THE PBR THEOREM AND ITS IMPLICATIONS

YEMIMA BEN-MENAHEM

The Hebrew University of Jerusalem

Abstract - The PBR theorem (published in 2012) purports to undermine epistemic interpretations of the quantum state (wave function) and support a realist interpretation thereof. The aim of this paper is to show that the theorem can actually be read as pointing to the opposite conclusion, that is, as a critique of the realist interpretation of the quantum state. The paper represents a radical epistemic interpretation that is not undermined by the theorem and is, arguably, strengthened by it.

Index Terms - PBR Theorem, Epistemic Interpretation of quantum mechanics

On most of its interpretations, Quantum mechanics (QM) is a fundamentally probabilistic theory. As such, it raises questions about the interpretation of the concept of probability in its context. One of the alternatives that have attracted a great deal of attention in the last few decades is an epistemic interpretation (also known as the information-theoretic, or Bayesian interpretation), according to which quantum states (wave functions) represent states of knowledge or belief or information. [1] The recent PBR theorem purports to show that the epistemic interpretation is untenable. It is therefore taken to confirm a realist interpretation of quantum states, according to which the quantum state (wave function) represents the physical state of the system, or is at least a function of this physical state. It is this realist message that is the source of the theorem's attraction. A point sometimes missed, though noted by PBR and elaborated, for example, in Leifer 2014, is that one should distinguish between two kinds of epistemic interpretations: epistemic interpretations that presuppose a well-defined state of the system—its 'real' state—and epistemic interpretations that decline this assumption. Let us call the latter radical epistemic interpretations. Once the distinction is in place, it becomes clear that the theorem targets the former variant of the epistemic interpretation, not the radical version. Even those who are aware of the distinction, however, associate the radical interpretation with positivism or instrumentalism. They therefore give the impression that for non-instrumentalists the theorem does in fact rule out epistemic interpretations and leaves the realist interpretation as the only viable option. Examining the radical interpretation in light of the PBR theorem, I will show that the association with instrumentalism is unfounded and that the theorem makes room for a radical epistemic interpretation distinct from instrumentalism. Furthermore, we will see that it is the radical interpretation, rather than the one undermined by the theorem, that many of the leading epistemic theorists have in mind. My conclusion is therefore that rumors about the death of the epistemic interpretation were exaggerated. Along the way, I hope to clarify the following issues involved in the epistemic approach.

1. If, as the epistemic theorist has it, quantum states represent knowledge, or belief or information, what is the right way of completing this statement--knowledge (belief, information) about what?
2. What is the difference between current epistemic interpretations of QM and earlier interpretations of quantum probabilities, for example, the ensemble interpretation favored by Einstein?
3. What is the difference between an epistemic interpretation of probability in general, as applied in daily contexts, or in statistical mechanics, where it is also popular, and the epistemic interpretation of QM?

I will start with an outline of the PBR theorem (I) and then turn to the roots of the controversy over the meaning of quantum states in early debates about the probabilistic interpretation of the wave function (II). I return to the lessons of the theorem in the concluding section (III).

I. THE PBR THEOREM

In their 2012 paper "On the reality of the quantum state", Pusey, Barrett and Rudolph summarize the no-go argument that has come to be known as the PBR theorem:

We show that any model in which a quantum state represents mere information about an underlying physical state of the system, and in which systems that are prepared independently have independent physical states, must make predictions that contradict those of quantum theory. (Pusey et al, 2012, 475)

Clearly, the theorem purports to target the epistemic interpretation of the wave function, thereby supporting its realist interpretation. This is indeed the received reading of the theorem and the source of its appeal. Under the title "Get Real" Scott Aaronson (2012) announced the new result as follows:

Do quantum states offer a faithful representation of reality or merely encode the partial knowledge of the experimenter? A new theorem illustrates how the latter can lead to a contradiction with quantum mechanics. (2012, p.443)

[1] See, for example, Caves *et al* (2001)(2002), Pitowsky(2006), Bub and Pitowsky (2010) Spekkens (2005)

Aaronson distinguishes between the question of whether a quantum state corresponds to different physical states and the PBR question--whether a physical state corresponds to different quantum states. It is the latterthat the PBR theorem purports to answer in the negative.

