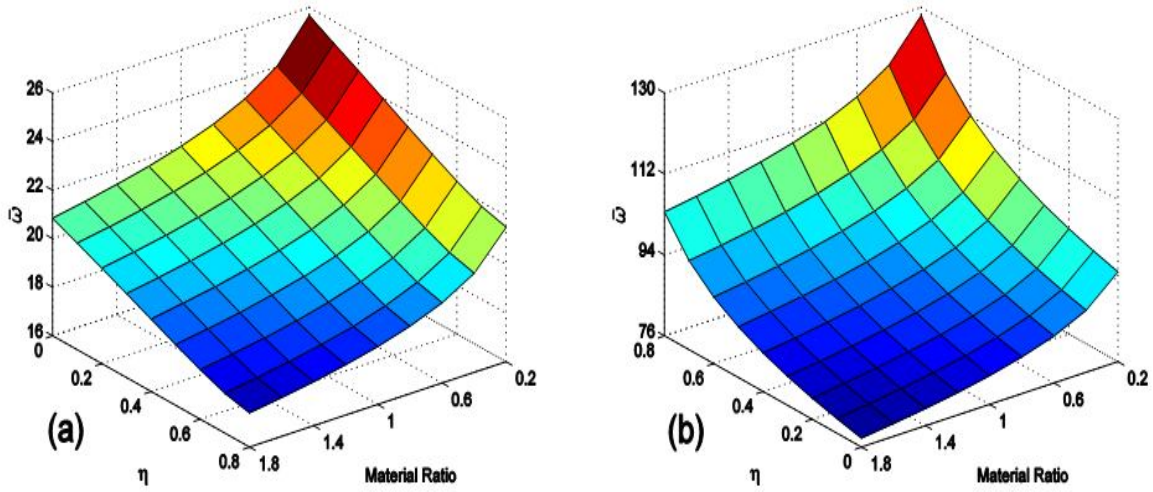


Fig 3. *Variation of non-dimensional frequencies of AFG double tapered micro beam for different values of material and taper ratio(second Mode)($f_h = f_b = \eta$), (a)CT, (b) MSGT*

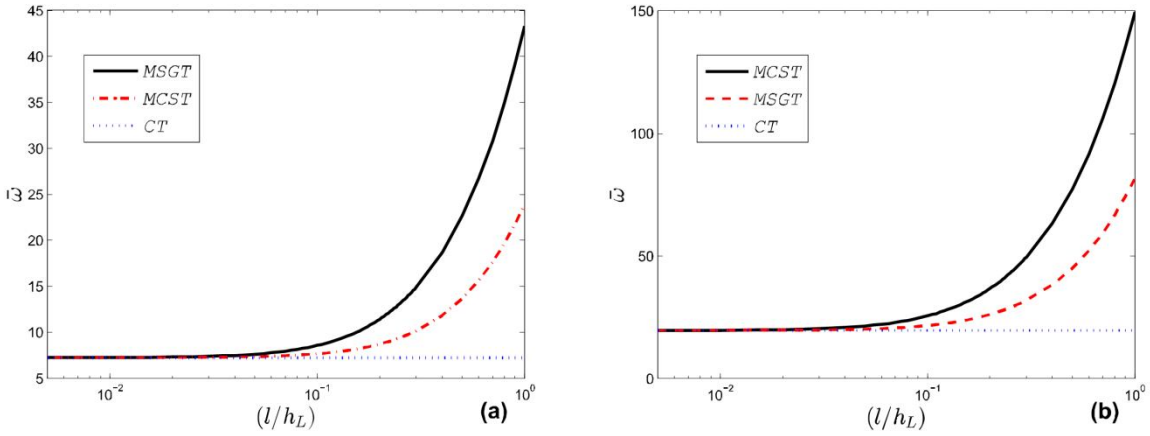


Fig 5. *Variation of non-dimensional frequencies of AFG double tapered micro beam for different values of dimensionless length scale parameters* ($f_h = f_b = 0.8, m_R/m_L = 0.5,\ l_R = l_L = l$), *(a)First mode, (b) Second mode.*

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