PRICING QUANTO OPTIONS WITHIN THE MARKOV-MODULATED, CROSS-CURRENCY MARKET MODEL

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Abstract - In this study, we investigate the fair valuation of quanto options under stochastic interest rates when the foreign equity price dynamics and the foreign exchange rate dynamics are governed by a Markov-modulated, cross-currency market model with both individual jumps and concurrent jumps. In addition, the random Esscher transform technique is developed to determine the pricing kernel in an incomplete market economy. Finally, the explicit analytical formula of European-style quanto options is given.

Keywords - Quanto, Foreign Equity Price, Foreign Exchange Rate, Markov-Modulated, Cross-Currency Market Model, Random Esscher Transform.

I. INTRODUCTION

A quanto option is a kind of cash settled derivatives that has an underlier denominated in foreign currency, but settles in domestic currency at a fixed exchange rate. It is constructed the same as a regular option, the only difference is it is purchased in its own domestic currency, although denominated in a foreign currency. At initiation, the investor fixes an exchange rate, which is then kept for the duration of contract. The quanto options have been created for hedging the fluctuating risk in the cross-currency market. Among which, a necessary step is to set up a dynamic model for the cross-currency fluctuations, and then the pricing of the quanto options might be given explicitly. Reference [8] prices the foreign equity options when both the foreign equity price process and the foreign exchange rate process are driven by multi-dimensional Lévy dynamics. Reference [13] applies the Gram–Charlier series expansion for incorporating the influence of skewness and kurtosis on foreign equity option prices. Reference [14] sets up the stochastic volatility model with double jumps for pricing foreign equity options. Reference [6] investigates the valuation of two types of foreign equity options when the underlying asset price dynamics are governed by a regime-switching mean-reversion log-normal model. In line with these aforementioned works in cross-currency markets, we thus dive into the joint dynamic modeling problem.

In asset price modeling, concurrent jumps refer to the occurrence of contemporaneous discontinuities in the price of more than one asset. The so-called concurrent jump phenomenon may be associated with some economically meaningful events, such as macroeconomic news announcements, as in [2]–[3], and [7]. In this study, we characterize the jump intensities of both the individual jump and the concurrent jump by two independent compound Poisson processes with allowing for simultaneous jump activities in two asset price dynamics. That is, if a rare event corresponds to an individual jump of the i th asset price, then only the i th asset price will jump; otherwise, if a rare event corresponds to a concurrent jump, then both asset prices will jump with a positive probability. Due to the structural changes in business cycles, the time-inhomogeneity should be allowed in the dynamic models, as in [4]–[5]. Reference [11] studies the valuation of currency options within a two-factor stochastic volatility model. Reference [1] prices the currency options under a Markov-modulated jump-diffusion model where the time-varying transition rates are modulated by a hidden Markov chain. Reference [12] investigates the pricing of foreign exchange options under Markov-modulated Lévy dynamics. In this study, we evaluate the European-style quanto option prices under a Markov-modulated, cross-currency market model by incorporating both the concurrent jumps and the time-inhomogeneity. In particular, the proposed model parameters are all modulated by a continuous-time, finite-state Markov chain, and whose state can be interpreted as the hidden states of an economy. Since the security economy described by this dynamic model is incomplete, we employ the random Esscher transform to solve such an option pricing problem within our framework, as in [9]–[10]. As a consequence, the closed-form solutions for the European-style quanto options are formally obtained.

The remainder of this study is organized as follows. The next section shows the dynamic model. Section 3 presents the change of measures and the valuation of European-style quanto options. The final section concludes this study.