It is crucial to understand that we're not discussing whether the same wavefunction can be compatible with multiple states of reality, but a different and less familiar question: whether the same state of reality can be compatible with multiple wavefunctions. Intuitively, the reason we're interested in this question is that the wavefunction seems more 'real' if the answer is no, and more 'statistical' if the answer is yes (2012, p. 443).
Since the PBR theorem does indeed answer the said question in the negative, the conclusion is that the wave function has gained (more) reality.

Why is the negative answer to this question a refutation of the epistemic interpretation? Here Pusey et al. build on a distinction betweenΨ -ontic models and Ψ-epistemic models introduced by Harrigan and Spekkens (2010): InΨ -ontic models theΨ function corresponds to the physical state of the system; inΨ-epistemic models,Ψrepresents knowledge about the physical state of the system. Consequently, there are also two varieties of incompleteness:Ψ could give us a partial description of the physical state or a partial representation of our knowledge about that state. If aΨ-ontic model is incomplete, it is conceivable thatΨ could be supplemented with further parameters – 'hidden variables'. In this case, the same Ψ function could correspond to various physical states of the system, distinguishableby means of the values of the additional hidden variables. Presumably,Ψ-epistemic models can also be complete or incomplete but completing them cannot be accomplished by hidden variables of the former kind. Note that this analysis presupposes an answer to question no. 1 (in the introduction to this paper)–what is the knowledge (belief, information) represented by the quantum state about? The answer given here is that it is knowledge about the physical state of the system. So far this is merely a terminological distinction, but Harrigan and Spekkens also offer a criterion that distinguishes Ψ-epistemic from Ψ-ontic models: If the Ψ function is understood epistemically, it can stand in a non-functional relation to the physical state of the system, that is, the same physical state may correspond to two different (non-identical but also not orthogonal) Ψfunctions. Or, when probabilities rather than sharp values are considered, the supports of the probability distributions corresponding to different Ψ functions can overlap for some physical states. This possibility, they claim, only makes sense under the epistemic interpretation ofΨ , for it is conceivable that knowledge about the physical state could be updated without any change to the physical state itself. In short, a criterion for a model being Ψ-epistemic is precisely the possibility of such overlap of the supports of different probability functions.

The authors of the PBR theorem make two assumptions: 1.The system has a definite physical state—not necessarily identical with the quantum state. 2. Systems that are prepared independently have independent physical states. PBR prove that for distinct quantum states,if epistemic models are allowed, that is (using the Harrigan Spekkens criterion), if an overlap of the probability distributions is allowed so that the intersection of the supports has a non-zero measure--then a contradiction with the predictions of QM can be derived. They derive the contradiction first for the special case in which there are two systems such that $|\langle\Psi_0|\Psi_1\rangle| = 1/\sqrt{2}$ and then generalize. In the special case the proof is based on the following steps.

1. The authors consider two independent preparations of a system with states $|\Psi_0\rangle$ and $|\Psi_1\rangle$.
2. They choose a basis for Hilbert space for which $|\Psi_0\rangle = |0\rangle$ and $|\Psi_1\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. (Such a basis can always be found).
3. They suppose that the probability distributions for the two states overlap in a non-zero region Δ so that there is a positive probability $q>0$ to get a physical state from that region in each one of the systems and, because of independence, to get such a value from the region Δ for both systems together with probability $q^2>0$.
4. They assume that the two systems are brought together and their entangled state measured.
5. They show that QM assigns zero probability to all four orthogonal states onto which such a measurements projects, whereas by assumption, the probability is non-zero.

Contradiction!
The conclusion PBR draw is that epistemic models are incompatible with the predictions of QM, and thus ruled out. We are therefore left with ontic models, which are "more real," to use Aaronson's formulation.

