RELIABILITY ANALYSIS IN 2-COMPONENT STANDBY REDUNDANT SYSTEM WITH PRIORITY THROUGH MAXIMUM ENTROPY PRINCIPLE

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Abstract - Ina previous study, under the situation that only the average and variance of failure and repair time of each component are provided, the mean time to failure(MTTF) and variance of failure time in a 2-component standby redundant system with priority have been formulated. In this study, we addressa method for evaluating the reliability of the system using MTTF and variance of failure time. In particular, based on maximum entropy principle, we derive the reliability function.

Index Terms - method of Lagrange, maximum entropy principle, system reliability, 2-component standby redundant system with priority

I. INTRODUCTION

In the reliability filed, redundancy of systems is a common method of improving system reliability and availability. Then, a 2-component redundant system with priority is known as one of the most fundamental redundant systems. In this study, we consider a 2-component standby redundant system composed of Component 1 having the priority and Component 2 which is the standby component. Buzacott [1] and Osaki [2, 3] have derived the evaluation method of mean time to failure (MTTF) of the entire system for the 2-component redundant system with priority. Note that any probability density functions (PDFs) and/or cumulative distribution functions (CDFs) about failure and repair times in the respective components are required to evaluate MTTF of the entire system on using the formulae derived by Buzacott and Osaki. However, in practical situations, it might be difficult to know the exact probability functions for failure and repair times in the respective components.

In recent years, Take moto and Arizono [4] have proposed the evaluation method of MTTF in the 2-component standby redundant system with priority under the limited information that only averages and variances about failure and repair times in the respective components are provided. Furthermore, through expanding the evaluation procedure considered by Take moto and Arizono [4], Oigawa et al. [5] have proposed the evaluation method of the variance of failure time in the 2-component standby redundant system with priority in the case that only averages and variances about failure and repair times of each component are provided. It is clear that MTTF is effective as an index for evaluating the reliability of the system. Similarly, the variance of failure time in the 2-component standby redundant system with priority is also an important index. Through the above researches, we can evaluate MTTF and variance of failure time in the entire system quantitatively. Then, we can evaluate qualitatively the reliability of a 2-component standby redundant system with priority by MTTF and variance of failure time in the entire system.

In this study, we consider a reliability function in order to evaluate quantitatively the system reliability in the situation that only averages and variances about failure and repair times in the respective components are provided as limited information. We consider developing the probability density function about failure time in the entire system based on the average and variance of failure time in the entire systeme valuated by Takemoto and Arizono [4] and Oigawa et al. [5]. In concrete, the maximum entropy principle is adopted to identify aprobability density function about failure time in the entire system under the constraints of the average and variance of failure time in the entire system, and then, the system reliability function is developed.Remark that the procedure based on the maximum entropy principle in the information theory is widely known as an elegant approach for deriving aprobability density function under someconstraints of the values of moments in aprobability distribution.

II. 2-COMPONENT STANDBY REDUNDANT SYSTEM WITH PRIORITY

In this section, details of 2-component standby redundant system with priority are described. Suppose the 2-component standby redundant system composed of Component 1 having priority and Component 2 which is a standby component. The priority component denotes a component which is always used whenever it is available. If the priority component falls in failure, it starts to be repaired immediately and the standby component starts to operate. When the repair of the priority component is completed, the priority component operates immediatelyand the standby component returns to standby state. On the other hand, if the standby component falls in failure