II.MARKOV-MODULATED,CROSS-URRENCY MARKET MODEL

Let (Ω, F, P) be a complete probability space, where P is the physical probability measure. We define a continuous-time, finite-state Markov chain $x(t)$ on (Ω, F, P) with state space $\acute{A} = \{1, 2, K, I\}$

for all $t \in [0,T]$. The states of $x(t)$ can be interpreted as the hidden states of an economy. Without loss of generality, we can take the state space Á for $x(t)$ to be a finite set of unit vectors $(e_1, e_2, \mathrm{K}, e_I)$ with $e_j = (0, \mathrm{K}, 1, \mathrm{K}, 0)$Î ¡^{I}. Review the Reference [4], in which the semi-martingale representation for $x(t)$ is given by:

$$dx(t) = \mathrm{P}\, x(t)dt + dM(t), \qquad (1)$$

where $M(t)$ is an ¡^{I}-valued martingale with respect to the natural filtration generated by $x(t)$ under P. Let $\Pi = (\pi(i,j))_{i,j=1,2,\ldots,I}$ denote the generator or rate matrix of $x(t)$ under P, where $\pi(i,j)$ is the transition intensity of $x(t)$ from state e_i to state e_j. Here, we let $B_v(t) = \exp\left(\int_0^t r_v(s)ds\right)$ denote the vth country's money market account at time t with initial value $B_v(0) = 1$ for $v \in \{d, f\}$. The domestic and foreign risk-free instantaneous interest rates are, respectively, defined by $r_d(t)$ and $r_f(t)$ through the following equation:

$$r_v(t) = \langle r_v, x(t) \rangle, \qquad (2)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in ¡^{I}. For all $t \in [0,T]$, we assume that the appreciation rate $a_k(t)$ and the stochastic volatility $s_k(t)$ of these two asset price dynamics and the stochastic jump intensity $l_k(t)$ of the Poisson process $N_k(t)$ are all modulated by a continuous-time, finite-state Markov chain $x(t)$ as follows:

$$a_k(t) = \langle a_k, x(t) \rangle, \qquad (3)$$

$$s_k(t) = \langle s_k, x(t) \rangle, \qquad (4)$$

and

$$l_k(t) = \langle l_k, x(t) \rangle, \qquad (5)$$

for k Î $\{Sf, X\}$.

Let $S_f(t)$ and $X(t)$ represent the foreign equity price at time t and the domestic price at time t of one unit of the foreign currency, respectively. We now specify the Markov-modulated, cross-currency market model for the dynamics of the foreign equity price and the foreign exchange rate as follows:

$$\frac{dS_f(t)}{S_f(t-)} = (a_{Sf}(t) - l_{Sf}(t)t_{Sf} - l_c(t)t_{Sf})dt + s_{Sf}(t)dB_{Sf}(t) + (\exp(Z_{Sf}(t-)) - 1)dN_{Sf}(t), \qquad (6)$$

and

$$\frac{dX(t)}{X(t-)} = (a_X(t) - l_X(t)t_X - l_c(t)t_X)dt + s_X(t)dB_X(t) + (\exp(Z_X(t-)) - 1)dN_X(t), \qquad (7)$$

where the correlated Brownian motions of these two underlying asset price dynamics are defined by $dB_X(t) = \rho(t)dB_{Sf}(t) + \sqrt{1-\rho^2(t)}dB_G(t)$ for $B_{Sf}(t) \perp B_G(t)$ and that the instantaneous correlation coefficient at time t is given by $r(t) = \langle r, x(t) \rangle$ with $-1 < r(j) < 1$ for $j = 1, 2, \mathrm{K}, I$. Under such dynamics, we suppose that the two jump terms $N_{Sf}(t)$ and $N_X(t)$ are partially correlated, meaning that if the foreign equity price jumps then the foreign exchange rate will jump with a positive probability p. We construct them in such a manner using three independent Poisson processes denoted by $n_{Sf}(t)$, $n_X(t)$, and $n_c(t)$. For $k \in \{Sf, X\}$, the independent Poisson process $n_k(t)$ has the Markov-modulated arrival intensity denoted by $l_k(t)$ with the discrete probability density function, which is given by the following:

$$P(n_k(t) = l \,|\, x(t)) = \frac{\left(\int_0^t l_k(s)ds\right)^l}{l!} \exp\left(-\int_0^t l_k(s)ds\right), \; P\text{-a.s.} \qquad (8)$$