Naturally, the strength of the theorem depends on the strength of its assumptions. The second assumption of PBR--independence--has in fact been called into question (Schlosshauer and Fine (2012, 2014), but one could also question the first assumption, namely, the assumption that the system has a definite physical state and that in epistemic models the quantum state represents the experimenter's knowledge (belief, information) about that physical state. Clearly, the assumption regarding the reality of a physical state of the system is an essentially realist assumption. The theorem therefore only undermines a particular combination of a realist assumption with a non-realist interpretation of quantum states. The contradiction it derives may indicate that this particular combination of realist and non-realist ingredients does not work.

From the logical point of view, one could also take the realist assumption itself to be responsible for the contradiction, in which case the theorem could be taken to challenge, rather than support a realist interpretation of quantum states.

II. THE MEANING OF QUANTUM STATES DIFFICULTIES

pertaining to the meaning of quantum states go back to the formative years of quantum mechanics, when it was realized that the wave function cannot represent an ordinary three-dimensional wave. As early as 1926, right after offering the probabilistic interpretation of Schrödinger's Ψ function, Born diagnosed the curious character of Ψ under his interpretation: Although particles follow probabilistic laws, the probability itself transforms "in accordance with the causal principle," i.e., deterministically. Furthermore, QM only answers well-posed statistical questions, and remains silent about the individual process. Born therefore characterized QM as a "peculiar blend of mechanics and statistics,"--"eine eigenartige Verschmelzung von Mechanik und Statistik".[2] This formulation is almost identical to that given by Jaynes and quoted in the PBR paper

Our present [quantum mechanical] formalism … is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature, all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. (Pusey et al 2012, p. 475).

The majority of quantum physicists were willing to tolerate the paradoxical situation diagnosed by Born. They treated QM both as a probabilistic theory and as a fundamental physical theory in which the wave equation plays the role of the equations of motionin classical mechanics. The question of whether the concept of probability could sustain this Verschmelzung was left hanging in midair. A dissident minority, including Einstein, preferred to bite the bullet and understand QM as a full-blown probabilistic theory, that is, a statistical description of an ensemble of similar systems that makes no definite claims about individual members of the ensemble.[3] Hence, the immediate analogy with statistical mechanics.

Statistical mechanics does indeed hold promise for peaceful coexistence between the probabilistic description of an ensemble and the precise physical description of each individual system comprising the ensemble. Even if this precise description is inaccessible in practice, it is taken to be possible in principle. The analogy between statistical and quantum mechanics suggested that quantum states correspond to macrostates in statistical mechanics, implying that in QM too, individual systems could occupy determinate physical states. These determinate states, though, like microstates in statistical mechanics, are not directly represented by quantum states; the latter only represent the statistical behavior of the ensemble. The picture of an underlying fundamental level, the description of which bears to QM the relation that classical mechanics bears to statistical mechanics, seemed to diminish the novelty and uniqueness of QM. Advocates of the Copenhagen interpretation opposed this picture and the analogy with statistical mechanics on which it was based. For them, QM was a novel theory--complete, fundamental and irreducible--that replaces classical mechanics rather than supplementing it with a convenient way of treating multi-particle systems. They therefore disagreed with ensemble theorists about the prospects of completing QM by means of 'hidden variable' theories.

Neither of these competing interpretations of quantum probabilities was epistemic. In this respect, there is a significant difference between the alternatives debated in the first decades of QM and the contemporary alternatives juxtaposed in the PBR argument, where the divide is between objective (physical, ontic) and epistemic interpretations. Given that epistemic interpretations of probability in general (outside of the quantum context) came of age later than objective interpretations, this difference should not surprise us.[4] It is significant, though, that when, in the wake of work by de Finetti and Savage, epistemic interpretations became popular, they were understood as diverging from traditional interpretations in their conceptual basis, not in their calculations. Indeed, it was considered to be the great achievement of these epistemic interpretations that despite their subjective starting point, they recovered the rules of objective theories of probability and proved to be equivalent to them. The equivalence between subjective and objective interpretations of the concept of probability holds for most applications of probability theory. In ordinary contexts, such as gambling or insurance, one could think of probabilities as reflecting our ignorance about the actual result rather than in terms of frequencies, ensembles or propensities, and it wouldn't matter. The equivalence also holds in statistical

[2] Born [1926] (1963), p. 234.

