A STOCHASTIC VESSEL SCHEDULING TRANSPORTATION-INVENTORY PROBLEM WITH GAMMA AND UNIFORM DEMANDS

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Abstract—This paper addresses a stochastic scheduling transportation-inventory problem to transport crude oil from a source to adestination over a finite time horizon via a fleet of heterogeneous vessels. The demands have either gammaor uniform distribution, and a penalty structure isimposed on the shortages and excesses in daily storage levels at the destination. The expected overall cost of such a fleet operation is composed of the total vessels' operational expenses, expected total penalties incurred for violating pre-specified lower and upper on storage levels, and vessel chartering expenses.The objective is to optimize the fleet schedules and to minimize the total expected cost, which can be achieved by simultaneously maintaining desirable storage levels and meeting the stochastic demands with acceptable reliability levels. For each demand distribution, the problem is formulatedas a stochastic optimization model and then chance constrained programming is used to convert the model into an exact mixed-integer nonlinear program. Some computational results are reported.

Keywords: Stochastic scheduling, inventory/shortages, transportation, chance constraints, integer nonlinear programming, gamma, uniform

I. INTRODUCTION

Oceanic transportation is the major transportation mode worldwide in terms of volume [1]. Crude oil and related products account to about 4.8% of the global value of all exported goods, and hence it constitutes the number one transported product in terms of tonnage. During recent years, oil prices dropped from $120 per barrel (in 2012) to a current price of about $60 per barrel (in 2018).Due to the high cost associated with owning and chartering oil tankers and the complexity, uncertainty, dynamic and combinatorial nature of vessel operation, it is imperative to utilize efficient quantitative methodologies and to analyze demand trends in order to reduce the overall cost of such an operation ([2]-[5]).

This paper studies an oil tanker vessel scheduling transportation-inventory problem to transport crude oil (a product) from a port in Middle East (a source) to an overseasstorage facility(a destination) that has its own demand structure. Self-owned and chartered vessels of different types are available, where each vessel type is characterized by its size, speed, loading and unloading times, etc.

Daily demands at the destination are determined based on historical, current, and expected daily consumption rates. The demand probability distributions can highly affect the expected total costs and overall operations ([6]-[7]), highlighting the significance to accommodate a sufficient knowledge of demand distributions into modeling considerations. The expected total cost associated with the above described vessel operation consists of the total vessels' operational expenses, expected total penalties incurred when violating specified lower and upper bounds at the destination storage facility, and expenses chartering vessels. The thrust of this research effort is to simultaneously optimize the fleet schedules and maintain the desirable daily storage levels to meet the stochastic demand requirements at the destination with acceptable reliability levels. This work aims to better understand and analyze the stochastic demand aspects of the studied vessel scenarios extending the authors' earlier work [8], which focuses on normally distributed demands, to explore two other demand probability distributions: gamma and uniform distributions. For the proposed problem, our statistical analysis of an oil company's simulated demands for a six-year period shows that the daily demands for the product at adestination have either gammaor uniform distributions.

The existing studies on stochastic transportation-inventory problems commonly use classical policies to model shortage and inventory levels, and apply methods such as simulation-based optimization, simulation-based heuristics, and robust optimization to approximate the solutions. To the best of our knowledge, norelevant literature exists on optimally solving a stochastic vessel scheduling transportation-inventory problem that utilizes gamma and uniform demands and four types of penalties signifying the importance of falling the storage levels within some desirable lower and upper bounds.

II. PROBLEM FORMULATION

In this section, we present some notations, introduce four penalty types on storage levels and compute their expected values, and then develop our stochastic vessel-scheduling model (SVSM).

2.1 Notations

$H=\{1,\ldots,|H|\}$: Set of days in the time horizon,

where $|H|$ is the number of days in the horizon.

$T=\{1,\ldots,|T|\}$: Set of vessel types, where $|T|$ is the number of vessel types.

$h \in H, t \in T$: Indices for days in the time horizon, and vessel types, respectively.

n: Indices for self-owned and chartered vessels.

Ω_t: Capacity of a vessel of type $t \in T$.

O_t: Number of available self-owned vessels of type $t \in T$.

CH_t: Number of available chartered vessels of type $t \in T$.

TN_t: Total number of self-owned and chartered vessels of type $t \in T$, where $n=1,\ldots,O_t,O_{t+1},\ldots,TN_t$.

$D_{1,t}$: Sum of loading time at the source and voyage time from the source to the destination of a vessel of type $t \in T$.

$D_{2,t}$:Sum of unloading time at the destination and voyage time to the source of a vessel of type $t \in T$.

D_t: Total loading/unloading and roundtrip voyage time ofa vessel of type $t \in T$.

$UD_{t,n}$: Maximum number of days that a vessel $n=1,\ldots,TN_t$of type $t \in T$ used during the time horizon.

$OC_{t,n}$: Daily operational cost ofa vessel $n=1,\ldots,TN_t$of type $t \in T$.

$C_{t,n}$: Total operational cost of vessel a vessel $n=1,\ldots,TN_t$of type $t \in T, i.e., C_{t,n} = D_t \times OC_{t,n}$.

$\$_{t,n}$: Cost for chartering a vessel $n=1,\ldots,TN_t$of type $t \in T$.

Q: Production capacity or product quota at the source in any day of the time horizon.

w: Product storage level at the destination at the beginning of the time horizon.

S_h: Product storage level at the destination on day $h \in H$.

R_h: Product demand at the destination on day $h \in H$.

TC_h: Product cumulative demand at the destination up to day $h \in H, i.e., TC_h = \sum_{h_1=1}^{h} R_{h_1}$.

SL_1: Minimum daily storage levels at destination to avoid penalties resulting from the product shortages.

SL_2: Maximum daily storage levels at destination to avoid penalties for the product excesses.

A_1: Allowable shortages at the destination with respect to SL_1.

A_2: Allowable excesses at the destination with respect to SL_2.

α_1: Daily per unit penalties associated with the storage levels that lie within (SL_1-A_1, SL_1).

