STABILITY BEHAVIOR OF TWOINCLINED ELECTRIFIED FILMS IN THE PRESENCE OF INSOLUBLE SURFACTANTS AND HEAT TRANSFER

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Abstract: The mathematical model for two falling viscoelastic electrified films confined between two rigid plates in the presence of interfacial insoluble surfactants has been derived and investigated. The corresponding boundary conditions at the rigid substrates and interface between the two layers have been derived and illustrated. The non-dimensional equations and conditions for the imposed model have been done. The parameters governing the flow system, such as Marangoni, Reynolds, elasticity numbers and the inclined substrate have been defined.

Keywords: Films, Surfactants, Viscoelastic Fluids, Electric field, Porous Media.

I. INTRODUCTION

The surfactants effect in multiphase systems has a significant influence on the shape and the motion of the interfaces. A wide range of applications of surfactants in the oil industry, for example in the cleanup of oil spills and for enhanced oil recovery. This has motivated the increasing interest to understand the effect of surfactants on the interfacial behavior in recent years.

The mathematical model for two falling viscoelastic films confined between two rigid plates in the presence of interfacial insoluble surfactants is developed. The effect of the surfactants on the stability of the fluids flow under the effect of the normal electric field and heat transfer is taken into account.

The present article is organized as follows. The next section lays down the equations describing the motion and the corresponding boundary conditions. In the third section the basic flow solutions and perturbation equations are obtained. The fourth section includes the case of small Reynolds number, in which a seventh order dispersion relation is derived. Finally, the results of the present study are outlined in the last section.

II. TECHNICAL CONTENT

2.1. Governing Equations

Two heated viscoelastic fluid films flowing down an inclined substrate with an angle β with respect to the horizontal in a porous domain are considered. Free charges are accumulated at the interface due to the driving force of normal electric field.

The material properties of the two films are different, so we will use the superscripts 1 and 2 to refer quantities pertaining to the lower and upper film, respectively.A Cartesian coordinate system *(x,y),* is selected such that *x*−axis is parallel along the flow and *y*−axis perpendicular to the plane films. The lower plate is at *y=0* and the unperturbed interface is at *y=h,* while the upper plate is located at *y=H.*

In a viscoelastic nature, the non-dimensional fundamental equations governing the unsteady motion for an incompressible fluid can be listed as [1-5]:

$$\frac{\partial u^{(j)}}{\partial x}+\frac{\partial v^{(j)}}{\partial y}=0, \quad j=1,2, \tag{1}$$

$$R_e^{(j)}\left(1+D_e^{(j)}\frac{\partial}{\partial t}\right)\left\{\frac{\partial u^{(j)}}{\partial t}+u^{(j)}\frac{\partial u^{(j)}}{\partial x}+v^{(j)}\frac{\partial u^{(j)}}{\partial y}\right\}=$$
$$-\mu^{1-j}\left(1+D_e^{(j)}\frac{\partial}{\partial t}\right)\frac{\partial p^{(j)}}{\partial x}+\left(1+\Lambda^{(j)}D_e^{(j)}\frac{\partial}{\partial t}\right)\nabla^2 u^{(j)}$$
$$-\frac{1}{D_a^{(j)}}\left(1+\Lambda^{(j)}D_e^{(j)}\frac{\partial}{\partial t}\right)u^{(j)}+\mu^{1-j}\rho^{j-1}, \tag{2}$$

$$R_e^{(j)}\left(1+D_e^{(j)}\frac{\partial}{\partial t}\right)\left\{\frac{\partial v^{(j)}}{\partial t}+u^{(j)}\frac{\partial v^{(j)}}{\partial x}+v^{(j)}\frac{\partial v^{(j)}}{\partial y}\right\}=$$
$$-\mu^{1-j}\left(1+D_e^{(j)}\frac{\partial}{\partial t}\right)\frac{\partial p^{(j)}}{\partial y}+\left(1+\Lambda^{(j)}D_e^{(j)}\frac{\partial}{\partial t}\right)\nabla^2 v^{(j)}$$
$$-\frac{1}{D_a^{(j)}}\left(1+\Lambda^{(j)}D_e^{(j)}\frac{\partial}{\partial t}\right)v^{(j)}+\mu^{1-j}\rho^{j-1}\cot\beta, \tag{3}$$

$$P_e^{(j)}\left\{\frac{\partial T^{(j)}}{\partial t}+u^{(j)}\frac{\partial T^{(j)}}{\partial x}+v^{(j)}\frac{\partial T^{(j)}}{\partial y}\right\}=\nabla^2 T^{(j)} \tag{4}$$

$$\frac{\partial^2 \chi^{(j)}}{\partial x^2}+\frac{\partial^2 \chi^{(j)}}{\partial y^2}=0. \tag{5}$$