[3] The ensemble interpretation is recommended, for example by Blokhintsev (1968) Ballentine, (1970) Popper (1967).

[4] The epistemic interpretation already existed in 1930 but took some time to make an impact outside of the community of probability theorists. Leading advocates of the subjective/epistemic interpretation of probability as degree of belief are Ramsey (1931), de Finetti (1974) and Savage (1954). The delay in impact is clearly reflected in the fact that Finetti's works of the 1930's had to wait several decades to be collected and translated into English.

mechanics. In the above quotation from Jaynes, for instance, he adopts an epistemic interpretation of statistical mechanical probabilities, but this subjective interpretation does not yield different predictions than those of standard statistical mechanics. The epistemic interpretation of QM, however, presumes to make an empirical difference— this is the crux of the PBR theorem. Bearing this difference in mind (and returning to it later), let us probe a bit deeper into the controversy between the majority view that swallowed Born's Verschmelzung and the dissidents promoting the ensemble interpretation. The focal question on which the parties were divided was whether quantum states tell us anything about the individual system. Here are some of the pros and cons of the conflicting positions on this question.

The great merit of the ensemble interpretation was its take on the notorious 'collapse' of the wavefunction. If the wavefunction represents the physical state of an individual system, its instantaneous 'collapse' (to a sharp value) upon measurement is worrisome. As a physical process, it should be bound by the known constraints on physical processes, in particular, the spatiotemporal constraints of the special theory of relativity. But once quantum probabilities are thought to refer to an ensemble of similar systems, the fact that measurements on individual systems yield definite values is exactly what one would expect. The quantum collapse, under this interpretation, is no more troubling than the 'collapse' of flipped coin from a state of probability 0.5, which characterizes an ensemble, to either heads or tails when a single coin is flipped. In other words, since, on the ensemble interpretation, the Ψ function does not stand for a physical state of an individual system in the first place, worries about the physical properties of the collapse, its Lorentz invariance etc., are totally misplaced.
Despite this advantage, the ensemble interpretation also faces a number of serious questions.
1. Interference and other periodic effects.
2. The uncertainty relations.
3. Disturbance by measurement.

With these questions in mind, we can return to the contentious analogy between QM and statistical mechanics. The ensemble theorist who embraces this analogy, we now realize, is not only committed to a more fundamental level than the quantum level (i.e., underlying 'hidden variables'), but is also at variance with the standard interpretation about the meaning of central quantum laws such as the uncertainty relations. It is therefore slightly misleading to use the term 'interpretation' in this context; the two positions actually represent different theories diverging in their empirical implications.[5]

III. AN ALTERNATIVE EPISTEMIC INTERPRETATION

Ironically, it was Schrödinger who first exposed the flaws of the ensemble interpretation and its favorite analogy between QM and statistical mechanics. Ironically, because Schrödinger is generally considered an ally of Einstein in resisting the Copenhagen approach to QM. And yet, unlike Einstein, Schrödinger stressed the difference between classical probabilities of the kind we find in statistical mechanics and quantum mechanical probabilities. Furthermore, he used this insight to criticize the EPR argument! In his 1935 analysis of "the present situation in QM" (the 'cat' paper), he made decisive steps toward an epistemic interpretation of quantum states, even if not under that name.[6]

