EFFECT OF SCREENING STUDENTS ON KNOWLEDGE ACQUISITION USING DIFFERENTIAL EQUATIONS

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Abstract- The problem of screening students in learning according to ability is studied using differential equations. This study was performed in order to investigate whether a mathematical model normally used in disease models can be applied to knowledge acquisition. This study investigates the effect of varying dropout rates, forgetting rates and interaction with facilitators and other sources on knowledge acquisition. A unique mathematical model for knowledge acquisition is developed. In this study we adopt the mathematical model that is normally used in disease modelling and apply it to teaching and learning. The developed model is then solved using MATLAB ODE45 and nlinfit to estimate parameters for actual data. The results of this study show that increasing more teaching staff and learning materials under the current assumptions does improve knowledge acquisition. Increasing dropout rates and forgetting rates negatively affects knowledge acquisition as expected, showing that the mathematical model normally used for disease modelling can be applied to this area.

Index Terms- Differential equations, knowledge acquisition, least squares.

I. INTRODUCTION

According to the free Dictionary by Farlex, the meaning of the word screening is a systematic examination or assessment done, especially to detect an unwanted substance or attribute. In this paper the systematic assessment of students will be investigated. The What Works Clearing house and American institutes for research have shown that screening students helps identify those in need of intervention strategies. Early detection of weak students in mathematics is thus very important.

Some studies have shown that screening students according to ability does not facilitate learning while some argue that screening helps identify weak students and be able to apply appropriate intervention strategies. The teaching of mathematics in South Africa has been a problem that need attention. In many schools teaching mathematics has remained a challenge due to the shortage of teachers in this area. This is caused by a very old perception that mathematics is difficult. This has remained in people's minds for a very long time. As a result, this has caused failures in the area of mathematics in both schools and tertiary institutions.

For schools, recent research according to Hulac et al. [1] has provided many effective interventions addressing students' reading needs in the area of mathematics educators have other problems involving which appropriate intervention programs must be followed to assist students in need of interventions. Test data is necessary to track and monitor the progress of all students [2].

The learning difficulties encountered by students at university is as a result of poor back- ground in their early learning career especially in primary school [3], [6]. The teaching and learning of mathematics in South Africa need attention. Extensive research shows poor performance in mathematics [4]. The reasons for this high failure rate has been associated to historical background of the education system, poor educator training and lack of facilities; Spaull [7] pointed out that the effects of apartheid are still responsible for some of the failures experienced at this stage.

As a result, the national examinations in mathematics and science has failed to adequately prepare school leaving students for university education. The department of education has introduced extended programs in universities to cater for this deficit. However, this move is affected by students' lack of basic knowledge in mathematical concepts. The acquisition of certain cognitive, social and emotional capabilities helps students to learn better in later years [3].

Admission of students into the university is based on their performance in the matriculation (grade 12) examinations. Some students get into the university without meeting entry requirements [8]. The number of students passing mathematics and science is of concern [9]. This small number of students accepted into the department of mathematics and lack of basic concepts and poor career guidance has resulted in very low graduation rates. There is need for universities to adopt their own entry assessments as the NSC examination is believed to be insufficient for university entrance. In Tanzania, the school pass rate is high but the University of Dares Salaam have its own examination tests [10].

The aim of this paper is to determine the conditions which facilitate knowledge acquisition. We use differential equations to formulate the dynamics of knowledge acquisition for the students who are enrolled in the department of mathematics. We determine the progression of the numbers of students who understand or who acquire the knowledge required for them to graduate in the department of

mathematics. To do this we adopt mathematical model that is normally used in disease modelling in which lecturers or teachers" infect" students with knowledge. The law of mass action is also adopted. The" infected" students also can spread the knowledge they acquire to other students. The differential equations formed also take into consideration the dropout rates in both groups, all students recruited in the department of mathematics and those who have understood the concepts. In the next section we develop the mathematical model and perform stability analysis for the system. We then fit the given data to develop a mathematical model that predicts the progression of knowledge acquisition.

