MATHEMATICAL SOLUTIONS FOR MEAN RADIANT TEMPERATURE CALCULATION IN A RECTANGULAR OR NON-RECTANGULAR GEOMETRY

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Abstract - Thermal environments of human body are affected by the mutual radiate heat exchange among the internal surfaces. This study describes an effective calculation method for view factors between non-rectangular surfaces in non-rectangular enclosures. The impact of room shape and different surface temperatures (windows and floor) on thermal comfort for a human body is estimated by computer simulation. For this purpose are rooms with two different shapes, two different floor surface temperatures, and two different window sizes considered to see the impact. A theoretical procedure for numerical approach of MRT (Mean Radiant Temperature) or OT (Operative Temperature) in rectangular spaces can be found in some literature. Author developed the method and program (COMFORT) for MRT calculation in a non-rectangular space. MRT distribution is provided as a function of the boundary condition of a model room and room shape.

Key words - Angle factor, Mathematical solution, Geometry, Non-rectangular.

I. INTRODUCTION

The heat losses by radiation from the human body to the surrounding surfaces play an important role in the whole heat balance of a person situated in a given environment. Thermal radiation is quantitatively evaluated by angle factor, over surface temperature, and emissivity of surface among internal surfaces. Among other factors thermal comfort depends on mean radiant temperature (MRT) or operative temperature (OT). Calculation of MRT or OT is dependent upon room geometry and the position of the human body within the room. Almost all of this work in this area was focused on a method for calculating person to enclosure radiant heat exchange for simplistic and orthogonal enclosures based only on the generation of angle factors of a person and a surface within a rectangular enclosure. But, today's architecture is not limited to simple rectangular enclosures. Simplification of a geometry is time-consuming and inaccurate. This paper presents a numerical solution method with an analytical equations and calculation results of MRT and OT for non-rectangular and rectangular enclosures.

II. ANGLE FACTOR UNDER NON-RECTANGULAR GEOMETRIC SURFACE

There are some cases for angle factor solution in non-rectangular surfaces in a literature. Angle factors between parallel- and perpendicular surfaces has a special relations with one of inclined surface. The special some cases, which have not yet reported in the literature, are developed in this study. Firstly, angle factor solution method in non-linear surface like as triangle is shown in figure 1.

We can use the following equations to calculate an angle factor between non-rectangular surfaces for some special cases like as Fig.1

$$\Phi_{12,34}+\Phi_{12,5}+\Phi_{12,6}+\Phi_{12,7}=1 \text{ and } \Phi_{12,5}=\Phi_{12,6},$$
$$\text{therefore, } \Phi_{12,34}+2\,\Phi_{12,5}+\Phi_{12,7}=1$$

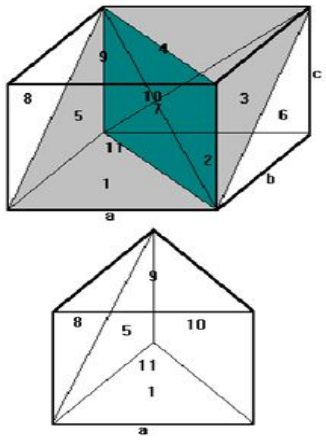
$$\Phi_{12,5}= 0.5\,(1-\Phi_{12,7}-\Phi_{12,34}) \quad (1)$$

$$\Phi_{1,58}+\Phi_{1,9}+\Phi_{1,11}+\Phi_{1,10}= 1,$$

$$\Phi_{1,10} = 1-(2\bullet\frac{c}{a}\bullet\Phi_{58,1}+\Phi_{1,9}+2\bullet\frac{c}{b}\bullet\Phi_{11,1}) \quad (2)$$

In equation (2), $\Phi_{1,9}$ is solved by the following equation (3).

$$\Phi_{1,9}=1-\frac{c\bullet(a+b+\sqrt{a^2+b^2})}{a\bullet b}+(\Phi_{10,11}+\Phi_{10,58})\bullet \frac{c\bullet\sqrt{a^2+b^2}}{0.5\bullet a\bullet b}+\frac{c}{0.5\bullet b}\bullet\Phi_{11,85} \quad (3)$$