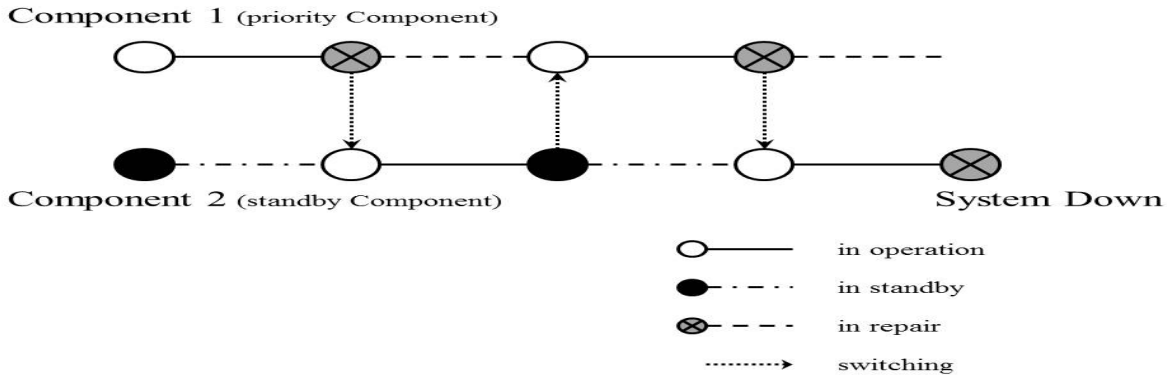


Fig. 1 2-Component Standby Redundant System with priority

during repairing the priority component, the entire system becomes down. That is, the standby component denotes a component which stands by whenever the component with priority is available.Initially, at time $t = 0$, Component 1 starts operation, and Component 2 is in the standby state.1) Fig.1 illustrates the outline of behaviour in this system. The solid line represents the operating state of each component,the dashed line represents the repair state of Component 1, and the dashed-dotted line represents2) the standby state of Component 2. In addition, the dotted arrowhead connecting Component 1 and3) Component 2 represents the switching of a component on operation. Note that the standby time of the Component 2 is not included in its lifetime. Also, it is assumed that switching time is negligibly short.

III. OUTCOME IN PREVIOUS LITERATURE

A. Derivation of MTTF

For the 2-component standby redundant system of Fig. 1, Buzacott [1] has assumed that the failure and repair time distributions of each component are explicitly prescribed as individual specific probability distributions.Under such an assumption, Buzacott [1] has derived the formula for evaluating MTTF. On the other hand, Osaki [2, 3] has derived the formula for evaluating MTTF of the 2-component standby redundant system in Fig. 1under the situation that the failure and repair time distributions of Component 1 are explicitly prescribed as individual specific probability distributions andComponent 2 has an exponential failure time distribution. Remark that all the distribution functions of the failure and repair times in each component have to be explicitly specified for evaluating MTTF onusing the theoretical formulae by Buzacott [1] and Osaki [2, 3].

However, in some practical situations, it might be difficult to know exactly the probability distributions of failure and repair times. On the other hand, in most practical cases, the averages and variances about failure and repair times in respective components will be at least known based on historical information and so on. At same time, it is impossible to evaluate the reliability of a system without fundamental information such as averages and variances on failure and repair times in each component.

In recent years, Takemoto and Arizono [4] have assumed the following situations in the 2-component standby redundant system illustrated in Fig. 1:

The failure time distributionof Component 1 has the cumulative distribution function (CDF) $F_1(t)$ with average $E_{F_1} = \lambda_1$ and variance V_{F_1}.

The repair time of Component 1 obeys the probability distribution $G_1(t)$ with average E_{G_1} and variance V_{G_2}.

The failure time of Component 2 obeys the probability distribution $F_2(t)$ with average E_{F_2} and variance V_{F_2}.

Then, Takemoto and Arizono [4] have developed a formula for evaluating MTTFin the 2-component standby redundant system illustrated in Fig. 1 by utilizingthe approximation technique of Keizer et al. [6]. The approximation technique of Keizer et al. [6] has been a procedure for approximating a general distribution with known average and variance witha mixed Erlang distribution composed of two Erlang distributions.Takemoto and Arizono have applied the approximation technique to the repair time distribution of Component 1 and the failure time distribution of Component 2.

As an example, based on the approximation technique of Keizer et al. [6], the probability density function(PDF) $g_1(t)$ of the repair time distribution of Component 1 is expressed as follows:

$$g_1(t) = p_1\mu_1^{n_1}\frac{t^{n_1-1}}{(n_1-1)!}e^{-\mu_1 t} + (1-p_1)\mu_1^{n_1+1}\frac{t^{n_1}}{n_1}e^{-\mu_1 t}, \quad (1)$$

where p_1, n_1 and μ_1 are parameters for the approximation and these parameters are provided as follows:

$$\frac{1}{n_1+1} < \varphi_1^2 \le \frac{1}{n_1}, \quad (2)$$

$$p_1 = \frac{(n_1+1)\varphi_1^2 - \sqrt{(n_1+1)(1-n_1\varphi_1^2)}}{\varphi_1^2+1}, \quad (3)$$

$$\varphi_1 = \sqrt{V_{G_1}/E_{G_1}}, \quad (4)$$

$$\mu_1 = \frac{n_1+1-p_1}{E_{G_1}}. \quad (5)$$

Similarly, note that the approximation parameters with respect to the probability density function $f_2(t)$ of the failure time distribution with $F_2(t)$ are obtained as p_2, n_2, φ_2 and λ_2 using E_{F_2} and V_{F_2}.