In these equations, the symbols *(u, v)* refer the components of the velocity vector**u**, *t* denotes the time, ρ is the density ratio, μ refers to the dynamical viscosity ratio, *p* is the pressure and the parameter Λ is the time constant ratio. The quantities

T and χ distinguish respectively the heat and the electric potential.Further R_e is the Reynolds number and D_e is the Deborah number, while D_a and D_e are respectively the Darcy and Peclet numbers.

2.2. Boundary Condition

In order to close the above system of the governing equation, the following boundary and interfacialconditions are imposed [6-10]

$$\boldsymbol{u}^{(j)} = 0 \ \ at \ \ y = 0, \ \boldsymbol{u}^{(j)} = 0 \ \ at \ \ y = 1, \quad (6)$$

$$T^{(j)} = 1 \ \ at \ \ y = 0, \ \ T^{(j)} = 0 \ \ at \ \ y = 1, \quad (7)$$

$$\chi^{(j)} = 1 \ \ at \ \ y = 0, \ \ \chi^{(j)} = 0 \ \ at \ \ y = 1, \quad (8)$$

$$v^{(j)} = \frac{\partial \xi}{\partial t} + u^{(j)} \frac{\partial \xi}{\partial x} \ \text{ at } \ y = \xi(x,t) \quad (9)$$

$$v^{(1)} - v^{(2)} = (u^{(1)} - u^{(2)}) \frac{\partial \xi}{\partial x} \ \text{ at } \ y = \xi(x,t) \quad (10)$$

$$u^{(1)} - u^{(2)} = (v^{(2)} - v^{(1)}) \frac{\partial \xi}{\partial x} \ \text{ at } \ y = \xi(x,t) \quad (11)$$

$$T^{(1)} = T^{(2)} \quad \text{at } \ y = \xi(x,t) \quad (12)$$

$$\frac{\partial T^{(1)}}{\partial y} - \kappa \frac{\partial T^{(2)}}{\partial y} = \left(\frac{\partial T^{(1)}}{\partial x} - \kappa \frac{\partial T^{(2)}}{\partial x}\right) \frac{\partial \xi}{\partial x} \ \text{ at } \ y = \xi(x,t) \quad (13)$$

$$\frac{\partial}{\partial t}(N\Gamma) + \frac{\partial}{\partial x}(N\Gamma u^{(1)}) = \frac{1}{P_{es}} \frac{\partial}{\partial x}\left(\frac{1}{N} \frac{\partial \Gamma}{\partial x}\right) \text{at } \ y = \xi(x,t), \quad (14)$$

$$N = \sqrt{1 + (\partial \xi / \partial x)^2},$$

$$\frac{\partial \chi^{(1)}}{\partial y} - \varepsilon \frac{\partial \chi^{(2)}}{\partial y} - \left(\frac{\partial \chi^{(1)}}{\partial x} - \varepsilon \frac{\partial \chi^{(2)}}{\partial x}\right) \frac{\partial \xi}{\partial x} = Nq \ \text{ at } \ y = \xi(x,t) \quad (15)$$

$$\frac{\partial \chi^{(1)}}{\partial x} - \frac{\partial \chi^{(2)}}{\partial x} + \left(\frac{\partial \chi^{(1)}}{\partial y} - \frac{\partial \chi^{(2)}}{\partial y}\right) \frac{\partial \xi}{\partial x} = 0 \ \text{ at } \ y = \xi(x,t) \quad (16)$$