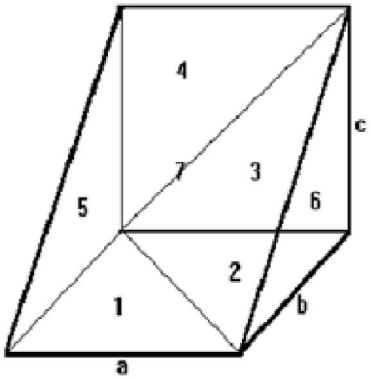


Figure 1 Non-rectangular surfaces

In an ambient non-rectangular special case, the angle factor can be also obtained by a point factor method for an infinite number of division surfaces with elements of equal area. But division surface into elements of equal area is not necessary for all surfaces. In this case, area weighted average method (equation (5)) is used to obtain the final result rather than a simple average method (equation (4)).

$$\Phi_{o,u} = \frac{E(1,u) + E(2,u) + ... + E(n,u)}{n} \quad (4)$$

$$\Phi_{o,u} = \frac{E(1,u)\cdot f_1 + E(2,u)\cdot f_2 + ... + E(n,u)\cdot f_n}{f} \quad (5)$$

In equation (4) and (5), u and f can be divided into many small surfaces like as figure 2 for precise results. In the equation (2), (3), (4) and (5), angle factors between any surfaces or E(n,u) can be obtained from the following three equations (6,7,8).

For perpendicular surface,

$$\varphi_{pe(fn,un)} = (z/(2\cdot\pi))\cdot [(1/\sqrt{x_1^2+z^2})\cdot \tan^{-1}(y_2/\sqrt{x_1^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_1^2+z^2}) - (1/\sqrt{x_2^2+z^2})\cdot \tan^{-1}(y_2/\sqrt{x_2^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_2^2+z^2})] \quad (6)$$

For parallel surface,

$$\varphi_{pa(fn,un)} = (1/(2\cdot\pi))\cdot [(x_2/\sqrt{x_2^2+z^2})\cdot \tan^{-1}(y_2/\sqrt{x_2^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_2^2+z^2}) + (y_2/\sqrt{y_2^2+z^2})\cdot \tan^{-1}(x_2/\sqrt{y_2^2+z^2}) - (y_1/\sqrt{y_1^2+z^2})\cdot \tan^{-1}(x_2/\sqrt{y_1^2+z^2}) - (x_1/\sqrt{x_1^2+z^2})\cdot \tan^{-1}(y_2/\sqrt{x_1^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_1^2+z^2}) - (y_2/\sqrt{y_2^2+z^2})\cdot \tan^{-1}(x_1/\sqrt{y_2^2+z^2}) + (y_1/\sqrt{y_1^2+z^2})\cdot \tan^{-1}(x_1/\sqrt{y_1^2+z^2})] \quad (7)$$

For inclined surface,

$$\varphi_{in(fn,un)} = kp\cdot[(1/(2\cdot\pi))\cdot(x_2/\sqrt{x_2^2+z^2})\cdot(\tan^{-1}(y_2/\sqrt{x_2^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_2^2+z^2})) + (y_2/\sqrt{y_2^2+z^2})\cdot \tan^{-1}(x_2/\sqrt{y_2^2+z^2}) - (y_1/\sqrt{y_1^2+z^2})\cdot \tan^{-1}(x_2/\sqrt{y_1^2+z^2}) - (x_1/\sqrt{x_1^2+z^2})\cdot(\tan^{-1}(y_2/\sqrt{x_1^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_1^2+z^2})) - (y_2/\sqrt{y_2^2+z^2})\cdot \tan^{-1}(x_1/\sqrt{y_2^2+z^2}) + (y_1/\sqrt{y_1^2+z^2})\cdot \tan^{-1}(x_1/\sqrt{y_1^2+z^2})] + ks\cdot[(z/(2\cdot\pi))\cdot(1/\sqrt{x_1^2+z^2})\cdot(\tan^{-1}(y_2/\sqrt{x_1^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_1^2+z^2})) - (1/\sqrt{x_2^2+z^2})\cdot(\tan^{-1}(y_2/\sqrt{x_2^2+z^2}) - \tan^{-1}(y_1/\sqrt{x_2^2+z^2}))] \quad (8)$$

In equation (8), if $\beta<90^o$ or $\beta>0^o$,

kp=cos(β)=z/B and ks=sin(β)= $\frac{\sqrt{B^2-z^2}}{B}$,

if $\beta>90^o$ and $\beta<180^o$, kp = -sin(β-90) and ks = cos(β-90)

Compareing with equation (6), (7) and (8), the following equation (9) and (10) can be found.