Further, Takemoto and Arizono [4] have divided the entire system into four subsystems by paying attention to the fact that PDFs $g_1(t)$ and $f_2(t)$ are defined by the probabilistic weighted sum of PDFs of two Erlang distributions, respectively. In concrete, the entire system has been divided into the four subsystems with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$, $(n_1+1, n_2, \mu_1, \lambda_2, (1-p_1)p_2)$, $(n_1, n_2+1, \mu_1, \lambda_2, p_1(1-p_2))$ and $(n_1+1, n_2+1, \mu_1, \lambda_2, (1-p_1)(1-p_2))$, respectively. Note that the notation $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$ represents that the repair time distribution of Component 1 obeys the Erlang distribution with phase n_1 and phase transition rate μ_1 and the failure time distribution of Component 2 obeys the Erlang distribution with phase n_2 and phase transition rate λ_2 and the probabilistic weight of this subsystem is given as $p_1 p_2$, others are also similar.

Since PDFs of individual components in each subsystem are given as an Erlang distribution, the state transitions in the respective subsystems are formulated based on the Markov process. Therefore, the MTTF of each subsystem can be evaluated using conventional research on the Markov process. Describe MTTF in the subsystem with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$ as $MTTF_{(n_1, n_2)}$. Then, Takemoto and Arizono [4] have derived MTTF in the entire system as follows:

$$\begin{aligned} MTTF = {} & p_1 p_2 MTTF_{(n_1, n_2)} + (1-p_1) p_2 MTTF_{(n_1+1, n_2)} \\ & + p_1 (1-p_2) MTTF_{(n_1, n_2+1)} + (1-p_1)(1-p_2) MTTF_{(n_1+1, n_2+1)}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} MTTF_{(n_1, n_2)} &= \tilde{R}_{(n_1, n_2)}(0) \\ &= \frac{1/\lambda_1}{1-\tilde{b}^{n_1}(0)} \sum_{j=0}^{n_2-1} \tilde{d}_{j(n_1)}(0) + \frac{\tilde{a}(0)}{1-\tilde{b}^{n_1}(0)} \\ &\quad \times \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} \binom{i+k}{k} \tilde{b}^i(0) \tilde{c}^k(0) \tilde{d}_{j(n_1)}(0), \end{aligned} \tag{7}$$

$$\begin{aligned} \tilde{R}_{(n_1, n_2)}(s) = {} & \tilde{u}_{(n_1)}(s) \sum_{i=0}^{n_2-1} \tilde{d}_{i(n_1)}(s) + \tilde{a}(s) \tilde{v}_{(n_1)}(s) \\ & \times \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} \binom{i+k}{k} \tilde{b}^i(s) \tilde{c}^k(s) \tilde{d}_{j(n_1)}(s), \end{aligned} \tag{8}$$

$$\tilde{a}(s) = \frac{1}{s+\mu_1+\lambda_2}, \tag{9}$$

$$\tilde{b}(s) = \frac{\mu_1}{s+\mu_1+\lambda_2}, \tag{10}$$

$$\tilde{c}(s) = \frac{\lambda_2}{s+\mu_1+\lambda_2}, \tag{11}$$

$$\tilde{d}_{0(n_1)}(s) = 1, \tag{12}$$

$$\tilde{d}_{j(n_1)}(s) = \tilde{w}_{(n_1)}(s) \sum_{k=0}^{j-1} \binom{n_1-1+j-k}{j-k} \tilde{c}^{j-k}(s) \tilde{d}_{k(n_1)}(s), \tag{13}$$

$$\tilde{u}_{(n_1)}(s) = \frac{\tilde{R}_1(s)}{1-\{1-s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}, \tag{14}$$

$$\tilde{v}_{(n_1)}(s) = \frac{1-s\tilde{R}_1(s)}{1-\{1-s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}, \tag{15}$$

$$\tilde{w}_{(n_1)}(s) = \frac{\{1-s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}{1-\{1-s\tilde{R}_1(s)\}\tilde{b}^{n_1}(s)}. \tag{16}$$

Further, $R_1(t) = 1 - F_1(t)$ and then, $\tilde{R}_1(s)$ denotes the Laplace transformation for $R_1(t)$.