α_2: Daily per unit penalties associated with the storage levels that lie within (SL_2, SL_2+A_2).

LB: Minimumallowable storage level.

UB: Maximum allowable storage level.

β_1: Sufficiently large daily per unit penalties associated with the storage levels that lie within (LB, SL_1-A_1).

β_2: Sufficiently large daily per unit penalties associated with the storage levels that lie within (SL_2+A_2, UB).

$g(t)$: Probability density function (pdf) of a gamma random variable with shape and slope parameters θ and λ, *i.e.*, $G(\theta, \lambda)$.

$u(t)$: The pdf of a uniform random variable over (a,b), i.e., $U(a,b)$.

$E(.)$: Expected value operator.

2.2 Expected penalties on storage levels

The imposed penalties on $S_h$for each day $h \in H$ is classified into four types, noting that when $S_h \in [SL_1, SL_2]$, no penalty is incurred.

Type I penalty: $P_{1,h} = \alpha_1(SL_1 - S_h)$ if $S_h \in [SL_1 - A_1, SL_1)$. (1)

Type II penalty: $P_{2,h} = \alpha_1 A_1 + \beta_1(SL_1 - A_1 - S_h)$, if $S_h \in (LB, SL_1 - A_1)$. (2)

Type IIIpenalty: $P_{3,h} = \alpha_2(S_h - SL_2)$ if $S_h \in (SL_2, SL_2 + A_2]$. (3)

Type IVpenalty: $P_{4,h} = \alpha_2 A_2 + \beta_2[S_h - (SL_2 + A_2)]$, if $S_h \in (SL_2 + A_2, UB)$. (4)

Accordingly, the expected daily penalties of Types I-IV (denoted by $E[P_{i,h}]$, $i=1,\ldots,4$, respectively), using (1)-(4), are defined as

$$E[P_{1,h}] = \alpha_1 \int_{SL_1 - A_1}^{SL_1} (SL_1 - t) f_h(t)\, dt, \qquad (5)$$

where $f_h(t)$ is the pdf of S_h,

$$E[P_{2,h}] = \int_{LB}^{SL_1 - A_1} [\alpha_1 A_1 + \beta_1(SL_1 - A_1 - t)] f_h(t)\, dt, \quad (6)$$

$$E[P_{3,h}] = \alpha_2 \int_{SL_2}^{SL_2 + A_2} (t - SL_2) f_h(t)\, dt, \qquad (7)$$

and

$$E[P_{4,h}] = \int_{SL_2 + A_2}^{UB} [\alpha_2 A_2 + \beta_2[t - (SL_2 + A_2)]] f_h(t)\, dt. \quad (8)$$

Thus, the total expected penalty $E[TP_h]$on day $h \in H$, using (5)-(8), is $E[TP_h] = \sum_{i=1}^{4} E[P_{i,h}]$.

It is desired that S_h for each day $h \in H$ falls within the no penalty interval $[SL_1, SL_2]$ with a probability of at least$(1-r_h)$,

where $0 < r_h < 1$ is called the confidence parameter in chance constrained programming[9],i.e. $\text{Prob}(SL_1 \le S_h \le SL_2) \ge (1-r_h)$.

The $(1-r_h)$ can be regarded as a level of service that should be maintained, namely, at least $100(1-r_h)\%$ of thedaily storage levels should fall within the no penalty interval $[SL_1, SL_2]$.

2.3 Model SVSM

To formulate SVSM, we define the following binary decision variables.

$X_{h,t,n} = \begin{cases} 1, & \text{if vessel } n \text{ of type } t \text{ departs the source on day } h, \\ 0, & \text{otherwise,} \end{cases}$

$Y_{h,t,n} = \begin{cases} 1, & \text{if vessel } n \text{ of type } t \text{ is available at the source on day } h, \\ 0, & \text{otherwise,} \end{cases}$

and

$V_{t,n} = \begin{cases} 1, & \text{if vessel } n \text{ of type } t \text{ is selected for chartering during } H, \\ 0, & \text{otherwise,} \end{cases}$

SVSM: Minimize

$$\sum_{h\in H}\sum_{t\in T}\sum_{n=1}^{TN_t} C_{t,n}X_{h,t,n} + \sum_{h\in H} E[TP_h] + \sum_{t\in T}\sum_{n=O_t+1}^{TN_t} \$_{t,n}V_{t,n}$$

subject to:

$$S_h = w + \sum_{t\in T}\sum_{n=1}^{TN_t}\sum_{h_1\in\{1,..,h-D_{1,t}\}} \Omega_t X_{h_1,t,n} - TC_h, \ \forall h \in H, \quad (9)$$

$$\text{Prob}\left(SL_1 \le S_h \le SL_2\right) \ge (1-r_h), \forall h \in H, \quad (10)$$

$$Y_{h,t,n} = Y_{h-1,t,n} - X_{h-1,t,n} + X_{h-D_t,t,n}, \quad \forall h \ge 2 \in H,\ t \in T,\ n = 1,...,TN_t, \quad (11)$$

$$X_{h,t,n} \le Y_{h,t,n}, \ \forall h \in H,\ t \in T,\ n = 1,...,TN_t, \quad (12)$$

$$Y_{h,t,n} \le V_{t,n}, \ \forall h \in H,\ t \in T,\ n = O_t + 1,...,TN_t, \quad (13)$$

$$\sum_{h_1=1}^{h}\sum_{t\in T}\sum_{n=1}^{TN_t} \Omega_t X_{h_1,t,n} \le hQ, \ \forall h \in H, \quad (14)$$

$$\sum_{h\in H} D_t X_{h,t,n} \le UD_{t,n}, \ \forall t \in T,\ n = 1,...,TN_t, \quad (15)$$

$$X_{h,t,n} \in \{0,1\} \text{ for } X_{h,t,n} \ne \varphi_X, \ \forall h \in H,\ t \in T,\ n, \quad (16)$$