If $\beta<90^o$ and $\beta>0^o$ *in Fig.2*

$$\varphi_{in(fn,un)} = \cos(\beta)\varphi_{pa(fn,un)} + \sin(\beta)\varphi_{pe(fn,un)} \quad (9)$$

If $\beta>90^o$ and $\beta<180^o$ *in Fig.2*

$$\varphi_{in(fn,un)} = \cos(\beta-90)\varphi_{pe(fn,un)} - \sin(\beta-90)\varphi_{pa(fn,un)} \quad (10)$$

Equation (8) is a general solution of angle factor for any geometric angle. Equation (6) and (7) are respectively special solution for equation (8), namely angle factor for perpendicular surface and parallel surface. It is very interesting that the special relation among equation (6), (7) and (8) is found, like as (9).

III. ANGLE FACTOR CALCULATION METHOD UNDER NON-RECTANGULAR SURFACE AND ANGLE

The following Fig. 2 shows the surface division for the area weighted average method.

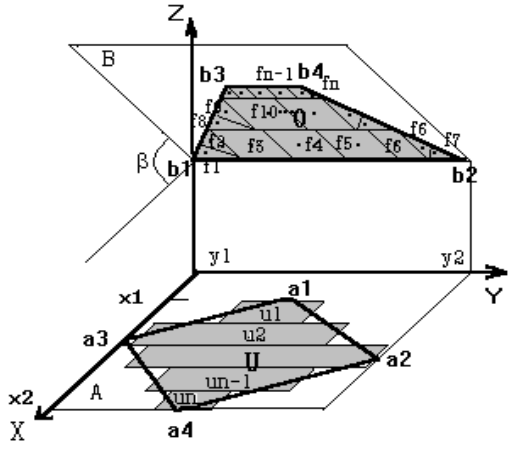


Figure 2 Area weighted average method

This area weighted average method has merits that a number of division is less than the simple average method. This method divides into a small area only at the edge area of a surface and the inner area can divide into relatively twice of the edge area. Therefore the

calculation speed and accuracy is higher than the other. Errors may relatively happen in edge area . That's why because the number of division surface is so many and calculation speed is too slow. The simple arithmetical average method needs many division surfaces to reduce the errors especially in every edge area. The figure 3 shows the principle of the weighted average of point shape factor method. This method doesn't need to make so many division surfaces. This method is needed more division surface only in edge area like as figure 3 (L1 and L2).

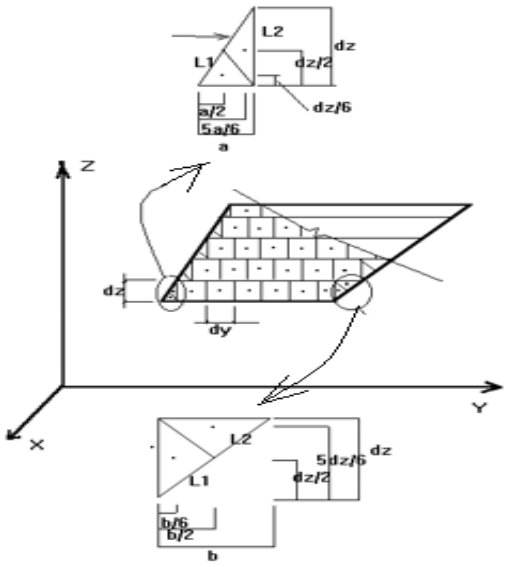


Figure 3 Division surface at the edge area

The following table (1) shows the angle factor calculation speed comparison between two different methods. Calculation has been fulfilled by the use of Program "VIEW17" which has been developed by author since 1994. Two different method used the same room geometry (room geometry: 6m•3.5m•2.6m) with same divided small surface ($1cm^2$) to compare with the calculation speed.