B. Derivation of the variance of the failure time

Oigawa et al. [5] have addressed the derivation of the variance of the failure time of the considered 2-component standby redundant system with priority.We summarize the derivation process of variance considered byOigawaet al. [5].When PDF of the failure time distribution of the subsystem with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$ is denoted as $f_{(n_1, n_2)}(t)$, we obtain the Laplace transformation $\tilde{f}_{(n_1, n_2)}(s)$ for $f(t)$ as follows:

$$\tilde{f}_{(n_1, n_2)}(s) = \int_0^\infty e^{-st} f_{(n_1, n_2)}(t) dt. \tag{17}$$

Since the reliability function $R_{(n_1, n_2)}(t)$ of the subsystem with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$ is expressed as

$$R_{(n_1, n_2)}(t) = 1 - \int_0^t f_{(n_1, n_2)}(\tau) d\tau, \tag{18}$$

the relationship between the Laplace transformations $\tilde{f}_{(n_1, n_2)}(s)$ and $\tilde{R}_{(n_1, n_2)}(s)$ can be given as

$$\tilde{f}_{(n_1, n_2)}(s) = 1 - s\tilde{R}_{(n_1, n_2)}(s). \tag{19}$$

Then, by applying the final-value theorem and the L'Hopital's rule to the differential calculus forms for this relationship, we have

$$\frac{d}{ds} \tilde{f}_{(n_1, n_2)}(s) \Big|_{s=0} \equiv \tilde{f}^{(1)}_{(n_1, n_2)}(0) = -E[t \,|\, (n_1, n_2)] = -MTTF_{(n_1, n_2)}, \tag{20}$$

$$\frac{d^2}{ds^2} \tilde{f}_{(n_1, n_2)}(s) \Big|_{s=0} \equiv \tilde{f}^{(2)}_{(n_1, n_2)}(0) = E[t^2 \,|\, (n_1, n_2)], \tag{21}$$

where superscript (i) of the function $\tilde{f}^{(i)}_{(n_1, n_2)}(0)$ indicatesi-th differentiation of the function $\tilde{f}_{(n_1, n_2)}(s)$, and $E[t \,|\, (n_1, n_2)]$ and $E[t^2 \,|\, (n_1, n_2)]$ are indicated as the 1st and 2nd moments of the failure time in the subsystem with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1 p_2)$. Therefore, the 2nd moment of the entire system $\tilde{f}^{(2)}(0)$ can be obtained as

$$\begin{aligned} \tilde{f}^{(2)}(0) = {} & p_1 p_2 \tilde{f}^{(2)}_{(n_1, n_2)}(0) + (1-p_1) p_2 \tilde{f}^{(2)}_{(n_1+1, n_2)}(0) \\ & + p_1 (1-p_2) \tilde{f}^{(2)}_{(n_1, n_2+1)}(0) + (1-p_1)(1-p_2) \tilde{f}^{(2)}_{(n_1+1, n_2+1)}(0), \end{aligned} \tag{22}$$

where

$$\tilde{f}^{(2)}_{(n_1,n_2)}(0) = -2\tilde{u}^{(1)}_{(n_1)}(0)\sum_{j=0}^{n_2-1}\tilde{d}_{j(n_1)}(0) - 2\tilde{u}_{(n_1)}(0)\sum_{j=0}^{n_2-1}\tilde{d}^{(1)}_{j(n_1)}(0) + 2\tilde{a}(0)\tilde{v}_{(n_1)}(0)\sum_{i=0}^{n_1-1}\sum_{j=0}^{n_2-1}\sum_{k=0}^{n_2-1-j}\binom{i+k}{k}\tilde{b}^i(0)\tilde{c}^k(0) \times\left\{\tilde{d}_{j(n_1)}(0)\,\tilde{a}(0)\left(1+i+k+\frac{h_{(n_1,n_2)}(0)}{\tilde{a}(0)}\right)-\tilde{d}^{(1)}_{j(n_1)}(0)\right\}, \quad (23)$$

$$\tilde{u}^{(1)}_{(n_1)}(0) = -\frac{V_{F_1}+\left(\frac{1}{\lambda_1}\right)^2}{2\left\{1-\tilde{b}^{n_1}(0)\right\}} - \frac{\frac{1}{\lambda_1}\tilde{b}^{n_1}(0)\left\{n_1\tilde{a}(0)+\frac{1}{\lambda_1}\right\}}{\left\{1-\tilde{b}^{n_1}(0)\right\}^2}, \quad (24)$$

$$\tilde{d}^{(1)}_{j(n_1)}(0) = \tilde{w}^{(1)}_{(n_1)}(0)\sum_{k=0}^{j-1}\binom{n_1-1+j-k}{j-k}\tilde{c}^{j-k}(0)\tilde{d}_{k(n_1)}(0) - \tilde{w}_{(n_1)}(0)\sum_{k=0}^{j-1}\binom{n_1-1+j-k}{j-k}(j-k)\tilde{a}(0)\tilde{c}^{j-k}(0)\tilde{d}_{k(n_1)}(0) + \tilde{w}_{(n_1)}(0)\%\sum_{k=0}^{j-1}\binom{n_1-1+j-k}{j-k}, \quad (25)$$