we define the random variable $N_k(t) = n_k(t) + n_c(t)$ with $N_k(t) \sim Poisson(\lambda_k(t) + \lambda_c(t))$. In particular, there are two types of jumps, individual jumps only for the k th asset with the stochastic arrival intensity $l_k(t)$ and concurrent jumps with the arrival rate $l_c(t)$, for both assets. A change of variables and integrating out $n_c(t)$ results in the joint probability density function for $N_{Sf}(t)$ and $N_X(t)$, which is given as follows:

$$P(N_{Sf}(t) = m, N_X(t) = n \,|\, x(t)) = \sum_{q=0}^{\min(m,n)} \exp\left(-\int_0^t (l_{sf}(s) + l_X(s) + l_c(s))ds\right) \times \frac{\left(\int_0^t l_{Sf}(s)ds\right)^{m-q} \left(\int_0^t l_X(s)ds\right)^{n-q} \left(\int_0^t l_c(s)ds\right)^q}{(m-q)!(n-q)!q!}, \; P\text{-a.s.} \qquad (9)$$

In addition, if a rare event occurs at time t, the random variable $Z_k(t)$, controlling the jump size, is assumed to be normally distributed with mean u_k and

variance δ_k^2. As a consequence, the mean percentage jump size of the k th asset price is $t_k = E\left[\exp(Z_k(t)) - 1\right] = \exp\left(u_k + \frac{1}{2}d_k^2\right) - 1 = f_{Z_k}(1) - 1$. In Eqs. (6) and (7), we also assume that for $t \neq s$, $Z_k(t)$ and $Z_k(s)$ are independent, and all of the randomness $B_k(t)$, $N_k(t)$, $x(t)$, and $Z_k(t)$ are mutually independent.

III. MEASURE CHANGE AND OPTION VALUATION

3.1. Risk-Neutral Esscher Measure

To identify an equivalent martingale probability measure for the dynamic model, we decompose the Markov-modulated, cross-currency logarithmic return $R_k(t) = \log\left(k(t)/k(0)\right) = C_k(t) + J_k(t) + J_c(t)$ into a continuous diffusion component $C_k(t) = \int_0^t \left(a_k(s) - \frac{1}{2}s_k^2(s) - l_k(s)t_k - l_c(s)t_k\right)ds + \int_0^t s_k(s)dB_k(s)$, an individual jump component $J_k(t) = \int_0^t Z_k(s-)dn_k(s)$, and a concurrent jump component $J_c(t) = \int_0^t Z_k(s-)dn_c(s)$ for $k \in \{Sf, X\}$ and all $t \in [0,T]$. Here, let $\{F_t^{R_k}\}_{t\in[0,T]}$ and $\{F_t^x\}_{t\in[0,T]}$ be the right-continuous, P-complete, natural filtrations generated by stochastic processes $\{R_k(t)\}_{t\in[0,T]}$ and $\{x(t)\}_{t\in[0,T]}$, respectively. In addition, we define $F_t = F_t^{R_k} \cup F_t^x$ as the enlarged σ-algebra for each $t \in [0,T]$. We assume that trading takes place continuously in time over an interval $[0,T]$, and the uncertainty is described by a filtered probability space $\left(\Omega, F, P, \{F_t\}_{t\in[0,T]}\right)$. The Radon-Nikodym derivative of the Markov-modulated Esscher transform is then formally given by the following:

$$h^{q^h}(t) = \left.\frac{dQ^{q^h}}{dP}\right|_{F_t} = \frac{\exp\left(\int_0^t q^{C_k}(s)s_k(s)dB_k(s)\right)}{E\left[\exp\left(\int_0^t q^{C_k}(s)s_k(s)dB_k(s)\right)\Big|_{F_0^x}\right]} \times \frac{\exp\left(\int_0^t q^{J_k}(s-)Z_k(s-)dn_k(s)\right)}{E\left[\exp\left(\int_0^t q^{J_k}(s-)Z_k(s-)dn_k(s)\right)\Big|_{F_0^x}\right]}$$