$$Y_{1,t,n} \in \{0,1\} \ \forall t \in T,\ n = 1,...,TN_t, \quad (17)$$

$$0 \le Y_{h,t,n} \le 1 \text{ for } Y_{h,t,n} \ne \varphi_Y, \ \forall h \ge 2 \in H, t \in T, n, \quad (18)$$

$$0 \le V_{t,n} \le 1, \ \forall t \in T,\ n = O_t + 1,...,TN_t, \quad (19)$$

$$S_h \ge 0, \ \forall h \in H. \quad (20)$$

III. SVSM WITH GAMMA DEMANDS

Assume that the daily demands R_h, $h \in H$, have independent gamma distributions with integer shape parameters k_h and a common slope parameter λ, i.e., $R_h \sim G(k_h, \lambda)$. Then, the cumulative demand TC_h up to day h has also a gamma distribution with the shape parameter $K_h = \sum_{h_1=1}^{h} k_{h_1}$ and slope parameter λ, i.e., $TC_h \sim G(K_h, \lambda)$, whose cdf is given by

$$\text{Prob}(TC_h \le x) = 1 - \exp(-\lambda x)\sum_{j=0}^{K_h-1}\frac{(\lambda x)^j}{j!}, \ x > 0.$$

The daily storage level S_h, using (9), can be written as $S_h = B_h - TC_h$, where

$$B_h = w + \sum_{t\in T}\sum_{n=1}^{TN_t}\sum_{h_1\in\{1,..,h-D_{1,t}\}} \Omega_t X_{h_1,t,n}. \quad (21)$$

Then, the cdf and pdf of S_h are respectively given by

$$G_h(t) = \text{Prob}(TC_h \ge B_h - t) = exp[-\lambda(B_h - t)]\sum_{j=0}^{K_h-1}\frac{[\lambda(B_h-t)]^j}{j!}, \quad -\infty < t < B_h,$$

and

$$g_h(t) = \frac{\lambda^{K_h}}{(K_h-1)!}(B_h - t)^{K_h-1}\exp[-\lambda(B_h - t)], \quad -\infty < t < B_h. \quad (22)$$

For the vessel scheduling transportation-inventory problem with gamma demands (SVSM-G), since $0 < S_h < B_h$, then thetruncated pdf $f_h(t)$ of S_h, using (22), is defined as

$$f_h(t) = \frac{\left[\lambda^{K_h}/(K_h-1)!\right](B_h-t)^{K_h-1}\exp(\lambda t)}{\exp(\lambda B_h) - \sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!}, \ 0 < t < B_h. \quad (23)$$

Hence, the constraint (10), using (23), can be written as

$$\text{Prob}(SL_1 \le S_h \le SL_2) = \exp[-\lambda(B_h - SL_2)]\sum_{j=0}^{K_h-1}\left[\begin{array}{l}[\lambda(B_h-SL_2)]^j - [\lambda(B_h-SL_1)]^j \\ \times \exp[-\lambda(SL_2-SL_1)] \\ + (\lambda B_h)^j(1-r_h)\exp(-\lambda SL_2)\end{array}\right]\Big/ j! \ge (1-r_h), \quad (24)$$

where $0 = LB < SL_1 - A_1 < SL_1 < SL_2 < SL_2 + A_2 \le B_h < UB.$

3.1 Expected penalties and Model SVSM-G

We derive, in this section, closed-form expressions for the four types of expected daily penalty costs (5)-(8) when the daily demands have gamma distributions.

The expected daily Type I penalty $E[P_{1,h}]$, using (5) and (23), is computed as

$$E[P_{1,h}] = \frac{\alpha_1\exp(\lambda SL_1)}{\lambda\left[\exp(\lambda B_h) - \sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}\left[\begin{array}{l}\left[\begin{array}{l}[\lambda(B_h-SL_1)]^j - \\ [\lambda(B_h-SL_1+A_1)]^j \\ \times\exp(-\lambda A_1)\end{array}\right]\Big/ j! \\ \qquad\sum_{j=0}^{K_h-1} \\ [(SL_1-B_h)+K_h] \\ + \left[[\lambda(B_h-SL_1)]^{K_h} - [\lambda(B_h-SL_1+A_1)]^{K_h}\right]/(K_h-1)! \\ \exp(-\lambda A_1)\end{array}\right], \quad (25)$$

where $B_h \ge SL_1$.

Expected Type II daily penalty $E[P_{2,h}]$ is associated with $(0, SL_1 - A_1)$; thus, plugging (23) into (6) gives:

$$E[P_{2,h}] = \frac{\exp[\lambda(SL_1 - A_1)]}{\lambda\left[\exp(\lambda B_h) - \sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]} \times$$

$$\left[\begin{array}{l}\left[\lambda[\alpha_1 A_1+\beta_1(SL_1-A_1)]-\beta_1(\lambda B_h-K_h)\right]\\ \times\sum_{j=0}^{K_h-1}\left(\begin{array}{l}[\lambda(B_h-SL_1+A_1)]^j\\ -(\lambda B_h)^j\exp[-\lambda(SL_1-A_1)]\end{array}\right)\Big/ j!\\ +\beta_1\left(\begin{array}{l}[\lambda(B_h-SL_1+A_1)]^{K_h}\\ -(\lambda B)^{K_h}\exp[-\lambda(SL_1-A_1)]\end{array}\right)\Big/(K_h-1)!\end{array}\right], \quad (26)$$

where $B_h \geq SL_1 - A_1$.

Substituting (23) into (7), the expected Type III daily penalty is calculated as

$$E[P_{3,h}] = \frac{\alpha_2\exp[\lambda(SL_2+A_2)]}{\lambda\left[\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}\times$$

$$\left[\begin{array}{l}[\lambda(B_h-SL_2)-K_h]\sum_{j=0}^{K_h-1}\left(\begin{array}{l}[\lambda(B_h-SL_2-A_2)]^j\\ -[\lambda(B_h-SL_2)^j\times\\ \exp(-\lambda A_2)\end{array}\right)\Big/ j!\\ -\left(\begin{array}{l}[\lambda(B_h-SL_2-A_2)]^{K_h}\\ -[\lambda(B_h-SL_2)]^{K}\exp(-\lambda A_2)]\end{array}\right)\Big/(K_h-1)!\end{array}\right], \quad (27)$$

where $B_h \geq SL_2 + A_2$.