II. MATHEMATICAL FORMULATION

In this section we develop a mathematical model for screening students according to ability. We assume that those who teach mathematics 'infect' their students with knowledge. In this regard we incorporate the use of disease models to this situation. We further assume that the population of study is divided into two main groups; those students who are considered to be more knowledgeable and those less knowledgeable. A more knowledgeable individual is introduced in both groups (Teachers or lecturers).

This is now a common practice in South African universities in which students are classified as either main stream (more knowledge- able) and ECP (extended curriculum programme) (less knowledgeable). In this model we further divide these groups into two of each; those who are recruited into a particular department and are considered to be more knowledgeable x_1 , those more knowledgeable students who have acquired the required knowledge for them to successfully complete the course x_2 , those who are recruited into the department and are considered to be less knowledgeable x_3 , those less knowledgeable students who have acquired the required knowledge for them to successfully complete the course x_4 as shown in Figure 1.

Their contact with lectures and other learning facilities is modeled by the mass action principle. The force of knowledge acquisition in more knowledgeable students is given as $l_1 = b_1 x_2$ and the force of knowledge acquisition for less knowledgeable students is $l_2 = b_2 x_4$. The contribution from other students across streams is neglected.
The differential equations describing the knowledge acquisition is given in the compartmental model in Figure 1 by mimicking the model of Huppert and Katriel [5].

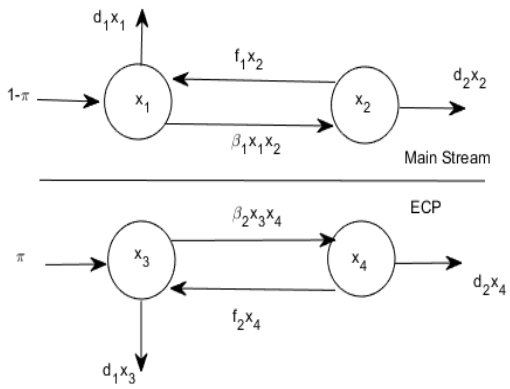


Figure 1: Compartmental model for knowledge flow

$$x_1' = (1 - p) + f_1x_2 - b_1x_1x_2 - d_1x_1 \quad (1)$$

$$x_2' = b_1x_1x_2 - (d_2 + f_1)x_2 \quad (2)$$

$$x_3' = p + f_2x_4 - b_2x_3x_4 - d_1x_3 \quad (3)$$

$$x_4' = b_2x_3x_4 - (d_2 + f_2)x_4 \quad (4)$$

$$x_1(0) = x_{01}, x_2(0) = x_{02}, x_3(0) = x_{03}, x_4(0) = x_{04} \quad (5)$$

Definition of parameters

p is the proportion of ECP students or the class considered to be less knowledgeable, b_1 is the rate of knowledge acquisition from other sources for example books, internet and teaching contacts, b_2 is the rate of knowledge acquisition for students classified as less knowledgeable from other sources and teaching contacts, d_1 is the dropout rate among students ready to acquire knowledge, d_2 is the dropout rate among students already understanding the taught concepts, f_1 is the rate of forgetting the acquired concepts among the knowledgeable students who will have acquired the knowledge. These then move back to the group that require more teaching contacts and other interventions.

f_2 the rate of forgetting the acquired concepts among less knowledgeable students who will have acquired the knowledge, these then move back to the group that require more teaching. The initial conditions

$x_1(0) = x_{01}, x_2(0) = x_{02}, x_3(0) = x_{03}, x_4(0) = x_{04}$ stipulate the initial values of the number of lecturers required to spread the knowledge to students.

III. ASSUMPTIONS OF THE MODEL

In this model we assume that more knowledgeable students have a higher understanding rate that less knowledgeable students $(b_1 > b_2)$ and that they also access learning material better than their less

knowledgeable counterparts. We further assume that the dropout rate in the beginning of the teaching period is less than one at a later stage $(d_1 < d_2)$; this is attributed to the fact that most students find out that the course is difficult for them at a later stage. These problems include failure to perform better in the course, financial problems and other related problems. We assume that more knowledgeable students have a better knowledge retention rate than their less knowledgeable counterparts $(f_1 < f_2)$. The learning model is that in which there is interaction between students. In this model only the knowledgeable students pass their knowledge to less knowledgeable students and not vice- versa. We also assume that more effort is applied in teaching less knowledgeable students.