$$\tilde{w}^{(1)}_{n_1}(0) = -\tilde{v}_{n_1}(0)h_{(n_1,n_2)}(0)\tilde{b}^{n_1}(0) - n_1\tilde{a}(0)\tilde{b}^{n_1}(0)\tilde{v}_{n_1}(0), \quad (26)$$

$$h_{(n_1,n_2)}(0) = \tilde{R}_{(n_1,n_2)}(0) + \frac{\tilde{b}^{n_1}(0)\tilde{R}_{(n_1,n_2)}(0) + n_1\tilde{a}(0)\tilde{b}^{n_1}(0)}{1-\tilde{b}^{n_1}(0)}. \quad (27)$$

Eventually, the variance of the failure time of the entire system can be evaluated as

$$V[t] = \tilde{f}^{(2)}(0) - MTTF^2. \quad (28)$$

IV. DERIVATION OF RELIABILITY FUNCTION

In this study, based on the limited information on MTTF and the variance of the failure time in the entire system, we evaluate the probability density function $f(t)$ of the failure time of the entire system. Then, it is widely known that maximum entropy principle brings an elegant solution for deriving the probability density function under the constraints of somevalues of moments. Therefore, we apply maximum entropy principle to obtain the probability density function $f(t)$ of the failure time in the entire system.

Then, from both of equations (6) and (28), it is seen that the probability density function $f(t)$ of the failure time of the entire system is calculated by the probability weighted sum for $f_{(n_1,n_2)}(t)$, $f_{(n_1+1,n_2)}(t)$, $f_{(n_1,n_2+1)}(t)$ and $f_{(n_1+1,n_2+1)}(t)$ as follows:

$$f(t) = p_1p_2f_{(n_1,n_2)}(t) + (1-p_1)p_2f_{(n_1+1,n_2)}(t) + p_1(1-p_2)f_{(n_1,n_2+1)}(t) + (1-p_1)(1-p_2)f_{(n_1+1,n_2+1)}(t). \quad (29)$$

Accordingly, we derive $f_{(n_1,n_2)}(t)$, $f_{(n_1+1,n_2)}(t)$, $f_{(n_1,n_2+1)}(t)$ and $f_{(n_1+1,n_2+1)}(t)$ to find $f(t)$.

As an example, we show the derivation of $f_{(n_1,n_2)}(t)$ based on maximum entropy principle. As for derivation of $f_{(n_1,n_2)}(t)$, we suppose the variance of the subsystem with parameters $(n_1, n_2, \mu_1, \lambda_2, p_1p_2)$ under $f_{(n_1,n_2)}(t)$ as

$$\sigma^2_{(n_1,n_2)} = \tilde{f}^{(2)}_{(n_1,n_2)}(0) - MTTF_{(n_1,n_2)}. \quad (30)$$

Based on maximum entropy principle, the problem of deriving a probability density function of $f_{(n_1,n_2)}(t)$ under constraints of given average and variance is formulated as follows:

$$\underset{f_{(n_1,n_2)}(t)}{\text{Maximize}} H(f_{(n_1,n_2)}(t)) = -\int_0^\infty f_{(n_1,n_2)}(t)\log f_{(n_1,n_2)}(t)\,dt, \quad (31)$$

where

$$\int_0^\infty f_{(n_1,n_2)}(t)\,dt = 1, \quad (32)$$

$$\int_0^\infty t f_{(n_1,n_2)}(t)\,dt = MTTF_{(n_1,n_2)}, \quad (33)$$

$$\int_0^\infty \left(t-MTTF_{(n_1,n_2)}\right)^2 f_{(n_1,n_2)}(t)\,dt = \sigma^2_{(n_1,n_2)}. \quad (34)$$

By using equations(32)-(34), a Lagrange function is defined with introducing Lagrange multipliers α_1, α_2 and α_3 as

$$L_{(n_1,n_2)} = H(f_{(n_1,n_2)}(t)) - \alpha_1\left\{\int_0^\infty f_{(n_1,n_2)}(t)\,dt - 1\right\} - \alpha_2\left\{\int_0^\infty t f_{(n_1,n_2)}(t)\,dt - MTTF_{(n_1,n_2)}\right\} - \alpha_3\left\{\int_0^\infty \left(t-MTTF_{(n_1,n_2)}\right)^2 f_{(n_1,n_2)}(t)\,dt - \sigma^2_{(n_1,n_2)}\right\}. \quad (35)$$