Finally, the expected value of Type IV daily penalty is computed by substituting (23) into (8) which give:

$$E[P_{4,h}] = \frac{1}{\lambda\left[\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}\times$$

$$\left[\begin{array}{l}\left(\begin{array}{l}\lambda[\alpha_2 A_2-\beta_2(SL_2+A_2)]\\ +\beta_2(\lambda B_h-K_h)\end{array}\right)\left(\begin{array}{l}\exp(\lambda B_h)-\exp[\lambda(SL_2+A_2)]\\ \times\sum_{j=0}^{K_h-1}[\lambda(B_h-SL_2-A_2)]^j/j!\end{array}\right)\\ +\beta_2[\lambda(B_h-SL_2-A_2)]^{K_h}\exp[\lambda(SL_2+A_2)]/(K_h-1)!\end{array}\right]$$

(28)

where $B_h \geq SL_2 + A_2$.

Using (25)-(28), we obtain the expected total daily penalty $E[TP_h] = \sum_{i=1}^{4} E[P_{i,h}]$ as

$$E[TP_h] = \frac{1}{\lambda\left[\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}\times$$

$$\left[\begin{array}{l}\alpha_1\exp(\lambda SL_1)\left[\begin{array}{l}[\lambda(SL_1-B_h)+K_h]\sum_{j=0}^{K_h-1}\left(\begin{array}{l}[\lambda(B_h-SL_1)]^j\\ -[\lambda(B_h-SL_1+A_1)]^j\\ \times\exp(-\lambda A_1)\end{array}\right)\Big/ j!\\ +\left[\begin{array}{l}[\lambda(B_h-SL_1)]^{K_h}-[\lambda(B_h-SL_1+A_1)]^{K_h}\\ \times\exp(-\lambda A_1)\end{array}\right]\Big/(K_h-1)!\end{array}\right]\\ +\exp[\lambda(SL_1-A_1)]\left[\begin{array}{l}\left[\lambda[\alpha_1 A_1+\beta_1(SL_1-A_1)]-\beta_1(\lambda B_h-K_h)\right]\\ \times\sum_{j=0}^{K_h-1}\left(\begin{array}{l}[\lambda(B_h-SL_1+A_1)]^j\\ -(\lambda B_h)^j\exp[-\lambda(SL_1-A_1)]\end{array}\right)\Big/ j!\\ +\beta_1\left(\begin{array}{l}[\lambda(B_h-SL_1+A_1)]^{K_h}\\ -(\lambda B_h)^{K_h}\exp[-\lambda(SL_1-A_1)]\end{array}\right)\Big/(K_h-1)!\end{array}\right]\\ +\alpha_2\exp[\lambda(SL_2+A_2)]\left[\begin{array}{l}[\lambda(B_h-SL_2)-K_h]\sum_{j=0}^{K_h-1}\left(\begin{array}{l}[\lambda(B_h-SL_2-A_2)]^j\\ -[\lambda(B_h-SL_2)]^j\\ \times\exp(-\lambda A_2)\end{array}\right)\Big/ j!\\ -\left(\begin{array}{l}[\lambda(B_h-SL_2-A_2)]^{K_h}\\ -[\lambda(B_h-SL_2)]^{K_h}\exp(-\lambda A_2)]\end{array}\right)\Big/(K_h-1)!\end{array}\right],\\ +\left(\begin{array}{l}\lambda[\alpha_2 A_2-\beta_2(SL_2+A_2)]\\ +\beta_2(\lambda B_h-K_h)\end{array}\right)\left(\begin{array}{l}\exp(\lambda B_h)-\exp[\lambda(SL_2+A_2)]\\ \times\sum_{j=0}^{K_h-1}[\lambda(B_h-SL_2-A_2)]^j/j!\end{array}\right)\\ +\beta_2[\lambda(B_h-SL_2-A_2)]^{K_h}\exp[\lambda(SL_2+A_2)]/(K_h-1)!\end{array}\right]$$

(29)

where $K_h = \sum_{h_1=1}^{h} k_{h_1}$ and $B_h \geq SL_2 + A_2$.

We now give our mixed-integer nonlinear programming model for SVSM-G with a nonlinear objective function (which is shown to be convex) and linear constraints; thus, the proposed model yields a global optimal solution.

SVSM-G: Minimize

$$\sum_{h\in H}\sum_{t\in T}\sum_{n=1}^{TN_t} C_{t,n}X_{h,t,n} + \sum_{h\in H}E[TP_h] + \sum_{t\in T}\sum_{n=O_t+1}^{TN_t}\$_{t,n}V_{t,n}$$

where $E[TP_h]$ is given by (29),

subject to (11)-(19), (24), and $B_h \geq SL_2 + A_2, \forall h \in H$, where B_h is defined by (21).

The objective function of SVSM-G minimizes the sum of the total vessel operation cost, total expected penalty cost, and total chartering cost. Constraints (11)-(19) are operation-related constraints.Constraint (24), which equivalently show the chance constraint (10), guarantee thatthe daily storage levels lie within the no penalty interval with the pre-specified probability.

3.2 Storage levels and expected penalties

We prove here that if for a given $h \in H$ the random storage level S_h falls with certainty within one of the five possible intervals for storage levels, then the expected total penalty for SVSM-G of the stochastic problem reduces to the total penalty of the deterministic model (DVSM). Here, the deterministic S_h is set equal to the expected daily storage level $E[S_h]$ for SVSM-G which, using (23), is computed as

$$E[S_h] = B_h - \frac{K_h\left(\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right)-(\lambda B_h)^{K_h}/(K_h-1)!}{\lambda\left[\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}.$$

(30)

Case 1. If $\text{Prob}(SL_1 \leq S_h \leq SL_2) = 1$, then $\text{Prob}(S_h \leq SL_1) = \text{Prob}(S_h \geq SL_2) = 0$. Based on (25)-(28), we have $E[P_{i,h}] = 0$, i=1,…,4; thus, $E[TP_h]$, given by (29), reduces to zero, which is essentially the total penalty $TP_h = \sum_{i=1}^{4} P_{i,h} = 0$ for DVSM, where$P_{i,h}$, i=1,…,4 are given by (1)-(4).

