BLACK-SCHOLES AND PITFALLS OF UNDERLYING

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Abstract: In this financial engineering research,we confirmthat the valuation of European styled options using Black-Scholesformula is not inthe accordance with the market valuation(we study the situation of DAX options - ODAX, traded on EUREX exchange); we also suggest how to obtain the correct value, whichdoes not provide longtime advantagebetween holding a long ora short option position till expiration; or in other words, if we repetitively hold till expiration a long call (by the way of example) position, we will obtain zero P/L in longtime average. The discrepancy between market price and result of Black-Scholes formula is usually considered to be caused bya problematic underlying asset volatility expectation but from the research it is obvious that the presumption about the process which is used for adescription of underlying price development could be also the reason.The purpose of this research is also to simplify this theoretically complicated topic for financial practitioners.

Keywords: call/put option; Black-Scholes formula, risk neutrality, DAX options

I. INTRODUCTION

In this financial engineering study, we acknowledge that the valuation of European styled option using Black-Scholes formula is not in the accordance with the market valuation (we study the situation on DAX options, traded on EUREX exchange); we also suggest how to obtain the correct valuation which provides in average no advantage between holding a long or short option position at the moment of expiration; or in other words, if we hold till expiration long call (by the way of example) position repetitively we will obtain zero P/L in longtime average. The discrepancy between market price and result of Black-Scholes formula is usually considered to be caused by problematic underlying volatility expectation but from the research it is obvious that the presumption about the process which is used for the description of underlying price development could play an important role.

The process of formula deriving which is well known as Black-Scholes is a quite dramatic. Black and Scholes 1973obtained the solution using advantage of previous research on option pricing that gave an idea of what the solution should look like. Briefly described: Luis Bachelier 1964 had derived the solution using Arithmetic Brownian Motion as the process according which the underlying asset is moving. Sprenkle 1962 was the first to assume a lognormal distribution, i.e., geometric Brownian motion as the process of underlying movements, but Sprenkle attempted to adjust for risk, which was neither necessary nor correct. In addition, he assumed a zero-interest rate and Boness 1964 added a non-zero interest rate but also improperly adjusted for risk, also Samuelson 1965 also incorrectly attempted to adjust for risk. Each of these formulas resembles the correct solution as found by Black and Scholes. Black and Scholes, however, originally derived the equation by using the Capital Asset Pricing Model, which provides the equation for the expected return on a risky asset as a function of its risk. At approximately the same time, however, a colleague of Black's, Robert Merton (1973), wrote a paper developing the model with virtually the full mathematical solution. The solution fully respect risk neutrality concept and Geometric Brownian Motion is expected process behind the underlying development. This process is described by stochastic differential equation:

$$\boldsymbol{S_t = \mu S_t dt + \sigma S_t dW_t} \qquad (1)$$

where *W*is a Wiener process or Brownian motion, $\boldsymbol{\mu}$ is the percentage annualized drift and $\boldsymbol{\sigma}$ is the annualized volatility. Illustrative simulation is in the figure 1. Important feature of the process is increasing change of price while value is higher which cause the same value of changes in yield which is arising from financial logic. The price distribution is in the figure 2.