Through adopting the variational method to the Lagrange function $L_{(n_1,n_2)}$, we have

$$f_{(n_1,n_2)}(t) = e^{-1-\alpha_1}\,e^{-\alpha_2 t}\,e^{-\alpha_3\left(t-MTTF_{(n_1,n_2)}\right)^2}. \quad (36)$$

At the same instant, by partially differentiating $L_{(n_1,n_2)}$ with α_1, α_2 and α_3, we have equations (32)-(34). Based on equation (36), $f_{(n_1,n_2)}(t)$ is rewritten as

$$f_{(n_1,n_2)}(t) = C_{(n_1,n_2)}\,e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}, \quad (37)$$

where

$$C_{(n_1,n_2)} = e^{-1-\alpha_1-\alpha_3MTTF^2_{(n_1,n_2)}}\,e^{\alpha_3\left(\frac{2MTTF_{(n_1,n_2)}\alpha_3-\alpha_2}{2\alpha_3}\right)^2}, \quad (38)$$

$$m_{(n_1,n_2)} = \frac{2MTTF_{(n_1,n_2)}\alpha_3-\alpha_2}{2\alpha_3}, \quad (39)$$

$$\delta^2_{(n_1,n_2)} = \frac{1}{2\alpha_3}. \quad (40)$$

Accordingly, the derivation of $f_{(n_1,n_2)}(t)$ becomes a problem of finding $C_{(n_1,n_2)}$, $m_{(n_1,n_2)}$ and $\delta^2_{(n_1,n_2)}$.

By substituting equation (37) for equations (32)-(34), the following equations can be obtained:

$$C_{(n_1,n_2)}\int_0^\infty e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}\,dt = 1, \quad (41)$$

$$C_{(n_1,n_2)}\int_0^\infty t e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}\,dt = MTTF_{(n_1,n_2)}, \quad (42)$$

$$C_{(n_1,n_2)}\int_0^\infty \left(t-\mu_{11}\right) e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}\,dt = \sigma^2_{(n_1,n_2)}, \quad (43)$$

Then, form equation (41), we can derive as follows:

$$\int_0^{\infty} e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n1,n2)}}} dt = \frac{1}{C_{(n_1,n_2)}}, \quad (44)$$

Moreover, by multiplying both sides of equation (44) by $1\Big/\sqrt{2\pi\delta^2_{(n_1,n_2)}}$, we obtain the following equation:

$$\frac{1}{\sqrt{2\pi\delta^2_{(n_1,n_2)}}}\frac{1}{C_{(n_1,n_2)}} = \frac{1}{\sqrt{2\pi\delta^2_{(n_1,n_2)}}} \times\left(\int_0^{m_{(n_1,n_2)}} e^{\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}} dt+\int_{m_{(n_1,n_2)}}^{\infty} e^{\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}} dt\right). \quad (45)$$

The right side represents the upper probability at $t=0$ of the normal distribution with average $m_{(n_1,n_2)}$ and variance $\delta^2_{(n_1,n_2)}$. Through the variable transformation of $\tau = \left(t-m_{(n_1,n_2)}\right)\Big/\delta^2_{(n_1,n_2)}$, equation(45) is converted to

$$\frac{1}{\sqrt{2\pi\delta^2_{(n_1,n_2)}}}\frac{1}{C_{(n_1,n_2)}} = \frac{1}{2} - erf\left(-\frac{m_{(n_1,n_2)}}{\sqrt{2\delta^2_{(n_1,n_2)}}}\right), \quad (46)$$

where $erf(z)$ denotesthe following error function:

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2} dt. \quad (47)$$

The error function is frequently used in probability theory, statistics, material science, and partial differential equation. Further, the error function and confluent hypergeometric function have the following relationship:

$$erf(x) = \frac{2x}{\sqrt{\pi}}\,{}_1F_1\left(\frac{1}{2};\frac{3}{2};-x^2\right), \quad (48)$$

where the confluent hypergeometric function is defined as

$${}_1F_1(\beta;\gamma;z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(\beta+n)}{\Gamma(\gamma+n)}\frac{z^n}{n!}. \quad (49)$$

Furthermore, the following Kummer transformation can be applied to be confluent hypergeometric function:

$${}_1F_1(\beta;\gamma;z) = e^{-z}\,{}_1F_1(\gamma-\beta;\gamma;-z). \quad (50)$$

Consequently, based on equations (48) and (50), equation **Error! Reference source not found.** is transformed as follows

$$C_{(n_1,n_2)} = \frac{1}{\sqrt{\frac{\pi\delta^2_{(n_1,n_2)}}{2}} + m_{11}e^{\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}}\,{}_1F_1\left(1;\frac{3}{2};\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}\right)}. \quad (51)$$