$$\frac{\partial q}{\partial t} + u^{(1)} \frac{\partial q}{\partial x} - N^{-2} q \left\{ \frac{\partial u^{(1)}}{\partial x} \left(\frac{\partial \xi}{\partial x}\right)^2 - \left[\frac{\partial u^{(1)}}{\partial y} + \frac{\partial v^{(1)}}{\partial x} \right] \frac{\partial \xi}{\partial x} + \frac{\partial v^{(1)}}{\partial y} \right\} = N^{-1} \left\{ S^{(1)} \left(\frac{\partial \chi^{(1)}}{\partial y} - \frac{\partial \chi^{(1)}}{\partial x} \frac{\partial \xi}{\partial x} \right) - S^{(2)} \left(\frac{\partial \chi^{(2)}}{\partial y} - \frac{\partial \chi^{(2)}}{\partial x} \frac{\partial \xi}{\partial x} \right) \right\} \quad (17)$$

Here, in the previous conditions ξ is the elevation of the unperturbed interface, κ is the thermal conductivity ratio, ε is the dielectric constant ratio, P_{es} denotes surfactant surface Peclet number, q is the surface charge and S refers to dimensionless electrical conductivity. Beside these conditions the continuity of the normal and tangential stress tensors is involved [11]. The above governed equations and the corresponding conditions are made dimensionless by the height of channel H as the characteristic length and $U_c = (\rho^{(1)} g H^2 \sin\beta) / \mu^{(1)}$ as a characteristic velocity scale in the channel, g is the gravity.

STABILITY ANALYSIS

In dealing with the linear stability of the above system of the governing equations and the associated boundary conditions. Each quantity f is analyzed into two parts, the basic state quantity f_0 and a perturbation profile f_1. Hence we obtained base state solutions further the perturbed solutions to give the dispersion relation between the wave number and the corresponding growth rate:

$$f = f_0 + f_1. \quad (18)$$

By substituting the decomposition of each of the unknowns into the governing equations and boundary conditions, the basic and the perturbation equations and hence the solutions are obtained

3.1. Base State Solutions

The solutions of the basic equations for the velocity, heat and the electric potential are respectively given by

$$\frac{\partial^2 u_0^{(j)}(y)}{\partial y^2} - \frac{1}{D_a^{(j)}} u_0^{(j)}(y) + \mu^{1-j} \rho^{j-1} = 0, \quad (19)$$

$$\frac{\partial^2 T_0^{(j)}(y)}{\partial y^2}=0,\quad \frac{\partial^2 \chi_0^{(j)}(y)}{\partial y^2}=0, \quad (20)$$

which have the solutions

$$u_0^{(j)}=a_1^{(j)}e^{-\frac{1}{\sqrt{D_a^{(j)}}}y}+a_2^{(j)}e^{\frac{1}{\sqrt{D_a^{(j)}}}y}+\mu^{1-j}\rho^{j-1}D_a^{(j)}, \quad (21)$$

$$T_0^{(1)}=1-\frac{\kappa y}{1+(\kappa-1)r}, \quad (22)$$

where, $r=h/H$ is the thickness ratio,

$$T_0^{(2)}=\frac{1-y}{1+(\kappa-1)r}, \quad (23)$$

$$\chi_0^{(1)}=1-\frac{(\varepsilon+(r-1)q_0)y}{1+(\varepsilon-1)r}, \quad (24)$$

$$\chi_0^{(2)}=\frac{(1+r\,q_0)(1-y)}{1+(\varepsilon-1)r}, \quad (25)$$

where the coefficients $a_1^{(j)}$, $a_2^{(j)}$ are obtained depending on the previous conditions

3.2. Perturbed Flow and Solutions

The solution of the above system of governing equations and boundary conditions can be obtained in terms of the stream function:

$$u_1^{(j)}=\partial\psi_1^{(j)}/\partial y,\ v_1^{(j)}=-\partial\psi_1^{(j)}/\partial x \quad (26)$$