According to Schrödinger, the Ψ function does not represent an existing physical state, but a maximal catalogue of possible measurements. It embodies "the momentarily-attained sum of theoretically based future expectations, somewhat as laid down in a catalog. ... It is the determinacy bridge between measurements and measurements" ([1935]1983.p.158). Schrödinger's interpretation of the wavefunction resembles that of later proponents of the epistemic interpretation who use terms such as "a device for the bookkeeping of probabilities" (Pitowsky 2006, p. 214) or "maximal information" (Caves et al 2001, p.1) to refer to what Schrödinger called a "catalog of future expectations". Without using epistemic language, Schrödinger, much like later epistemic theorists, makes the following moves: First, he does not think of quantum states as representing physical states of the system (or functions of such physical states). Second, he does not think of quantum states as reflecting the observer's ignorance, but rather as objective constraints on the amount of information that could, in principle, be obtained. The latter move is mostly ignored in the philosophical literature, but is, in my view, of crucial importance for it means that QM does, after all, refer to the individual system, setting objective limits to what can be gleaned, from measurements made on it, about its future. This is certainly not the picture suggested by the ensemble interpretation. Together, the two moves imply a different answer to the 'knowledge about what'

[5] Construing the diverging interpretations as different theories does not mean that the existing empirical tests are conclusive, but that in principle, such tests are possible. Even the evidence accumulated so far, however, makes violation of the uncertainty relations in the individual case highly unlikely.

[6] As the title of the paper indicates,Schrödinger analyzed QM as it was at the time. One should not conclude that he had no reservations about it, or that he gave up on the prospect of finding better alternatives. But Schrödinger's remarkable attempt at a fair exposition of standard QM led to new insights, in particular his analysis of entanglement, and to the identification of flaws in Einstein's critique.

question than that ascribed to the epistemic theorist by PBR. It is not knowledge about the physical state of the system, but about possibilities of future measurement results. And it is not the knowledge or belief of the experimenter that is at issue, but the objective constraints on that knowledge. The subjectivity often associated with epistemic interpretations (recall Jayness) vanishes when the quantum state is interpreted as an objectiveupper bound on probabilities the experimenter is allowed to assign.

Schrödinger was also explicit about the non-classical nature of quantum probability, stressing that the difference between classical and quantum probability does not originate in a particular dynamics, but in structural features of the space of events.
Consequently, Schrödinger took the uncertainty relations to apply to individual systems rather than ensembles. They do not express ignorance about the individual system or practical obstacles to precise measurements or preparation, but fundamental constraints on the very assignment of values to all physical magnitudes (at a particular moment). It follows that "The classical notion of state becomes lost in that at most a well-chosen half of a complete set of variables can be assigned definite numerical values" ([1935]1983, p.153). Schrödinger therefore rules out the possibility that quantum probabilities and uncertainties are analogous to probabilities in statistical mechanics, or that they merely reflect ignorance about a completely definite physical reality.

Schrödinger, who was the first to identify entanglement, argued that this quantum phenomenon also follows from the maximal nature (completeness) of the Ψ function. A complete catalog for two separate systems is, ipso facto, also a complete catalog of the combined system, but the reverse does not follow. "Maximal knowledge of a total system does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all" (Ibid.p. 160, italics in original). The reason is that the maximal catalog of the combined system may contain conditional statements of the form: if a measurement on the first system yields the value x, a measurement on the second will yield the value y, and so on. Schrödinger's sums up: "Best possible knowledge of a whole does not necessarily include the same for its parts…The whole is in a definite state, the parts taken individually are not" (p.161). In other words, separated systems can be correlated or entangled via the Ψ function of the combined system, but this does not mean that their individual states are fixed!

Schrödinger tells us (in f.n. no. 7) that his 1935 paper was written in response to the EPR paper published earlier that year. Due to the common tendency to assimilate Schrödinger's position with that of Einstein, one can easily miss the fact that in this paper Schrödinger actually launches a critique of Einstein's understanding of QM in general and the EPR argument in particular. The EPR argument purports to show that correlations between the remote parts of the system (expressed as conditional statements of the above-mentioned type), entail that each individual state already had a determinate value prior to measurement. By contrast, Schrödinger argued first, that such determinacy is precluded by the uncertainty relations properly understood, and second, that given his reading of the Ψ function as a maximal catalog of possible measurements, the indeterminacy of individual outcomes makes perfect sense. The implication was that the EPR thought experiment had failed to indicate a way of getting around the uncertainty relations.