In addition,based on equations (42) and (43), we have respectively the following equations:

$$C_{(n_1,n_2)} = \frac{MTTF_{(n_1,n_2)} - m_{(n_1,n_2)}}{\delta^2_{(n_1,n_2)}e^{\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}}}, \quad (52)$$

$$C_{(n_1,n_2)} = \frac{MTTF^2_{(n_1,n_2)} + \sigma^2_{(n_1,n_2)} - m^2_{(n_1,n_2)} - \delta^2_{(n_1,n_2)}}{m_{(n_1,n_2)}\delta^2_{(n_1,n_2)}e^{-\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}}}. \quad (53)$$

Then, from equations (52) and (53), the following equations can be shown:

$$\delta^2_{(n_1,n_2)} = -MTTF_{(n_1,n_2)}m_{(n_1,n_2)} + MTTF^2_{(n_1,n_2)} + \sigma^2_{(n_1,n_2)}. \quad (54)$$

Accordingly, by substituting equation (54) for equations (51) and (52) (or (53)), we can obtain the following function for $m_{(n_1,n_2)}$:

$$\left(MTTF_{(n_1,n_2)} - m_{(n_1,n_2)}\right)\left\{\sqrt{\frac{\pi\delta^2_{(n_1,n_2)}}{2}} + m_{(n_1,n_2)} \times e^{\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}}\,{}_1F_1\left(1;\frac{3}{2};\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}\right)\right\} - \delta^2_{(n_1,n_2)}e^{\frac{m^2_{(n_1,n_2)}}{2\delta^2_{(n_1,n_2)}}} = 0. \quad (55)$$

Although it is difficult to solve analytically equation (55) for $m_{(n_1,n_2)}$ because that $m_{(n_1,n_2)}$ is included in the confluent hypergeometric functionas an argument, we can derive the value of $m_{(n_1,n_2)}$ satisfying equation (55) numerically by appropriate search algorithms such as linear search. It is clear that we can get the values of $\delta^2_{(n_1,n_2)}$ and $C_{(n_1,n_2)}$ by calculated $m_{(n_1,n_2)}$. As a result, we can derive $f_{(n_1,n_2)}(t)$ by substituting calculated $m_{(n_1,n_2)}$, $\delta^2_{(n_1,n_2)}$ and $C_{(n_1,n_2)}$ for equation (37). After all, the reliability function $R_{(n_1,n_2)}(t)$ of the subsystem with parameters $(n_1,n_2,\mu_1,\lambda_2,p_1p_2)$ can be presented as

$$\begin{aligned} R_{(n_1,n_2)}(t) &= 1-\int_0^{\infty} f_{(n_1,n_2)}(t)dt \\ &= C_{(n_1,n_2)}\left\{\sqrt{\frac{\pi\delta^2_{(n_1,n_2)}}{2}} - \left(t-m_{(n_1,n_2)}\right)e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}\right. \\ &\quad \left. -\left(t-m_{(n_1,n_2)}\right)e^{-\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}}\,{}_1F_1\left(1;\frac{3}{2};\frac{\left(t-m_{(n_1,n_2)}\right)^2}{2\delta^2_{(n_1,n_2)}}\right)\right\}. \end{aligned} \quad (56)$$

Moreover, it is obvious that $f_{(n_1+1,n_2)}(t)$, $R_{(n_1+1,n_2)}(t)$, $f_{(n_1,n_2+1)}(t)$, $R_{(n_1,n_2+1)}(t)$, $f_{(n_1+1,n_2+1)}(t)$ and $R_{(n_1+1,n_2+1)}(t)$ in the corresponding subsystems are derived through the same manner as mentioned over. Eventually, based on equations (18)and (29), we can evaluate the reliability function $R(t)$ of the entire system as follows:

$$\begin{aligned} R(t) &= p_1p_2R_{(n_1,n_2)}(t) + (1-p_1)p_2R_{(n_1+1,n_2)}(t) \\ &\quad + p_1(1-p_2)R_{(n_1,n_2+1)}(t) + (1-p_1)(1-p_2)R_{(n_1+1,n_2+1)}(t). \end{aligned} \quad (57)$$

V. NUMERICAL ANALYSIS

In this section, we investigate the validity of our proposal on evaluating the reliability function.