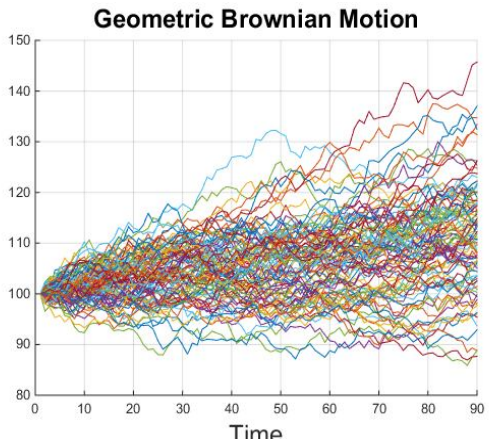


Fig. 1 Geometric Brownian motion-simulation source: own illustration

Many works have appeared to upgrade the formula later, especially in the field of volatility. Heston, Steven& Saikat 1997 developed a closed-form option pricing formula for a spot asset whose variance follows a GARCH process. The model allows for correlation between returns of the spot asset and variance and also admits multiple lags in the dynamics of the GARCH process. The single factor (one lag) version of this model contains Heston's 1993 stochastic volatility model as a diffusion limit and therefore unifies the discrete GARCH and continuous-time stochastic volatility literature of option pricing. The model provides the first option formula for a random volatility model that is solely a function of observables; all the parameters can be easily estimated from the history of asset prices, observed at discrete intervals.

Empirical analysis on S&P500 index options shows the single factor version of the GARCH model to be a substantial improvement over the Black-Scholes 1973 model. The GARCH model continues to substantiallyoutperform the Black-Scholes model even when the Black-Scholes model is updated every period while the parameters of the GARCH model are held constant. The improvement is due largely to the ability of the GARCH model to describe the correlation of volatility with spot returns. This allows theGARCH model to capture strike price biases in the Black-Scholes model that give rise to the skew in implied volatilities in the index options market.

Other significant contribution to the research area was done by Rubenstein 1967; Black 1989; Black, Scholes 1972; Chance 1994; Conine, Tamarkin 1984; Hull 2003, 2008; Donathan 1990; Duffie 1988, Garven 1986; Gibson 1991; Kutner 1998; Szpiro 2011.

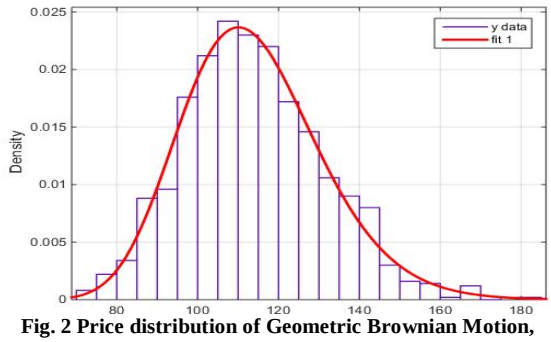


Fig. 2 Price distribution of Geometric Brownian Motion, source: own illustration

II. METHODOLOGY

Methodology could be summarized into the following steps:

1. Valuationof DAX options using Black-Scholes formula (market price of underlying is assumed to be a geometric Brownianmotion) and its comparison to the market price of DAX options (ODAX).
2. Valuation of option price if market price of underlying is assumed to follow a pure random walk (Brownian motion) and its comparison to market price of ODAX.
3. Valuation of option price if market price is assumed to follow ARIMA (1,1,1) process and its comparison to market price of ODAX.
4. Valuation of option price if market price is assumed to be a process according to a dynamic financial market model (Stádník 2014, 2014a, 2014b, 2014c) and its comparison to ODAX.
5. Assessment of valuation with respect to the above mentioned underlying development processes.
6. Thesuggestion of algorithm for correct valuation.

For valuation, instead of Black-Scholes formula for geometrical Brownian motion, we have usednumerical approach with MATLAB implementation (Appendix) and the option premium is found as an equilibrium price which ensures in longtime average zero P/L while opening repetitively speculative position and hold the option till its expiration.

Financial data are taken from EUREX website http://www.eurexchange.com/exchange-en/

RESULTSAs underlying asset, we use DAX index. Its price development is in the figure 3a, b. The core question is: if the price development follows Geometric Brownian Motion. From the figures it is not clear, mainly the right part of figure 3b.