Case 2. If $\text{Prob}(SL_1-A_1 \leq S_h \leq SL_1) = 1$, then $\text{Prob}(S_h \leq SL_1-A_1) = \text{Prob}(S_h \geq SL_1) = 0$. Setting $SL_1 - A_1 = 0, SL_1 = B_h$, based on (26)-(28), $E[P_{i,h}] = 0$, i=2,3,4, and using (25), (29) reduces to

$$E[TP_h] = \alpha_1\left[\frac{K_h\left(\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right)-(\lambda B_h)^{K_h}/(K_h-1)!}{\lambda\left[\exp(\lambda B_h)-\sum_{j=0}^{K_h-1}(\lambda B_h)^j/j!\right]}\right],$$

which, using (30), is

$E[TP_h] = \alpha_1 [B_h - E[S_h]] = \alpha_1 [SL_1 - E[S_h]]$, which isthe Type I penalty $P_{1,h}$ for DVSM when S_h is set equal to $E[S_h]$ (see Equation (1)).

Case 3. If $\text{Prob}(0 \le S_h \le SL_1\text{-}A_1) = 1$, then $\text{Prob}(S_h \ge SL_1\text{-}A_1) = 0$. Since $E[P_{i,h}] = 0$, i=1,3,4, using $E[P_{2,h}]$, (29) reduces to

$$E[TP_h] = \alpha_1 A_1 + \beta_1 \left[\frac{K_h \left(\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right) - (\lambda B_h)^{K_h} / (K_h\text{-}1)!}{\lambda \left[\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right]} \right],$$

which,using (30),we have $E[TP_h] = \alpha_1 A_1 + \beta_1 [B_h - E[S_h]] = \alpha_1 A_1 + \beta_1 [SL_1 - A_1 - E[S_h]]$. But, this is the Type II penalty $P_{2,h}$ for DVSM (see Equation (2)).

Case 4. If $\text{Prob}(SL_2 \le S_h \le SL_2 + A_2) = 1$, then $\text{Prob}(S_h \le SL_2) = \text{Prob}(S_h \ge SL_2 + A_2) = 0$.Based on (25), (26) and (28), $E[P_{i,h}] = 0$, i=1,2,4. Utilizing $E[P_{3,h}]$, (29)becomes:

$$E[TP_h] = \alpha_2 \left[B_h - \frac{K_h \left(\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right) - (\lambda B_h)^{K_h} / (K_h\text{-}1)!}{\lambda \left[\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right]} \right],$$

or, using (30), $E[TP_h] = \alpha_2 E[S_h] = \alpha_2 [E[S_h] - SL_2]$ since $SL_2 = 0$. But, this is the penalty $P_{3,h}$ of Type III for DVSM (see Equation (3)).

Case 5. If $\text{Prob}(S_h \ge SL_2 + A_2) = 1$, then $\text{Prob}(S_h \le SL_2 + A_2) = 0$. Since $E[P_{i,h}] = 0$, i=1,2,3, and, using $E[P_{4,h}]$, (29) reduces to

$$E[TP_h] = \alpha_2 A_2 + \beta_2 \left[B_h - \frac{K_h \left(\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right) - (\lambda B_h)^{K_h} / (K_h\text{-}1)!}{\lambda \left[\exp(\lambda B_h) - \sum_{j=0}^{K_h-1} (\lambda B_h)^j / j! \right]} \right],$$

which, using (30), $E[TP_h] = \alpha_2 A_2 + \beta_2 E[S_h] = \alpha_2 A_2 + \beta_2 [E[S_h] - (SL_2 + A_2)]$ since $SL_2 + A_2 = 0$, which isType IV penalty $P_{2,h}$ for DVSM (see Equation (4)).

IV. SVSM WITH UNIFORM DEMANDS

For a given $h \in H$ assume that the cumulative daily demandTC_h is uniformly distributed over (a_h, b_h) where $0 < a_h < b_h$(i.e., $TC_h \sim U(a_h, b_h)$). Using(9) $S_h = B_h - TC_h$ where B_h is given by (21). For the vessel scheduling transportation-inventory problem with uniform demands (SVSM-U), we can showthat S_h is also uniformly distributed over $(B_h - b_h, B_h - a_h)$ (i.e., $S_h \sim U(B_h - b_h, B_h - a_h)$) where $0 < B_h - b_h < B_h - a_h$ due to$S_h > 0$.

4.1 Expected penalties and Model SVSM-U

The chance constraint (10) for $S_h$with pdf $f_h(t) = 1/(b_h - a_h)$, $0 < B_h - b_h < t < B_h - a_h$, which implies $B_h > b_h$, where B_h is given by (21), can be written as

$$\text{Prob}(SL_1 \le S_h \le SL_2) = \frac{SL_2 - SL_1}{b_h - a_h} \ge (1 - r_h), \quad \forall h \in H.$$

However, this implies

$$\max_{h \in H} \{(1 - r_h)(b_h - a_h)\} \le SL_2 - SL_1 \le \min_{h \in H} \{b_h - a_h\}. \quad (31)$$