IV. STABILITY ANALYSIS

In this section we investigate the 'knowledge free state', the situation in which teaching has not started or done. We also compute the conditions under which knowledge acquisition progress. Unlike in the case of disease modeling, we desire the knowledge acquisition to progress which would correspond to the progression of a disease. We compute the equilibrium points for the system (1) - (5). We note the region of demographical interest

$$W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4_+ \mid x_1 + x_2 \leq \frac{1-p}{d_1}, x_3 + x_4 \leq \frac{p}{d_1}\} \quad (6)$$

is positively invariant for the system (1) -(5) since the vector field on the boundary Ω does not point to the exterior of Ω. The positivity and boundedness of solutions is not discussed here. We study the stability of the steady states namely the knowledge free state and the knowledge acquisition state.
The knowledge free state is given by

$$E_0 = (\frac{1-p}{d_1}, 0, \frac{p}{d_1}, 0) \quad (7)$$

The condition under which knowledge acquisition progress is expressed by the second equilibrium point $(x_1^*, x_2^*, x_3^*, x_3^*)$.
At this stage we define the knowledge acquisition number S_0.
Definition 4.1. The basic knowledge acquisition number S_0, is the number of secondary teaching contacts produced by one knowledgeable teacher/student introduced in the recruited group of students. If $S_0 < 1$, then knowledge acquisition does not progress and if $S_0 > 1$ then knowledge acquisition progresses.

In the above definition, it is assumed that the knowledge acquired by students is shared amongst themselves. It is therefore necessary at this stage to state that not all contact amongst students result in successful knowledge transfer. For knowledge transfer to be successful, adequate contact should take place. According to Van den Driessche and Watmough [11], we calculate the value of S_0 by considering the matrix of new knowledgeable individuals.

From a rearranged system of equations (1) - (5) starting with the one responsible for knowledge acquisition. The second generation matrix was calculated and the value of S_0 being the dominant eigenvalue of this matrix. We obtain two knowledge acquisition numbers; one from the main stream student population S_{01} and the other from the ECP student population S_{02}. These values were found to be

$$\left\{S_{01} = \frac{b_1(1-p)}{d_1(d_2 + f_1)}, S_{02} = \frac{b_2 p}{d_1(d_2 + f_2)}\right\} \quad (8)$$

We now investigate the stability of the knowledge-free state (Condition under which no learning take place) and the state in which knowledge progress. The equilibrium point corresponding to the knowledge-free state is shown in equation (7). The stability of the knowledge-free state is obtained by computing the Jacobian matrix evaluated at the point (7). The eigenvalues are;

$$a_{1,2} = -d_1, a_3 = (d_2 + f_1)(S_{01} - 1), \quad (9)$$
$$a_4 = (d_2 + f_2)(S_{02} - 1).$$

The state in equation (7) is asymptotically stable if $S_{01} < 1$ and $S_{02} < 1$. This means that if there is no teaching and learning going on, the system tends to admit this state. The equilibrium point corresponding to the knowledge acquisition state is given by

$$E_1 = \left[\frac{d_2 + f_1}{b_1}, x_2^*, \frac{d_2 + f_2}{b_2}, x_4^*\right] \quad (9)$$
$$\left[x_2^* = \frac{d_1(d_2 + f_1)}{d_2 b_1}(S_{01} - 1), x_4^* = \frac{d_1(d_2 + f_2)}{d_2 b_2}(S_{02} - 1)\right]$$

To determine the stability of the knowledge acquisition state, we compute the Jacobian matrix evaluated at this point (9).