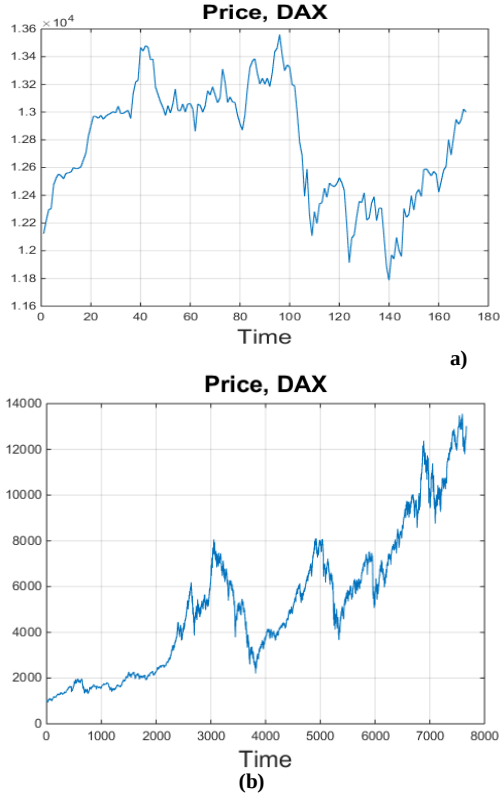


Fig. 3(a&b) Price development of DAX index (1/2 year back) a), (34 years back) b), source: own illustration

Also, the price distribution in the figure 4 does not support geometric Brownian motion as there is no any significant skewness in comparison to figure 2.

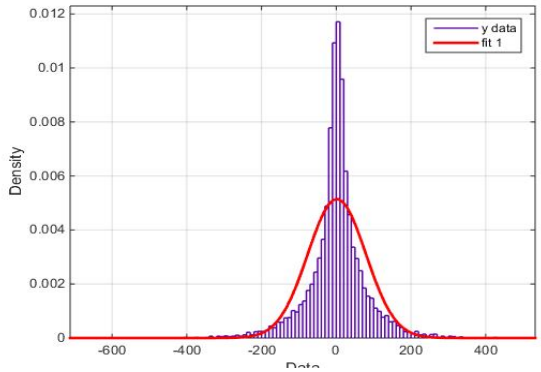


Fig. 4 DAX price distribution, kurtosis 8.3311source: own illustration

We may observe higher values of kurtosis which is typical for investment instruments.
Figure 5a representsthe simulation of future DAX development if we assume that it follows geometric Brownian motion according formula (1).

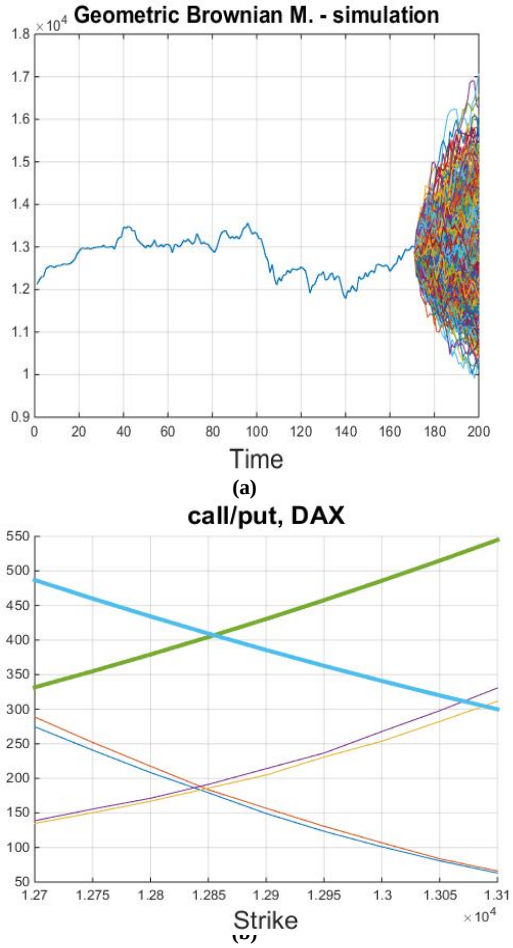


Fig. 5(a&b) Simulation of geometric Brownian motion on DAX (a), DAX option prices (b), source: own illustration

The drift and volatility are calculated based on historical data (figure 1a). Important point is that volatility is assumed to be of constant value. Figure 1b illustrate the valuation using Black-Scholes formula (thick lines) and market bid-ask lines (thin lines) with respect to strike price.