Since $0 < LB = B_h - b_h \le SL_1 - A_1$ and $UB > B_h - a_h \ge SL_2 + A_2$ or

$$SL_2 + A_2 + a_h \le B_h \le SL_1 - A_1 + b_h, \quad (32)$$

then, using (5)-(8),the expected penalty costs $E[P_{i,h}]$, i=1,2,3, 4, are respectively defined as:

$$E[P_{1,h}] = \frac{\alpha_1 A_1^2}{2(b_h - a_h)}, \quad (33)$$

$$E[P_{2,h}] = \frac{(SL_1\text{-}A_1 + b_h\text{-}B_h)[2\alpha_1 A_1 + \beta_1 (SL_1\text{-}A_1 + b_h\text{-}B_h)]}{2(b_h - a_h)}, \quad (34)$$

where $B_h \le SL_1 - A_1 + b_h$,

$$E[P_{3,h}] = \frac{\alpha_2 A_2^2}{2(b_h - a_h)}, \quad (35)$$

and

$$E[P_{4,h}] = \frac{(B_h\text{-}SL_2\text{-}A_2\text{-}a_h)[2\alpha_2 A_2 + \beta_2 (B_h\text{-}SL_2\text{-}A_2\text{-}a_h)]}{2(b_h - a_h)}, \quad (36)$$

where $B_h \ge SL_2 + A_2 + a_h$. Thus, the total expected daily penalty cost $E[TP_h] = \sum_{i=1}^{4} E[P_{i,h}]$, using (33)-(36), is given by

$$E[TP_h]=\frac{1}{2(b_h-a_h)}\left[\begin{array}{l}\alpha_1A_1^2+\alpha_2A_2^2+(SL_1-A_1+b_h-B_h)\left[\begin{array}{l}2\alpha_1A_1+\\ \beta_1(SL_1-\\ A_1+b_h-B_h)\end{array}\right]\\ +(B_h-SL_2-A_2-a_h)\left[2\alpha_2A_2+\beta_2(B_h-SL_2-A_2-a_h)\right]\end{array}\right], \quad (37)$$

where $SL_2+A_2+a_h \le B_h \le SL_1-A_1+b_h$. We can write (37) as $E[TP_h]=G_h+L_h$, where

$$G_h=\frac{1}{2(b_h-a_h)}\left[(\beta_1+\beta_2)B_h^2-2B_h\left[\begin{array}{l}\alpha_1A_1-\alpha_2A_2+\\ \beta_1(SL_1-A_1+b_h)+\\ \beta_2(SL_2+A_2+a_h)\end{array}\right]\right], \quad (38)$$

and

$$L_h=\frac{1}{2(b_h-a_h)}\left[\begin{array}{l}\alpha_1A_1^2+\alpha_2A_2^2+(SL_1-A_1+b_h)[2\alpha_1A_1+\\ \beta_1(SL_1-A_1+b_h)]-(SL_2+A_2+a_h)\\ [2\alpha_2A_2-\beta_2(SL_2+A_2+a_h)]\end{array}\right].$$

Since $L_h$is a constant, it will have no impact on solving the stochastic scheduling problem to optimality and, thus, it will be omitted. The proposed mixed-integer nonlinear program, denoted by SVSM-U, for our vessel transportation-inventory problem with uniform demands is stated next, noting that (31) must be satisfied as discussed above. This model has a nonlinear objective function (which is shown to be convex) and linear constraints; therefore, it yields a global optimal solution.

SVSM-U: Minimize

$$\sum_{h\in H}\sum_{t\in T}\sum_{n=1}^{TN_t}C_{t,n}X_{h,t,n}+\sum_{h\in H}G_h+\sum_{t\in T}\sum_{n=O_t+1}^{TN_t}\$_{t,n}V_{t,n}$$

where $G_h$is given by (38),

subject to (11)-(19), (32), and $B_h \ge b_h$, $\forall h \in H$, where B_h is defined by (21).

4.2 Storage levels and expected penalties

As investigated for SVSM-G (see Section 3.2), we can also show that if $S_h = B_h - TC_h \ge 0$, $h \in H$, whereB_h is given by (21), falls with certainty within one of the five storage intervals, then the expected total daily penalty $E[TP_h]$ of SVSM-U reduces to the total daily penalty $TP_h$of DVSM in which the deterministic S_h is set equal to $E[S_h]=B_h-E[TC_h]=B_h-(a_h+b_h)/2$.

COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSES

This section presents some computational experiments and analysis for solving the proposed stochastic models SVSM-G and SVSM-U and their deterministic counterpart DVSM. Sensitivity analyses is also performed to assess the impacts of changes in the probabilities for meeting demands (*i.e.*, service levels) as well as fluctuations in the variations of demands on the optimal expected overall costs v^*(SVSM-G) and v^*(SVSM-U).

5.1 Test problems and randomly generated instances

We construct twelve test problems, denoted by TP_1-TP_{12}, based on which, we randomly generate one hundred instances of every problem for each of the models SVSM-G and SVSM-U. Tables 1 and 2respectively present the input data that are common to TP_1-TP_{12} and additional specific data for each test problem. We consider three types of vessels, *i.e.*, T = {1,2,3} (see Table 1), assuming that $OC_t = OC_{t,n}$, $UD_t = UD_{t,n}$, and $\$_t = \$_{t,n}$, $t \in T$, $\forall n$.

Four time horizons are chosen as indicated in Table 2, namely $|H|$ = 30, 60, 90, and 120 days which are respectively used for the set of test problems given by TP_1-TP_3, TP_4-TP_6, TP_7-TP_9, and TP_{10}-TP_{12}. For example, in TP_1 the fleet is composed of only three self-owned and three chartered vessels of type 1 (*i.e.*, (O_1,O_2,O_3) = (3, 0, 0) and (CH_1,CH_2, CH_3) = (3, 0, 0)), noting that (UD_1,UD_2,UD_3) are randomly sampled from ([19,21],[0,0],[0,0]). Also, specific data related to the gamma and uniform demands are provided in Table3. The main objective to is to ensure that the daily storage levels S_h, $h \in H$, fall within $[SL_1, SL_2]$ with the probability $(1-r_h)$ of at least 0.85 (i.e., 85% service level).