In a two-dimensional flow, we consider a harmonic displacement

$$f_1=\hat{f_1}(y)e^{ik(x-ct)}+\text{complex conjugate} \quad (27)$$

Finally, we can derive the equations governing the linear stability as

$$\frac{d^4\hat{\psi}_1^{(j)}}{dy^4}-\left(2k+\frac{1}{D_a^{(j)}}\right)\frac{d^2\hat{\psi}_1^{(j)}}{dy^2}+k^2\left(k^2+\frac{1}{D_a^{(j)}}\right)\times\hat{\psi}_1^{(j)}=\frac{ikR_e^{(j)}}{L^{(j)}(c)}\left\{(u_0^{(j)}-c)\left(\frac{d^2\hat{\psi}_1^{(j)}}{dy^2}-k^2\hat{\psi}_1^{(j)}\right)-\frac{d^2u_0^{(j)}}{dy^2}\hat{\psi}_1^{(j)}\right\} \quad (28)$$

$$\frac{d^2\hat{T}_1^{(j)}}{dy^2}-k^2\hat{T}_1^{(j)}=ikP_e^{(j)}\left\{(u_0^{(j)}-c)\hat{T}_1^{(j)}-\frac{d\hat{T}_1^{(j)}}{dy}\hat{\psi}_1^{(j)}\right\}, \quad (29)$$

$$\frac{d^2\hat{\chi}_1^{(j)}}{dy^2}-k^2\hat{\chi}_1^{(j)}=0, \quad (30)$$

where,

$$L^{(j)}(c)=\left(1-ickD_e^{(j)}\right)^{-1}\left(1-ick\,\Lambda^{(j)}D_e^{(j)}\right) \quad (31)$$

In the following we will solve this system in the limit of small Reynoldnumber.

SMALL REYNOLDS NUMBER APPROXIMATION

Before dealing with the case of small Reynolds number, it is useful to express the Reynolds bythe lower layersuch that

$$R_e^{(j)}=\mu^{1-j}\rho^{j-1}R_e. \quad (32)$$

Assuming that, the variables such as the stream function, heat, electric potential and the wave speed are expressed in terms of small Reynolds number as

$$\hat{f}_1^{(j)}(y)=\hat{f}_{1[0]}^{(j)}(y)+R_e\hat{f}_{1[1]}^{(j)}(y)+\square(R_e^2) \quad (33)$$

The solutions in zeroth order Reynolds number

$$\hat{\psi}_{1[0]}^{(j)}=A_1^{(j)}\cosh(ky)+A_2^{(j)}\cosh\left(\sqrt{\frac{1+k^2D_a^{(j)}}{D_a^{(j)}}}\,y\right)+A_3^{(j)}\sinh(ky)+A_4^{(j)}\sinh\left(\sqrt{\frac{1+k^2D_a^{(j)}}{D_a^{(j)}}}\,y\right), \quad (34)$$

$$\hat{T}_{1[0]}^{(j)}=B_1^{(j)}\cosh(ky)+B_2^{(j)}\sinh(ky), \quad (35)$$

$$\hat{\chi}_{1[0]}^{(j)}=C_1^{(j)}\cosh(ky)+C_2^{(j)}\sinh(ky). \quad (36)$$

When the solutions of the stream function, electric potential, and heat, are put into the associated boundary conditions, we have a linear homogeneous system of algebraic equations, which can be written in the following matrix form

$$\square\times\square=\mathbf{0}. \quad (37)$$

When determinant of matrix $\square$ set to be zero, we get a dispersion equation of seventh order between the wavenumber k and the perturbation frequency $c$for specified values of other parameters:

$$D(c,k)=c_{[0]}^{7}+(a_1+ib_1)c_{[0]}^{6}+(a_2+ib_2)c_{[0]}^{5}$$
$$+(a_3+ib_3)c_{[0]}^{4}+(a_4+ib_4)c_{[0]}^{3}+(a_5+ib_5)c_{[0]}^{2}$$
$$+(a_6+ib_6)c_{[0]}+(a_7+ib_7)=0 \qquad (38)$$

where the coefficients that appear in this equation are clear from the context.