In case of call option, the charts are increasing, in case of put option the charts are decreasing. We observe that Black-Scholes formula over valuates the market prices of option premium.
Figure 6a represents the simulation of future DAX development if we assume that it follows random walk. The drift and volatility are also taken from historical time series in figure 1a. Figure 6b illustrate the valuation using algorithm implemented in MATLABenvironment and the option premium is found as an equilibrium price which ensures zero P/Lin average while we open repetitively speculative option position and hold the option till its expiration (Appendix).

Thick lines in the figure represent the valuation according mentioned algorithm and thin lines are market bid-ask lines with respect to strike price.

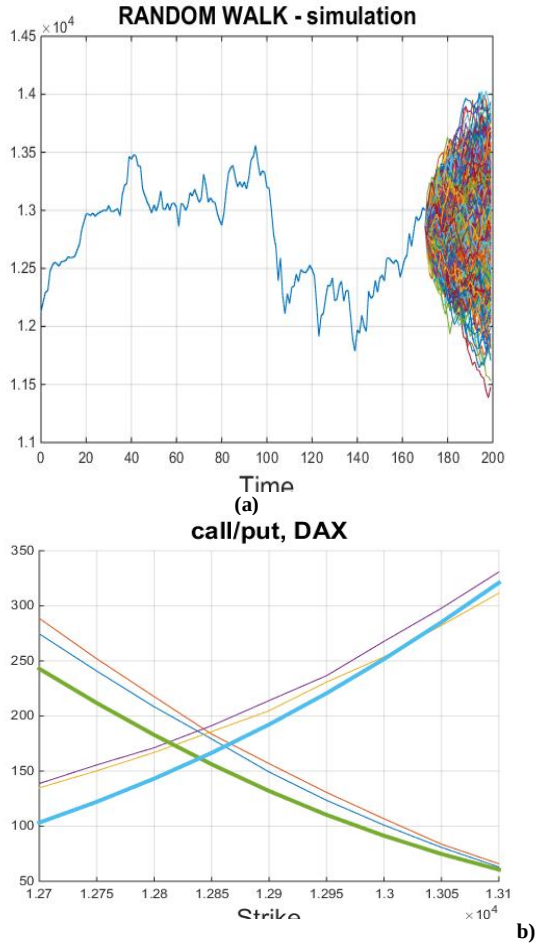
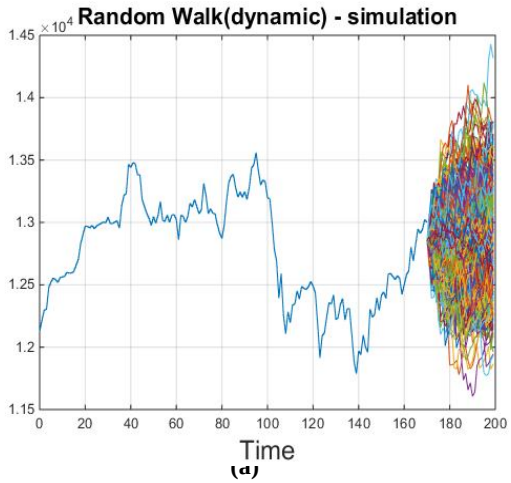