$$\times \frac{\exp\left(\int_0^t q^{J_c}(s-)Z_k(s-)dn_c(s)\right)}{E\left[\exp\left(\int_0^t q^{J_c}(s-)Z_k(s-)dn_c(s)\right)\Big|_{F_0^x}\right]}, \quad (10)$$

where $q^h(t) = \langle q^h, x(t)\rangle$ for $h = \{C_k, J_k, J_c\}$, in which $\theta^{C_k}(t)$, $\theta^{J_k}(t)$, and $\theta^{J_c}(t)$ are the Markov-modulated Esscher parameters of $C_k(t)$, $J_k(t)$, and $J_c(t)$, respectively. Q^{θ^h} is the so-called Esscher measure. Therefore, the mean percentage jump size becomes $t_k^{q^{J_k}(t)} = \exp\left(q^{J_k}(t)u_k + \frac{1}{2}(q^{J_k}(t)d_k)^2\right) - 1$. Here, the concrete form of the Markov-modulated Esscher transform density process $h^{q^h}(t)$ is an exponential F_t-martingale.

For determining a risk-neutral pricing measure of option pricing, it is possible to select the vth country's risk-neutral Esscher measure as $Q^{\theta^h,v}$ for $v \in \{d, f\}$ such that the discounted asset price dynamics are $Q^{\theta^h,v}$-martingales. This is obtained by determining the Markov-modulated Esscher parameters $\theta^{C_k}(t)$, $\theta^{J_k}(t)$, and $\theta^{J_c}(t)$ as solutions of the martingale condition $E^{q^h,f}\left[\exp\left(-\int_0^t r_f(s)ds\right)S_f(t)\middle|F_0\right] = S_f(0)$ and of the martingale condition $E^{q^h,d}\left[\exp\left(\int_0^t \left(r_f(s) - r_d(s)\right)ds\right)X(t)\middle|F_0\right] = X(0)$, respectively. We assume that the Markov-modulated Esscher transform is defined by Eq. (10), then these two martingale conditions are satisfied if and only if:

$$q^{C_{Sf},f}(t) = \frac{r_f(t) - a_{Sf}(t)}{s_{Sf}^2(t)} + \frac{l_{Sf}(t)t_{Sf} + l_c(t)t_{Sf}}{s_{Sf}^2(t)} \quad (11)$$

$$q^{C_X,d}(t) = \frac{r_d(t) - r_f(t) - a_X(t)}{s_X^2(t)} + \frac{l_X(t)t_X + l_c(t)t_X}{s_X^2(t)} \quad (12)$$

$$q^{J_k,v}(t) = = \frac{-u_k - \frac{1}{2}d_k^2}{d_k^2} \quad (13)$$

and

$$q^{J_c,\nu}(t) = \frac{-u_k - \frac{1}{2}d_k^2}{d_k^2}. \quad (14)$$

The derivation of solving the Markov-modulated Esscher parameters will be made available on request. For $\nu \in \{d, f\}$, an equivalent martingale probability measure can be treated as the risk-neutral Esscher measure $Q^{\theta^h,\nu}$ with respect to the measure P. We begin with identifying the underlying dynamics for the foreign equity price and the foreign exchange rate under $Q^{\theta^h,\nu}$. For $k \in \{Sf, X\}$ and all $t \in [0,T]$, let $\theta^{C_k,\nu}(t)$, $\theta^{J_k,\nu}(t)$, and $\theta^{J_c,\nu}(t)$ be the Markov-modulated Esscher parameters of the domestic/foreign risk-neutral Esscher measures. Under $Q^{\theta^h,\nu}$ and conditional on F_t^x, we then have:

$$B_{Sf}^{\theta^h,f}(t) = B_{Sf}(t) - \int_0^t \theta^{C_{Sf}}(s)\sigma_{Sf}(s)ds, \quad (15)$$

$$B_X^{\theta^h,d}(t) = B_X(t) - \int_0^t \theta^{C_X}(s)\sigma_X(s)ds, \quad (16)$$