Table 1. Intervals for randomly generating the common input data for test problems TP_1-TP_{12}

$\|T\|$	3
(OC_1,OC_2,OC_3)	([5,500, 6,500], [7,500, 8,500], [9,500, 10,500])
$(\$_1,\$_2,\$_3)$	([2,000,000, 2,500,000], [3,000,000, 3,500,000], [4,000,000, 4,500,000])
(D_1, D_2, D_3)	([1, 3], [4, 6], [7, 9])
Q	[400, 600]
α_1	[15, 25]
α_2	[15, 25]
LB	[0, 10]
UB	[1,900, 2,100]
β_1	[90, 110]
β_2	[90, 110]

We solve SVSM-G based on the generated twelve hundredinstances. The twelve hundred instances of DVSM relative to SVSM-G are obtained by setting the deterministic daily demands of DVSM equal to the expected daily demands k_h/λ

Table 2. Additionalspecific input data for TP_1-TP_{12}

TP	$\|H\|$	(O_1,O_2,O_3)	(CH_1,CH_2,CH_3)	(UD_1,UD_2,UD_3)
TP_1	30	(3,0,0)	(3,0,0)	([19,21], [0,0], [0,0])
TP_2	30	(2,1,0)	(2,1,0)	([19,21], [23,25], [0,0])
TP_3	30	(1,1,1)	(1,1,1)	([19,21], [23,25], [27,29])
TP_4	60	(4,0,0)	(4,0,0)	([42,46], [0,0], [0,0])
TP_5	60	(2,2,0)	(2,2,0)	([42, 46], [48,52], [0,0])
TP_6	60	(1,1,2)	(1,1,2)	([42,46], [48,52], [54,58])
TP_7	90	(5,0,0)	(5,0,0)	([68,72], [0,0], [0,0])
TP_8	90	(3,2,0)	(3,2,0)	([68,72], [74,78], [0,0])
TP_9	90	(1,1,3)	(1,1,3)	([68,72], [74,78], [80,84])
TP_{10}	120	(6,0,0)	(6,0,0)	([94,98], [0,0], [0,0])
TP_{11}	120	(3,3,0)	(3,3,0)	([94,98], [100,104], [0, 0])
TP_{12}	120	(1,2,3)	(1,2,3)	([94,98], [100,104], [106,110])

Table 3. Specific input data for TP_1-TP_{12} with gamma and uniform demands

	G(k_h, λ)	U(a_h, b_h)
(k_h, λ) [(a_h, b_h)]	([1, 4], [1/140, 1/160])	([90$\sqrt{h}$, 110$\sqrt{h}$], [190$\sqrt{h}$, 210$\sqrt{h}$])
w	[610, 620]	[1,100, 1,130]
(Ω_1, Ω_2, Ω_3)	([4, 6], [9, 11], [14, 16])	([35, 45], [55, 65], [75, 85])
SL_1	[15, 25]	[920, 940]
SL_2	[580, 620]	[980, 1,020]
A_1	[3, 7]	[8, 12]
A_2	[3, 7]	[8, 12]

for SVSM-G. Similarly, the twelve hundred instances of SVSM-U and their respective instances of DVSM are solved, where the deterministic cumulative daily demands of DVSM were set equal to the expected cumulative daily demands (a_h+b_h)/2for SVSM-U.

5.2Computational experience related to solving SVSM-G, SVSM-U, and DVSM

We used a HP Envy Beats 23-n010 computer with Intel(R) Core(TM) i7-4785T CPU, 2.2 GHz, 4 GB of RAM, 32-bit OS. We utilized GAMS-24.7 with Conopt3 (as MINLP solver) and Cplex Optimizer-12.6.1 to solve the generated instances of SVSM-G and SVSM-U and their deterministic counterparts DVSM. The termination limits were set at 1,000,000 iterations and the convergence criteria were set at the default "Global Optimality" for both GAMS and Cplex.

Our computational results are organized in Tables 4-5. Thefirst column of each table represents the twelve test problems. Columns 2-5 show summary statistics such as the minimum, maximum, median, and average of the percentages of the over-costs of the stochastic model relative to its deterministic counterpart DVMS obtained based on solving the instances of each test problem. (The percentage of over-cost for a stochastic model SM, where SM includes SVSM-G and SVSM-U, is defined as 100[v*(SM)-v*(DVSM)]/v*(DVSM) where v*(SM) and v*(DVSM) are the optimal objective values of SM and DVSM models, respectively.) Columns 6-7 of Tables4 and 5report the average CPU times (in seconds) for solvingSVSM-G and SVSM-U as well as their DVSM counterparts, respectively. We used the one-tailed or two tailed Wilcoxon rank-sum tests to statistically support the following conclusions.

- The over-cost of each stochastic model relative to its deterministic counterpart is significantly larger than zero (p-value < 0.00001, one-tailed test).
- The over-costs among instances with the same time horizon are not significantly different (p-value > 0.2194, two-tailed test); thus, the configurations of self-owned and chartered vessels have no impact on the over-cost.
- An increase in the days of time horizon leads to an increase in the over-cost (p-value < 0.0001, one-tailed test).
- The over-costs of SVSM-G and SVSM-U models relative to their deterministic counterparts are significantly different (p-value < 0.0001, two-tailed test).
- Any increase in the number of days in the time horizon leads to an increase in the CPU times of stochastic and deterministic models (p-value < 0.00001, one-tailed test).

5.3 Sensitivity Analyses

We have investigated the effects of changes in the probabilities (1-r_h) (i.e., service levels) for meeting the daily demands on the over-cost of test problems TP_1-TP_{12} associated with SVSM-G. The results ofTable 6 show that an increase in (1-r_h) triggers an increase in the over-cost (p-value < 0.0001, one-tailed test). Thus, higher probabilities for meeting demands consistently lead to higher over-costs. (Similar results can be obtained forSVSM-U.)

We have also examined the effects of changes in the variations of demands on the over-costs for SVSM-G.Table 7 shows the results based on test problems TP_1-TP_{12} in which the coefficients of variation (CV) for gammademands, givenby $k_h^{-1/2}$, are randomly sampled from various intervals. The results indicate that any decrease in demand variations diminishes the over-cost (p-value <0.0001, one-tailed test). When the demand variations reduce to zero, the over-cost will also decrease to zero, that is, v*(SVSM-G) reduces to v*(DVSM). Hence, variations in demands significantly affect the expected total cost.(Similar results can be obtained forSVSM-U.)