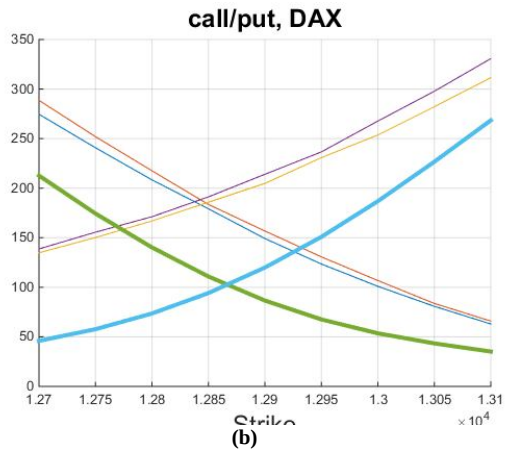
Fig. 6 (a&b) Simulation of random walk on DAX (a), DAX option prices (b), source: own illustration

From the results we may conclude that the valuation is closer to the market valuation if we assume that underlying follows random walk than in case of geometric Brownian motion presumption.

In the figure 7a we assume dynamic financial market model (Stádník 2014, 2014a, 2014b, 2014c) based on which is derived future DAX development.



(a)



(b)

Fig. 7(a&b) Simulation of random walk with memory on DAX (a), DAX option prices (b), source: own illustration

Dynamic financial market model is able to modelleptokurtic features of price distribution using feedbacks (figure 8). Thus, the process behind the development could be very similar to the empirically measured distribution (figure 4). Comparison of valuation using such underlying development is in the figure 7b. The idea of feedback processes is based on the observations that traders, investors and other market participants don't only watch present or historical data but according to them they are also placing buy or sell orders and thus influence future development. So, there is a feedback in the financial markets which also influences a future price development and cause the future direction dependence.The most usual examples are traders who use technical analysis. In the model we work with the feedback processes regardless whether they really help to make profit or not.

Feedback can increase or decrease the frequency of incoming buy or sell orders and therefore changes a probability of the next price step direction from 50% (in case of a pure symmetric random walk) to for example 51%. Then we can conclude that the probability of the next step direction depends on the past.

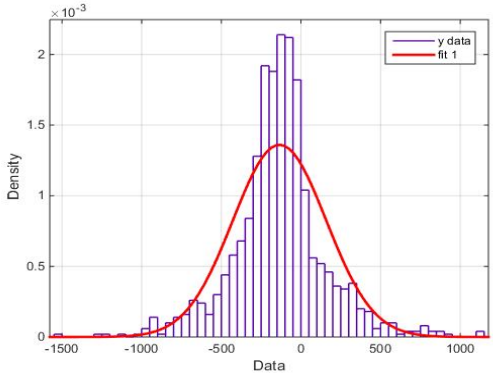
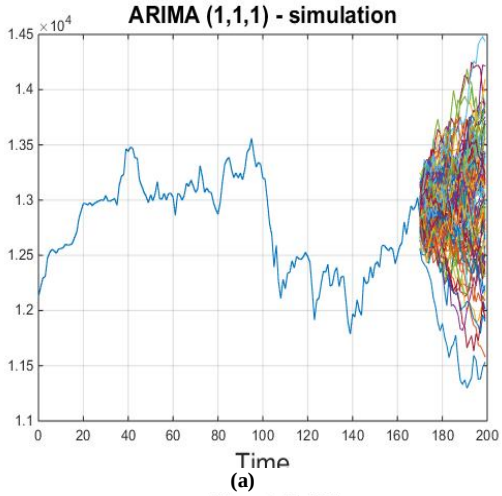
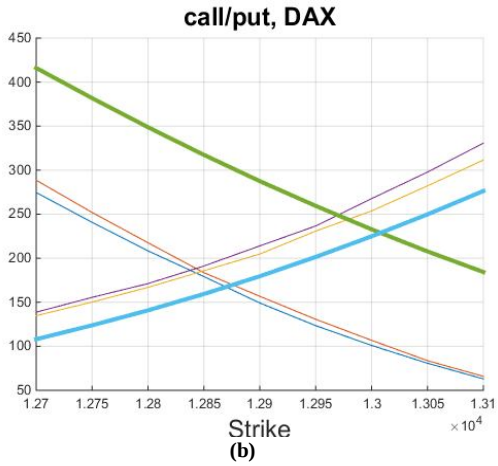


Fig. 8 kurtosis 5.4953,source: own illustration

Finally, we use ARIMA (1,1,1) for simulation of DAX future development, figure 9a. Results are in the figure 9b.