are Brownian motions. Furthermore, under $Q^{\theta^h,\nu}$, the arrival intensities $\lambda_k^{\theta^h,\nu}(t)$ and $\lambda_c^{\theta^h,\nu}(t)$ of the Poisson processes $n_k^{\theta^h,\nu}(t)$ and $n_c^{\theta^h,\nu}(t)$, and the jump size $Z_k^{\theta^h,\nu}(t)$ are, respectively, given by the following:

$$l_k^{q^h,\nu}(t) = l_k(t)\exp\left(q^{J_k}(t)u_k + \frac{1}{2}\left(q^{J_k}(t)d_k\right)^2\right), \quad (17)$$

$$l_c^{q^h,\nu}(t) = l_c(t)\exp\left(q^{J_k}(t)u_k + \frac{1}{2}\left(q^{J_k}(t)d_k\right)^2\right), \quad (18)$$

and

$$Z_k^{\theta^h,\nu}(t) \overset{i.i.d.}{\sim} N\left(u_k + \theta^{J_k}(t)\delta_k^2, \delta_k^2\right), \quad (19)$$

The derivation of distributional properties under $Q^{\theta^h,\nu}$ will be made available on request.

Using the solutions of Markov-modulated Esscher parameters given by Eqs. (11)–(14), we have:

$$B_{Sf}^{q^h,f}(t) = B_{Sf}(t) + \left(\frac{u_{Sf(t)} - r_f(t) - l_{Sf}(t)t_{Sf} - l_c(t)t_{Sf}}{s_{Sf}(t)}\right)t, \quad (20)$$

$$B_X^{q^h,d}(t) = B_X(t) + \left(\frac{u_X(t) - r_d(t) + r_f(t) - l_X(t)t_X - l_c(t)t_X}{s_X(t)}\right)t, \quad (21)$$

$$l_k^{q^h,\nu}(t) = l_k(t)\exp\left(-\frac{u_k^2}{2d_k^2} + \frac{d_k^2}{8}\right), \quad (22)$$

$$l_c^{q^h,\nu}(t) = l_c(t)\exp\left(-\frac{u_k^2}{2d_k^2} + \frac{d_k^2}{8}\right), \quad (23)$$

and

$$Z_k^{\theta^h,\nu}(t) \overset{i.i.d.}{\sim} N\left(-\frac{1}{2}\delta_k^2, \delta_k^2\right), \quad (24)$$

Consequently, the dynamics for the foreign equity price under $Q^{\theta^h,f}$ and for the foreign exchange rate under $Q^{\theta^h,d}$ are, respectively, given by the following:

$$\frac{dS_f(t)}{S_f(t-)} = r_f(t)dt + s_{S_f}(t)dB_{S_f}^{q^h,f}(t) + \left(\exp\left(Z_{S_f}^{q^h,f}(t-)\right) - 1\right)dN_{S_f}^{q^h,f}(t), \quad (25)$$

and

$$\frac{dX(t)}{X(t-)} = \left(r_d(t) - r_f(t)\right)dt + s_X(t)dB_X^{q^h,d}(t) + \left(\exp\left(Z_X^{q^h,d}(t-)\right) - 1\right)dN_X^{q^h,d}(t), \quad (26)$$

For $k \in \{Sf, X\}$ and all $t \in [0,T]$, the following formula is valid for the diffusion component of any F_t-measurable random variable Ψ:

$$E^{q^h,f}\left[Y \middle| F_0\right] = E^{q^h,d}\left[Y\exp\left(\int_0^t s_k(s)dB_k^{q^h,d}(s) - \frac{1}{2}\int_0^t s_k^2(s)ds\right)\middle| F_0\right], \quad (27)$$

Using Eq. (27) in Eq. (25), we obtain $B_{Sf}^{\theta^h,f}(t) = B_{Sf}^{\theta^h,d}(t) - \int_0^t \rho(s)\sigma_X(s)ds$ follows a Brownian motion under any foreign martingale measure $Q^{\theta^h,f}$. This transform and Eq. (24) are substituted in Eq. (25), we then have the foreign equity price dynamics under $Q^{\theta^h,d}$ as follows:

$$\frac{dS_f(t)}{S_f(t-)} = \left(r_f(t) - r(t)s_{Sf}(t)s_X(t)\right)dt + s_{Sf}(t)dB_{Sf}^{q^h,d}(t) + \left(\exp\left(Z_{Sf}^{q^h,d}(t-)\right) - 1\right)dN_{Sf}^{q^h,d}(t), \quad (28)$$