$$J(x_1^*, x_2^*, x_3^*, x_4^*) = \begin{pmatrix} A & B & 0 & 0 \\ C & 0 & 0 & 0 \\ 0 & 0 & D & E \\ 0 & 0 & F & 0 \end{pmatrix} \quad (10)$$

where

$$A = -\left(d_1 - \frac{d_1(d_2 + f_1)}{d_2}(1 - S_{01})\right)$$

$$B = E = -d_2$$

$$C = -\left(\frac{d_1(d_2 + f_1)}{d_2}(1 - S_{01})\right)$$

$$D = -\left(d_1 - \frac{d_1(d_2 + f_2)}{d_2}(1 - S_{02})\right)$$

$$F = -\left(\frac{d_1(d_2 + f_2)}{d_2}(1 - S_{02})\right) \quad (11)$$

Evaluating $|J - \lambda I| = 0$, the eigenvalues of the Jacobian matrix evaluated at the knowledge-acquisition state are

$$a_{1,2} = \frac{1}{2d_2^2}\left(-S_{01}d_1d_2^2 + H - \sqrt{K^2 + L}\right) \quad (11)$$

$$H = d_1d_2f_2(1 - S_{01}) \quad (12)$$

$$K = (S_{01}d_1d_2^2 - d_1d_2f_2(1 - S_{01}))^2 \quad (13)$$

$$L = 4(1 - S_{01})(d_1d_2^5 + d_1d_2^4f_2) \quad (14)$$

If $S_{01} > 1$ and $S_{02} > 1$, $a_{1,2}$ are negative eigenvalues, the knowledge acquisition state is asymptotically stable otherwise it is unstable. Note that the eigenvalues $a_{3,4}$ are exactly the same as shown in (11) -(14) with f_2 replaced with f_1 and S_{01} with S_{02}.

V. RESULTS AND DISCUSSION

In this section we discuss the results obtained from the differential equations. We investigate the effects of increasing the dropout rates in both groups, d_1 and d_2, teaching and other learning resource contact rates b_1 and b_2 respectively, and the forgetting rates f_1 and f_2 on the number of students acquiring knowledge. From stability analysis we have noted that if $(S_{01} < 1)$ and $(S_{02} < 1)$, there is no knowledge acquisition and when $(S_{01} > 1)$ and $(S_{02} > 1)$ knowledge acquisition is facilitated.

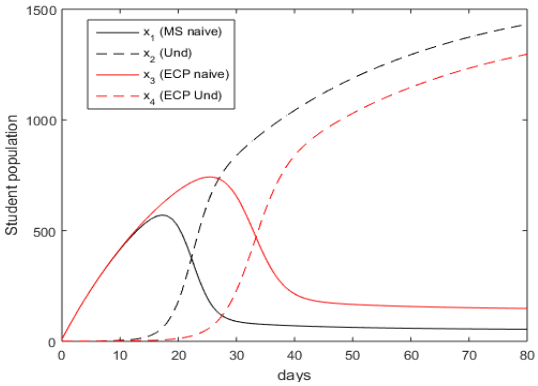


Figure 2: Knowledge acquisition profiles when equal numbers of students are enrolled in both MS and ECP.

In Figure 2 the student learning profile is shown in the case when equal numbers of students are enrolled in both streams; the main stream and the ECP. The student population from the main stream learn much faster than those in the ECP under given assumptions.

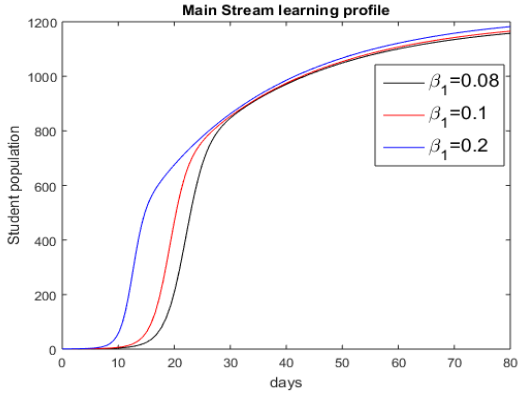


Figure 3: Effect of increasing b_1 on Mainstream learning profiles

In Figures 3, increasing the teaching contact and learning from other sources show rapid increase in the student population that understand taught concepts. The final number of students that understand the concepts also increase after 70 days.