(a)



(b)

Fig. 9(a&b) Simulation of ARIMA (1,1,1) on DAX (a), DAX option prices (b), source: own illustration

To conclude if the market valuates options correctly we have to wait till expiration and check if we are at zero P/Lin average while we open repetitively speculative option position. We may then calculate option premiums retroactively and construct the

appropriate model or asses which presumption of underlying development is the closest to reality.

CONCLUSIONS

In this financial engineering research, we affirm that the valuation of European styled option using Black-Scholes formula is not in the accordance with the market valuation (we study the situation on DAX options, traded on EUREX exchange).The discrepancy between market price and result of Black-Scholes formula (which expects that the process behind the underlying development, DAX index in our research, is geometric Brownian motion)is usually considered to be caused by a problematic expectation of underlying asset volatility.But from this research it is obvious that the presumption about the process, which is behind the underlying price development, could be also the reason for serious valuation difficulties.

The main finding is that the discrepancy is the lowest if we assume that the underlying asset follows pure random walk process (Brownian motion). In this case we evaluate options as equilibrium price,which ensures zero P/L in average, while we open repetitively speculative option position and hold the option till its expiration. The highest differences are if we use Black-Scholes formula. We have checked also ARIMA model and model with memory for underlying developmentwith problematic conclusions.

To asses if the market valuates options correctly (not only that there is discrepancy between market and theoretical valuation) we have to wait till option expiration and check if we are at zero P/L in average while we open repetitively speculative option positions. We may then calculate option premiums retroactively and construct the appropriate model or asses which presumption of underlying development is the closest to reality.

APPENDIX

Core MATLAB Code:

```
Strike_vektor=[12700:50:13100];
for  Strike=12700:50:13100
      Strike_poc=Strike_poc+1;
      poc=0;
for premium=5:0.02:1200
         poc=poc+1;
call_p(poc)=premium;
put_p(poc)=premium;
         uplatneni1=0;
         uplatneni2=0;
         PL1(1)=0;
         PL2(1)=0;
for k=1:pocet_cest;
              if (cesta(k,30)>Strike)
                  PL1(k+1)=cesta(k,30)-Strike-
                  call_p(poc)+PL1(k);
                  uplatneni1=uplatneni1+1;
else
                  PL1(k+1)=PL1(k)-call_p(poc);
              end;
              if (cesta(k,30)<Strike)
                   PL2(k+1)=-cesta(k,30)+Strike-
                  put_p(poc)+PL2(k);
                  uplatneni2=uplatneni2+1;
else
                  PL2(k+1)=PL2(k)-call_p(poc);
             end;
         end;
        PL_celkem1(poc)=PL1(k+1);
        PL_celkem2(poc)=PL2(k+1);
        if (abs(PL_celkem1(poc))<20)
              vitez1(Strike_poc)=premium;
        end;
         if (abs(PL_celkem2(poc))<20)
              vitez2(Strike_poc)=premium;
        end;
end;
end;
```

The code checksif we are at zero P/L in average while we open repetitively speculative option position. There are basically two cycles: the first one for strike price and the second one is checking if value of premium
fulfill the mentioned condition.

FUNDING

This work was supported by the Czech Republic GA 16-21506S and VSE IG102027.

DISCLOSURE STATEMENT

I do not have any competing financial, professional, or personal interests from other parties.