Using Eq. (27) in Eq. (26), the Brownian motion $B_X^{\theta^h,f}(t) = B_X^{\theta^h,d}(t) - \int_0^t \rho(s)\sigma_{Sf}(s)ds$ under $Q^{\theta^h,f}$ is derived for $X(t)$. We substitute this process and Eq.

(24) into Eq. (26), the foreign exchange rate dynamics under $Q^{\theta^{h},f}$ is given by the following:

$$\frac{dX(t)}{X(t-)} = \left(r_d(t) - r_f(t) + r(t)s_{Sf}(t)s_X(t)\right)dt + s_X(t)dB_X^{q^{h},f}(t) + \left(\exp\left(Z_X^{q^{h},f}(t-)\right) - 1\right)dN_X^{q^{h},f}(t), \quad (29)$$

Furthermore, the coefficient of correlation between these two asset price dynamics under the Markov-modulated, cross-currency market model is defined as follows:

$$r^{MMCCM}(t) = Corr\left(\frac{dS_f(t)}{S_f(t-)}, \frac{dX(t)}{X(t-)}\right) = \frac{Cov\left(\frac{dS_f(t)}{S_f(t-)}, \frac{dX(t)}{X(t-)}\right)}{\sqrt{Var\left(\frac{dS_f(t)}{S_f(t-)}\right)}\sqrt{Var\left(\frac{dX(t)}{X(t-)}\right)}}, \quad (30)$$

Note that $Cov\left(\frac{dS_f(t)}{S_f(t-)}, \frac{dX(t)}{X(t-)}\right)$ is a measure of dependence between these two asset price dynamics. Eq. (30) implies that $r^{MMCCM}(t) = 0$ is a necessary condition for these two asset price dynamics to be independent. In addition, $r^{MMCCM}(t) = 1$, if and only if, these two asset price dynamics are linearly dependent.

3.2. Valuation Of Quanto Options

We note that, the parameters of the Markov-modulated, cross-currency market model can be expressed explicitly in terms of the occupation times of the state of Markov chain $x(t)$. Let $\Gamma_j(t,T)$ denote the occupation time of $x(t)$ in state j ($j = 1,2,\ldots,I$) over the time duration $[t,T]$ with $t < T$. For $v \in \{d, f\}$ and $k \in \{Sf, X\}$, we define:

$$R_v(t,T) = \int_t^T r_v(u)du = \sum_{j=1}^{I} r_v(j)\Gamma_j(t,T), \quad (31)$$

$$V_k(t,T) = \int_t^T s_k^2(u)du = \sum_{j=1}^{I} s_k^2(j)\Gamma_j(t,T), \quad (32)$$

$$D_{SX}(t,T) = \int_t^T r(u)s_{Sf}(u)s_X(u)du = \sum_{j=1}^{I} r(j)s_{Sf}(j)s_X(j)\Gamma_j(t,T), \quad (33)$$

$$l_k^{q^{h},v}(t,T) = \int_t^T l_k^{q^{h},v}(u)du = \sum_{j=1}^{I} l_k^{q^{h},v}(j)\Gamma_j(t,T), \quad (34)$$

and

$$l_c^{q^{h},v}(t,T) = \int_t^T l_c^{q^{h},v}(u)du = \sum_{j=1}^{I} l_c^{q^{h},v}(j)\Gamma_j(t,T), \quad (35)$$