The parts of Figs. 1 and 2investigate the stability behavior of some physical parameters (Marangoni, elasticity numbers and electrical conductivity for the lower and the above layer). In these figures the growth rate from relation (38) is plotted against the thickness ratio of the sheet films, in which the stability behavior corresponds to negative values of the disturbance growth rate

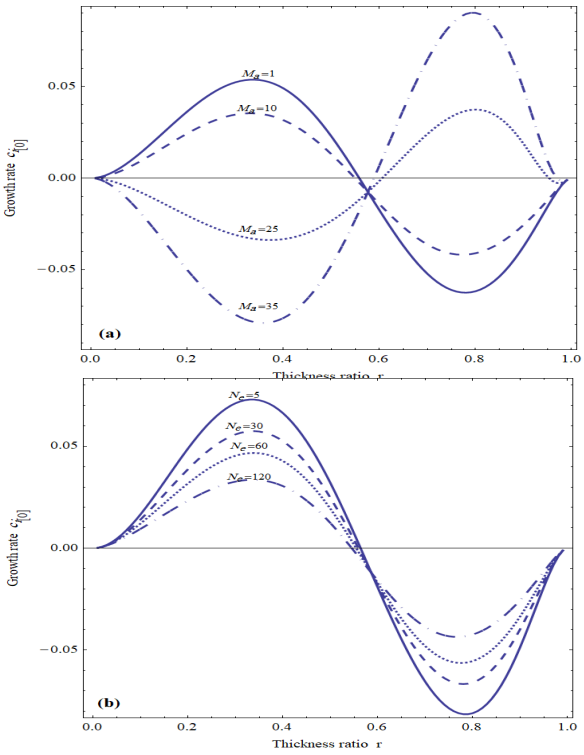
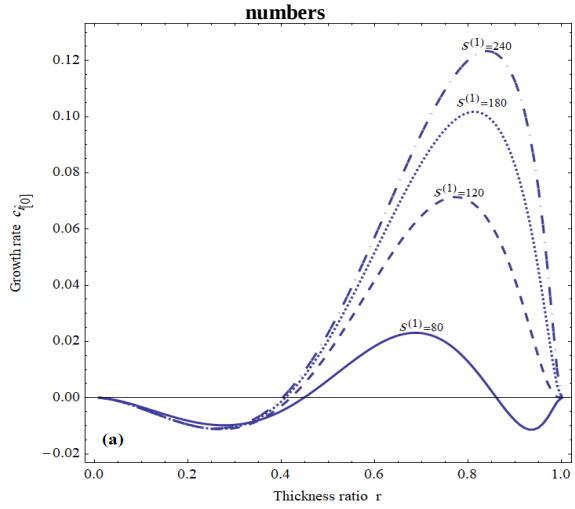


Fig.1. (a & b) Growth rate $c_{i[0]}$ of different fluids versus the thickness ratio*r*, for variation of Marangoni and elasticity numbers



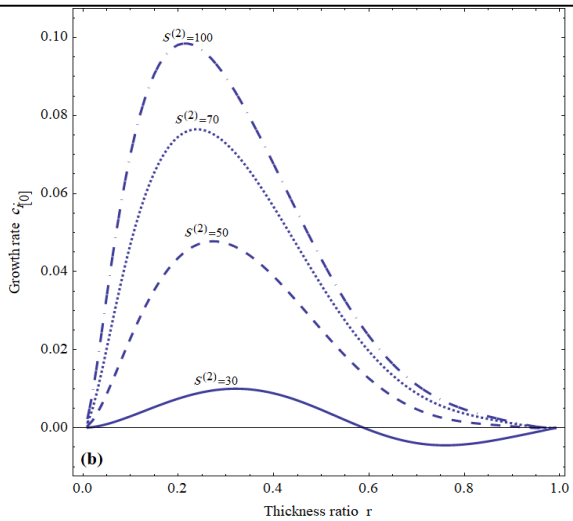


Fig.2. (a & b) Growth rate as a function of the thickness ratio for different values of electrical conductivity in the two films.

An inspection of the pictures of these graphs shows that, broadly speaking, the fluid sheet is stable for increasing both Marangoni, elasticity numbers, while the inverse holds for electrical conductivity in both layers.

CONCLUSIONS

In this work, we have illustrated the linear stability of the contaminated interface between two thin films flow of a viscoelastic electrified fluid inside an inclined impermeable channel.

The stability stage is investigated for the case of the small Reynolds number, in which we reached a seventh order dispersion relation satisfied by the growth rate and wave number depending on the other physical parameters.

Numerical results were performed graphically to determine the growth rate against the thickness ratio at different Marangoni and elasticity numbers as well as electrical conductivity for the lower and the above layer. It is found that the motion is stable for increasing both Marangoni and elasticity numbers, while the opposite behavior is happened for enlarging the electrical conductivities.

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