REFERENCES

[1] Bachelier, L. (1964).“Theory of Speculation.” English translation by A. J. Boness, The Random
[2] Character of Stock Market Prices, ed. P. Cootner. Cambridge: Massachusetts: The
[3] M.I.T. Press (1964), 17-78.
[4] Black, F. (1989)“How We Came Up with the Option Formula.” Journal of Portfolio Management 15
[5] (Winter, 1989), 4-8.
[6] Black, F. and M. Scholes.(1973). “The Pricing of Options and Corporate Liabilities.” Journal of
[7] Political Economy 81 (May-June, 1973), 637-654.
[8] Black, F. and M. Scholes.(1972). “The Valuation of Option Contracts and a Test of Market Efficiency.”
[9] The Journal of Finance 27 (May, 1972), 399-417.
[10] Boness, A. J. (1964),“Elements of a Theory of Stock-Option Value.” Journal of Political Economy 72
[11] (April, 1964), 163-175.
[12] Chance, D. M.(1994). “Translating the Greek: An Alternative Interpretation of Call Option
[13] Derivatives.” Financial Analysts Journal 50 (July-August, 1994), 43-49.
[14] Conine, T. E., Jr. and M. Tamarkin.(1984). “A Pedagogic Note on the Derivation of the Comparative

[15] Statics of the Option Pricing Model." The Financial Review (1984), 397-400.
[16] Dothan, M. U. (1990). Prices in Financial Markets. New York: Oxford University Press (1990), Ch.9.
[17] Duffie, D. (1988)Security Markets: Stochastic Models. Boston: Academic Press (1988), Ch. 22.
[18] Heston, Steven L.1993,"A Closed-Form Solution for Options with Stochastic Volatility, with Applications to Bond and Currency Options," Review of Financial Studies 6
[19] Heston, Steven L. and Nandi, Saikat.(1997).A Closed-Form GARCH Option Pricing Model (November 1997). 97-9. Available at SSRN: https://ssrn.com/abstract=96651 or http://dx.doi.org/10.2139/ssrn.96651
[20] Garven, J. R. (1986)."A Pedagogic Note on the Derivation of the Black-Scholes Option Pricing
[21] Formula." The Financial Review 21 (1986), 337-344.
[22] Gibson, R. (1991). Option Valuation: Analyzing and Pricing Standardized Option Contracts. New
[23] York: McGraw-Hill (1991), Chs. 4, 5.
[24] Hull, John C. (2008). Options, Futures and Other Derivatives (7th ed.). Prentice Hall. ISBN 0-13-505283-1.
[25] Hull, J. (2003).Options, Futures and Other Derivatives. Upper Saddle River, New Jersey: PrenticeHallCh. 12.
[26] Kutner, G. W.(1988). "Black-Scholes Revisited: Some Important Details." The Financial Review 23
[27] (February, 1988), 95-104.
[28] Merton, R. C., (1973). "Theory of Rational Option Pricing." Bell Journal of Economics and
[29] Management Science 4 (Spring, 1973), 141-183.
[30] Rubinstein, M. E. (1967). "The Valuation of Uncertain Income Streams and the Pricing of Options."
[31] Bell Journal of Economics 7 (Autumn, 1976), 407-425.
[32] Sprenkle, C. M.(1962). "Warrant Prices as Indicators of Expectations and Preferences." Yale
[33] Economic Essays 1 (1962), 172-231.
[34] Stádník, B. (2014). The Puzzle ofFinancial Market Distributions, *Ekonomický Časopis*, 7, 2014: 709-728 http://www.ekonom.sav.sk/sk/casopis/rocnik/7-2014, ISSN: 0013-3035
[35] Stádník, B. (2014a). Springoscillationswithinfinancialmarkets, *Procedia – Social and BehavioralSciences*110(24):1176–1184.
[36] Stádník, B. (2014b). Theriddleof volatility clusters, *Business: Theory and Practice*15(2): 140–148. http://doi.dx.org/10.3846/btp.2014.14 2014
[37] Stádník, B. (2014c). The puzzle offinancial market distributions, *Ekonomický Časopis* 7: 709–728.
[38] Szpiro, G. G. Pricing the Future: Finance, Physics, and the 300-Year Journey to the BlackScholes
[39] Equation. New York: Basic Books (2011).

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