Then, the no-arbitrage price of European-style quanto call options, in terms of the occupation times, obey the following semi-analytic form:

$$QO(t) = \int_{[t,T]^I} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P_{SX}^{q^{h},v} \times QO^{MMCCM}(t;j,m,n) \times F(\Gamma_1,\Gamma_2,\ldots,\Gamma_I)d\Gamma_1 d\Gamma_2 \ldots d\Gamma_I, \quad (36)$$

where

$$P_{SX}^{q^{h},v} = \sum_{q=0}^{\min(m,n)} \exp\left(-\left(l_{Sf}^{q^{h},v}(t,T) + l_X^{q^{h},v}(t,T) + l_c^{q^{h},v}(t,T)\right)\right) \times \frac{\left(l_{Sf}^{q^{h},v}(t,T)\right)^{m-q}\left(l_X^{q^{h},v}(t,T)\right)^{n-q}\left(l_c^{q^{h},v}(t,T)\right)^{q}}{(m-q)!(n-q)!q!}, \quad Q^{q^{h},v}\text{-a.s.} \quad (37)$$

and the no-arbitrage option price notation $QO^{MMCCM}(t;m,n)$, depends on the occupation times and jump numbers. The symbol $F(\Gamma_1,\Gamma_2,\ldots,\Gamma_I)$ denotes the joint probability distribution density for the occupation time. Write $\Gamma(t,T) = (\Gamma_1(t,T),\Gamma_2(t,T),\ldots,\Gamma_I(t,T))$ for the vector of occupation times. Note that $F(\Gamma_1,\Gamma_2,\ldots,\Gamma_I)$ can be completely determined by the following characteristic function:

$$E\left[\exp\left(i\langle e,\Gamma(t,T)\rangle\right)\middle|F_t^x\right] = \langle x(t)\exp((\Psi + iG)(T-t)),1\rangle, \quad (38)$$

where $i = \sqrt{-1}$, $1 = (1,1,\ldots,1) \in \mathbb{R}^I$ and the matrix G denotes a diagonal matrix consisting of the elements in the vector $e = (e_1,e_2,\ldots,e_I)$ as its diagonal, as in [4].

Let K_f be the foreign exercise price and T be the expiry date of an option. The terminal payoff of a quanto call (that is, a guaranteed foreign exchange rate foreign equity call option) at the expiry date is given as follows:

$$QO(T) = \bar{X}(S_f(T) - K_f)^+, \quad (39)$$

where $\bar{X}$ denotes the pre-specified foreign exchange rate at which the conversion of the option's payoff is made into domestic currency. Considering both the concurrent jumps and the time-inhomogeneity in the oscillating cross-currency market, the no-arbitrage price of an European-style quanto call option at time t is then given by the following:

$$QO^{MMCCM}(t; j, m, n) = \bar{X}\exp(-R_d(t,T)) \times [S_f(t)\exp(R_f(t,T) - D_{SX}(t,T))N(d_1) - K_f N(d_2)], \quad (40)$$

where

$$d_1 = \frac{\ln\left(\frac{S_f(t)}{K_f}\right) + (R_f(t,T) - D_{SX}(t,T))}{\sqrt{V_{Sf}(t,T) + md_{Sf}^2}} + \frac{\frac{1}{2}(V_{Sf}(t,T) + md_{Sf}^2)}{\sqrt{V_{Sf}(t,T) + md_{Sf}^2}} \quad (41)$$

and

$$d_2 = d_1 - \sqrt{V_{Sf}(t,T) + md_{Sf}^2}. \quad (42)$$

The derivation of the European-style quanto option pricing formula will be made available on request.

CONCLUSIONS

In this study, the dynamics of the foreign equity price and the foreign exchange rate are modeled by a Markov-modulated, cross-currency market model with capturing both individual jumps and concurrent jumps, and the domestic and foreign risk-free interest rates are assumed throughout to be stochastic rather than deterministic. Compared with most of the existing jump-diffusions, the main advantage is that we incorporate both the concurrent jumps and the time-inhomogeneity generated by the cross-currency market into the underlying asset price dynamics. After determining the Markov-modulated risk premiums, we formally derive the explicit valuation formula for European-style quanto call options. This study concludes that the correlations between the foreign equity price and the foreign exchange rate affect the option prices.

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