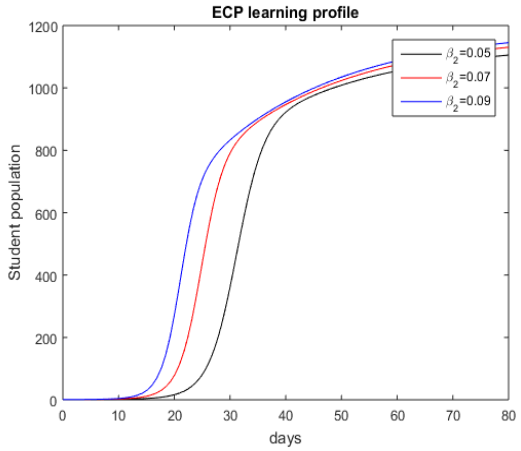


Figure 4: Effect of increasing b_{knk} on U_k & U_{nk}

In Figure 4, Increasing the teaching effort and learning from other material increase the student population that understand taught concepts. The trend is similar to the one observed in the main stream student population

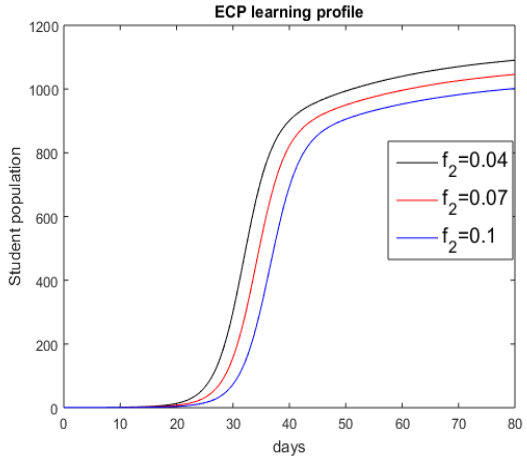


Figure 5: Effect of increasing f_2 on ECP learning profiles.

In Figure 5, the effect of increasing f_2 on the student learning profile is that of decreasing the student population who have understood taught concepts. The ability to retain knowledge is less that that shown in Figure 6 for the main stream students.

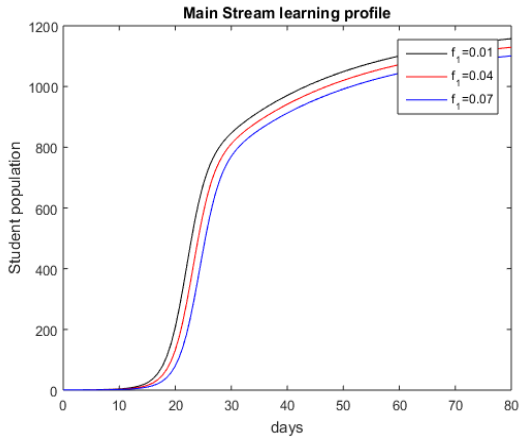


Figure 6: Effect of increasing f_1 on main stream learning profiles

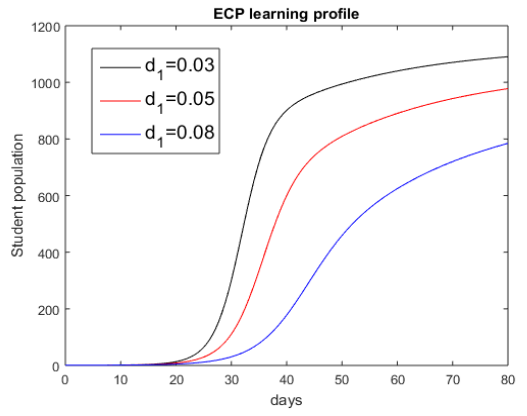


Figure 7: Effect of increasing d_1 on ECP learning profiles.