Table 4. Summary statistics on the percentages of over-costs of v*(SVSM-G) relative to v*(DVSM), and the average CPU times (in seconds) for SVSM-G and DSVM

TP	Over-cost (%)				CPU time	
(1)	Min (2)	Max (3)	Med (4)	Avg (5)	SVSM-G (6)	DVSM (7)
TP_1	2.31	38.15	17.83	19.52	2.512	0.443
TP_2	2.69	37.29	17.25	18.86	2.491	0.502
TP_3	2.55	39.08	17.98	20.42	2.705	0.528
TP_4	6.49	65.18	25.31	28.59	18.246	0.892
TP_5	6.71	62.41	26.52	27.15	18.739	0.887
TP_6	7.12	67.58	26.89	29.25	18.582	0.923
TP_7	10.67	94.22	40.24	45.66	59.803	2.691
TP_8	11.82	99.41	42.53	47.39	60.317	2.847
TP_9	11.63	104.06	43.37	47.85	60.684	2.809
TP_{10}	17.55	99.85	57.89	64.81	120.162	5.325
TP_{11}	18.69	109.11	59.75	66.52	122.518	5.641
TP_{12}	17.46	107.28	59.43	65.95	121.905	5.297

SUMMARY AND CONCLUSIONS

In this paper, we have investigated a stochastic scheduling transportation-inventory problem to ship crude oil from a port to an overseas storage facilitybya set of heterogeneous fleet of self-owned and time-chartered vessels over a finite time horizon. The demands at a destination have gamma or uniform distributions.A dailypenalty structure, including two penalty types for the shortages and two penalty types for the excesses in storage levels are adopted to respectively deal with moderate and extreme violations of some specified desirable storage levels.

The goalis to minimize the expected total cost, which includes the vessel operational expenses, expected penalty for violating the lower and upper bounds on storage levels, and the chartering expenses, tofind the optimal decisions related to the vessel schedules and the storage levels in order to meet the stochastic demands with acceptable levels ofreliability. We have formulated the problems with gamma and uniform demands as stochasticoptimization models andthen have converted them into exact

Table 5. Summary statistics on the percentages of over-costs of v^*(SVSM-U) relative to v^*(DVSM), and the average CPU times (in seconds) for SVSM-U and DSVM

TP	Over-cost (%)				CPU time	
	Min	Max	Med	Avg	SVSM-G	DVSM
(1)	(2)	(3)	(4)	(5)	(6)	(7)
TP_1	1.93	34.22	14.29	16.43	2.134	0.412
TP_2	1.85	34.31	14.43	16.35	2.108	0.514
TP_3	1.79	36.55	14.57	15.79	2.395	0.523
TP_4	5.47	61.35	23.52	25.47	16.302	0.742
TP_5	5.26	58.92	24.65	25.35	16.471	0.728
TP_6	5.18	62.48	24.51	26.22	17.210	0.759
TP_7	9.39	91.14	39.42	42.28	56.823	2.153
TP_8	10.12	96.47	40.68	43.59	57.458	2.245
TP_9	10.29	101.52	41.29	43.34	58.290	2.487
TP_{10}	12.34	93.88	55.96	59.34	116.364	4.428
TP_{11}	12.65	104.41	57.12	60.57	118.723	4.629
TP_{12}	13.43	101.28	57.43	61.85	117.309	4.802

Table 6. Percentages of over-costs of v^*(SVSM-G) relative to v^*(DVSM) when $(1\text{-}r_h)$ take various values

TP	$(1\text{-}r_h)$				
	0.80	0.85	0.90	0.95	0.99
TP_1	15.58	19.52	21.13	23.42	25.63
TP_2	14.12	18.86	20.49	23.65	25.78
TP_3	16.83	20.42	22.33	24.52	26.65
TP_4	23.45	28.59	32.25	34.19	37.26
TP_5	23.93	27.15	31.67	33.48	35.92
TP_6	24.05	29.24	33.59	34.21	36.57
TP_7	39.92	45.66	49.04	52.35	54.02
TP_8	40.65	47.39	49.13	51.97	54.74
TP_9	41.56	47.85	50.35	52.63	55.29
TP_{10}	57.34	64.81	67.29	69.88	71.05
TP_{11}	58.25	66.52	69.42	71.08	73.37
TP_{12}	58.94	65.95	68.02	70.23	72.94

Table 7. Over-cost percentages of v^*(SVSM-G) relative to v^*(DVSM) when CVs of gammademands are randomly from various intervals

TP	CV of gamma demands			
	[0.25, 0.277]	[0.289, 0.224]	[0.354, 0.447]	[0.5, 1]
TP_1	8.27	12.17	16.36	19.52
TP_2	8.15	12.35	15.49	18.86
TP_3	8.85	13.48	16.24	20.42
TP_4	11.48	19.82	24.53	28.59
TP_5	12.62	20.95	24.25	27.15
TP_6	11.28	19.73	25.84	29.24
TP_7	27.36	34.58	40.32	45.66
TP_8	27.18	35.29	41.18	47.39
TP_9	28.07	35.12	40.26	47.85
TP_{10}	44.11	53.06	58.65	64.81
TP_{11}	44.02	54.25	59.21	66.52
TP_{12}	45.18	53.14	60.17	65.95

mixed-integer nonlinear programs SVSM-G and SVSM-U. It is shown that when either the random storage levels fall with certainty within one of the allowable intervals for the penalty structure or there are no variations in demands, SVSM-G and SVSM-U reduce to their deterministic counterparts. Our experiments have indicated thatuncertainties in demands have substantial effects on the total expected cost. Discarding uncertainties in demands is likely to underestimate the total cost and provide unrealistic results. Also the nature of the demand distribution significantly impacts the expected total cost.Future research should focus on probabilistic vessel loading/unloading and travel times, and partially loaded/unloaded vessels.

REFERENCES

References are available upon request.

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