The PBR theorem targets an epistemic interpretation that combines a realist assumption--the existence of definite physical states underlying quantum states--with a non-realist construal of quantum probabilities. Inspired by Schrödinger, I have argued that this is not the only available epistemic interpretation, and not the one most commonly upheld by epistemic theorists. An interpretation polar to that of PBR is available: One can let go of the assumption of definite physical states, but construe quantum probabilities as objective constraints on the information made available by measurement. If I am right, the widespread reading of the theorem as dealing a fatal blow to the epistemic interpretation should be reconsidered.

REFERENCES

[1] Aaronson, S. (2012) "Get Real" Nature Physics**8** 443-444.
[2] Ballentine, L.E. (1970) "The Statistical Interpretation of Quantum Mechanics" Review of Modern Physics 42 358-381.
[3] Blokhintsev, D.I. (1968) The Philosophy of Quantum Mechanics (Amsterdam: Reidel)
[4] Born, M. [1926] (1963) "Quantenmechanik der Stossvorgange" Zeitschrift f. Physik 38 803-827, in: Ausgewahlte Abhandlungen**II** (Gottingen: Vandenhoek Ruprecht) 233-252.
[5] Caves, C.M., Fuchs, C.A. Schack, R. (2001) "Quantum probabilities as Bayesian probabilities" arXiv:quant-ph/0106133v2
[6] Clauser, J., Horne, M., Shimony, A., Holt, R. (1969). "Proposed Experiment to Test Local Hidden-Variable Theories" Physical Review Letters 23 (15) 880.
[7] De Finetti, B. (1974), Theory of Probability (New York: John Wiley)
[8] Einstein, A., Podolsky, B. and Rosen, N. (1935) "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" Phys. Rev. **47** 777–780.
[9] Harrigan, N. & Spekkens, R. W. (2010) "Einstein, Incompleteness, and the Epistemic View of Quantum States" Found. Phys. **40** 125-157.
[10] Leifer, M.S. (2014) "Is the Quantum State Real?" Quanta**3** 67-155.
[11] Pitowsky, I. (2006) "Quantum Mechanics as a Theory of Probability" in: Demopoulos and Pitowsky (2006) pp. 213-239.

[12] Popper, K. R. (1967) “Quantum Mechanics Without the Observer” in: M. Bunge: Quantum Theory and Reality (Berlin: Springer) 7-43.
[13] Pusey, M. F., Barrett, J. and Rudolph, T. (2012) ”On the Reality of the Quantum State” Nature Physics**8** 475-478.
[14] Redhead, M. (1987) Incompleteness, Nonlocality and Realism (Oxford: Clarendon)
[15] Savage, L.J. (1954) The Foundations of Statistics (New York: John Wiley).
[16] Schlosshauer, M. and Fine, A. (2012) “Implications of the Pusey-Barrett-Rudolph Quantum No-Go Theorem” Physical Review Letters**108** 260404
[17] Schlosshauer, M. and Fine, A. (2014) “No-go theorem for the Composition of Quantum Systems” Physical Review Letters **112** 070407
[18] Schrödinger, E. [1935](1983) “The Present Situation in Quantum Mechanics” trans. by J.D. Trimmer, in Wheeler and Zurek (1983), pp. 152-167.
[19] Spekkens, R.W. (2005) “In Defense of the Epistemic View of Quantum States: A Toy Theory” arXive:quant-ph/0401052v2
[20] Wheeler, J.A. and Zurek, W.H. (1983)Quantum Theory and Measurement (Princeton: Princeton University Press).

★★★