As an example, suppose that suppose $E_{\lambda} = 1/\lambda_1 = 100$, $V_{F_1} = 20.0^2$, $E_{G_1} = 5.0$, $V_{G_1} = 1.5^2$

and $V_{F_2} = 22.5^2$, respectively. Moreover, the failure and repair time distributions in both components are respectively defined as the log-normal and/or Weibull distributions. The log-normal distribution and Weibull distribution are well known in the reliability field. In this case, we obtain parameters for the approximation using a mixed Erlang distribution as $(n_1, n_2, p_1, p_2) = (11, 4, 0.673, 0.031)$.

Based on the above numerical values, $MTTF_{(n_1,n_2)}$ and the second moments $\tilde{f}_{ij}^{(2)}(0)$ are given in Table 1. Further, m_{ij}, δ_{ij}^2 and C_{ij} are given in Table 2. Thus, we can

Table 1: the numerical value of $MTTF_{(n_1,n_2)}$ and $\tilde{f}_{ij}^{(2)}(0)$

	(n_1, n_2)	(n_1+1, n_2)	(n_1, n_2+1)	$(n_1+1, n_2+$
$MTTF_{(n_1,n}$	923.8	1141.1	854.3	1054.4
$\tilde{f}_{ij}^{(2)}(0)$	1054190 .1	900365 .9	1552855 .4	132454 5.6

Table 2: the numerical value of m_{ij}, δ_{ij}^2 and C_{ij}

	(n_1, n_2)	(n_1+1, n_2)	(n_1, n_2+1)	(n_1+1, n_2+1)
m_{ij}	885.7	820.1	1119.1	1034.5
δ_{ij}^2	235948. 7	199753. 1	233808. 6	233808.6
C_{ij}	8.5×10^{-4}	9.2×10^{-4}	7.7×10^{-4}	8.4×10^{-4}

evaluate the reliability function by using the procedure proposed in Section IV.

The validity of our proposal on evaluating the reliability functionis confirmed with the following procedures:

I. The averages and variances of failure and repair time in Component 1 are respectively given. Further, the average and variance of failure time in Component 2 are also given.
II. The probability distributions for the failure and repair time in Component 1 and the probability distribution for the failure time in Component 2 are individually specified as a particular probability distribution.
III. Through a million times of trials iterated in the Monte Carlo simulation under the combination of the specified probability distributions, the reliability in the entire system on time is estimated.
IV. The proposed reliability function is compared with the reliability estimated through the Monte Carlo simulation.

One of the results of comparing the simulation and the proposed reliability is shown in Fig 2. In Fig 2, the failure time distribution $F_1(t)$ and the repair time distribution $G_1(t)$ in Component 1 and the failure time distribution $F_2(t)$ in Component 2 are respectively supposed as the Weibull distribution with average E_{F_1} and variance V_{F_1}, the log-normal distribution with average E_{G_1} and variance V_{G_2}, and the Weibull distribution with average E_{F_2} and V_{F_2}.

From Fig 2, it has been confirmed that the shape of the reliability function proposed in this study resembles that of simulation result. Moreover, remark that similar results havebeen obtained under different conditions. Consequently, it has been confirmed that the proposed reliability function based on maximum entropy principle is useful as an approximation method for the reliability evaluation under the condition that only the average and variance of failure time in the entire system can be used.

CONCLUSION

In this study, we have addressed the evaluation method of the reliability function in the 2-component standby

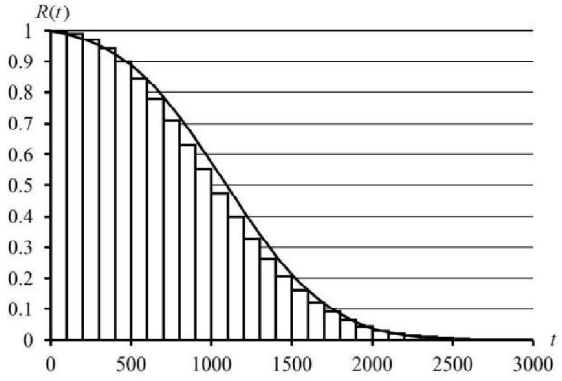


Fig. 2 Proposed reliability function and simulation result

redundant system with priority. Then, by utilizing the evaluation results for the mean time to failure by Takemoto and Arizono [4] and the variance of the failure time by Oigawa et al. [5], we have succeeded in deriving the evaluation method of the reliability function in the 2-component standby redundant system with priority. In concrete, by using only the average and variance of the failure time in the entire system, the reliability function in the 2-component standby redundant system with priority has been derived based on the maximum entropy principle. Then, it would like to be the future issue to establish the way of the optimal preventive maintenance of the system by using the reliability indices such as the reliability function.

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