In Figure 7, the dropout rate in the beginning of the term has a negative impact on the learning profiles than at the end of the term. In Figure 8 the final number of students who understand taught concepts are much less than those shown in Figure 7. This shows that student retention must be targeted towards the end of the term. This is usually done by doing revision classes and addressing other issues outside learning of taught concepts

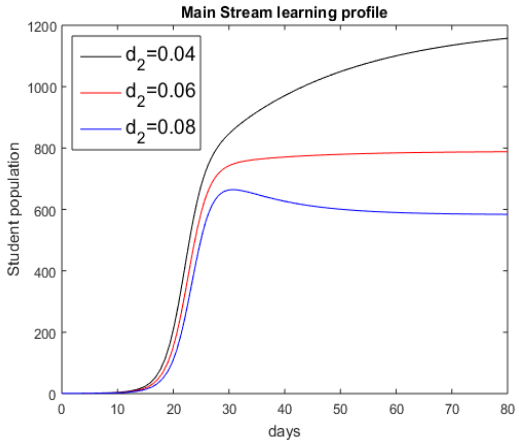


Figure 8: Effect of increasing d_1 on Mainstream learning profiles.

VI. DATA FITTING

In this section we fit the actual data of the performance of students taken from the department of mathematical and physical sciences at the Central University of Technology. The department realized that student performance was poor in mathematics. This was caused by various reasons among others poor lecture attendance, lack of tutorial practice and high dropout rate. To solve this problem, the department started administering weekly tutorial tests in which students were tested on concepts that were covered each week. In this section we used the data obtained from the performance of students in mathematics II. The number
of students who passed the tutorial tests were counted and the data is shown in the table below;

Table 1: Test results for knowledgeable and non-knowledgeable students.

Week	TT 1	TT 2	TT 3	TT 4	TT 5	TT 6	TT 7
Passes (U_k)	5	384	400	356	340	345	300
Passes (U_{nk})	10	118	120	130	122	130	116

We then perform parameter estimation by using nlinfit method in Matlab, where $p_1, p_2, p_3, p_4, p_5, p_6, p_7$ are unknown parameters. We start with initial guesses for the parameters. The methods use the Levenberg - Marquardt algorithm to estimate the parameters from

actual data. These parameters were obtained as $p = 0.62, d_1 = 0.83, b_1 = 0.25, d_2 = 0.45, f_1 = 0.99, f_2 = 1.89, b_2 = -1.85.$, with approximation error of 3.3×10^{-4}. The comparison of the actual data and theoretical data of the knowledge acquisition curve is shown in Figure 9.

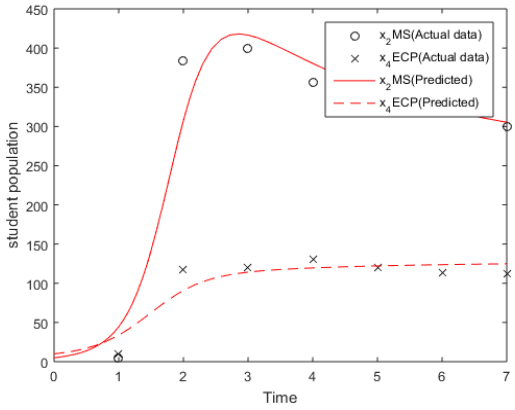


Figure 9: Predicted model on actual data for both streams

In Figure 9 it can be seen that the actual data trend is in agreement with the theoretical trend. The data was collected from a total population of 500 students. Each group had an initial number of knowledgeable individuals (teachers/lecturers) introduced. The estimated parameters can be used to predict knowledge acquisition in other similar groups.

CONCLUSION

In this paper we showed that indeed, the mathematical models that are normally used in disease models can be used in studying knowledge acquisition in education. This study reveals that The knowledge free state is asymptotically stable when $(S_{01} < 1)$ and $(S_{02} < 1)$ and the knowledge acquisition state is asymptotically stable when $(S_{01} > 1)$ and $(S_{02} > 1)$. Applying more teaching effort by providing more teaching contact in both streams ECP and main stream students under the current assumptions $b_2 < b_1$ help in improving knowledge acquisition. Early dropouts do not have more impact than late dropouts in knowledge acquisition.

Increasing the forgetting rate in more knowledgeable students affects both groups negatively. Forgetting of concepts is normally reduced by doing revision classes and weekly tutorials. A more detailed model which includes knowledge sharing between streams can be studied other studies. Estimated parameters can be used in other research work similar to the present one.

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