Table (1) Calculation speed comparison between two different methods(Samsung Note Book Computer, 5850 NT)

Calculation method	Simple average method	Area weighted average method
Speed (Second)	25	12

IV. THEORETICAL APPROACH FOR MRT AND OT CALCULATION

4.1 Development of simulation model "COMFORT"

The air temperature, without accounting for mean radiant temperature and operative temperature as means of evaluating individual comfort, is insufficient and inaccurate. The MRT as an indicator of thermal comfort has been calculated for prediction of indoor climate. Angle factor must be first solved to calculate MRT. A calculation method for angle factors among non-rectilinear surfaces was provided in author's previous work/3/. Programming procedure for MRT and OT is given. MRT is directly correlated to geometrical configuration of the enclosure and the location of the person. The MRT is calculated by the following equation (11).

$$T_{mrt} = \sqrt[4]{\sum_{i=1}^{n}((\Phi_{b,si}) \bullet T_{si}^4)} \qquad (11)$$

Otherwise the OT is calculated by the following equation (12).

$$OT = b \bullet T_{air} + (1-b) \bullet T_{mrt} \qquad (12)$$

Herein, the constant b is different from each researchers. The table 2 shows the constant b value for equation (12) according to researchers.

Table (2) Constant b-value

Author	B	Year
Mackey et al.*	0.5	1944
Neilsen and Pedersen*	0.42	1952
Gagge, Rapp and Dardy*	0.75	1967
P.Schmidt*	0.57	1981
Gehrlisch and Sachs*	0.5	1982
DIN ISO 7730*	0.5-0.7	1987
Bedford**	0.45	1936
Koch**	0.42	1962
McNall and Schlegel**	0.46	1968
McIntyre and Griffith**	0.41	1971
McIntyre and Griffiths**	0.56	1972

Physical approach , ** Subjective approach

There are various suggestions to evaluate the MRT in many literature from simple form of equation $\phi_{rp,wi} = \frac{A_{wi}}{\sum_{j=1}^{m} A_{wj}}$ to angle factor by experimental approach.

Many authors calculate the angle factors related to finite body and surface on a certain room point. In this study a cubic box is considered to simulate a human body. Figure 4 shows the principle of the cubic box to be calculated. Within the program the length of a,b and c can be changed under a limited condition.

In Fig. 4 , a $\leq$2xy(1), b$\leq$2xy(2), c$\leq$2xy(3) .

For this procedure, a form of equation $\phi_{rp,wi}$ =($\sum_{p=1}^{6} \Phi_{p,wi}$)/6 is given to calculate the total room angle factor. The value $\phi_{rp,wi}$ means the angle factor .

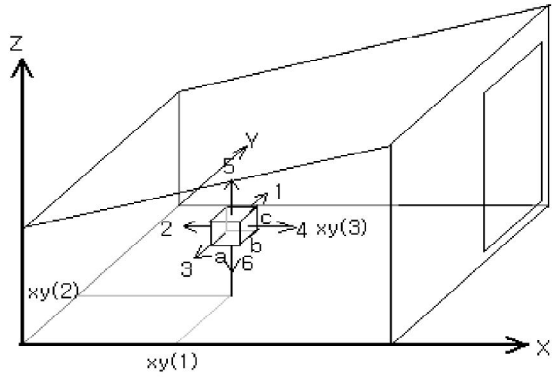


Figure 4 Principle of the cubic boxof each room inner surface for the room position.

According to this process, the simulation program "COMFORT" has been updated since the last version/4/. The room geometry of figure 5 is considered to validate the results according to the calculation method of this study against the results of Fanger et al/1/ and Schmidt/2/. The comparison shows a good agreement each other.

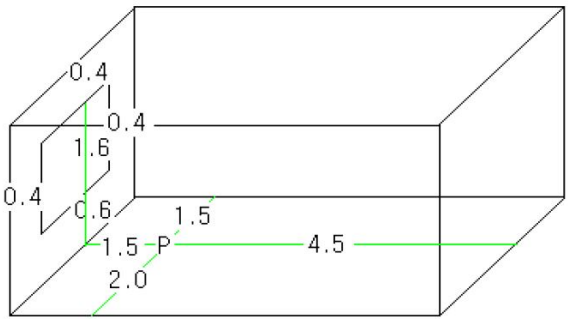


Figure 5 Room geometry for comparison

Table 3 shows the angle factor calculation results of the other suggestion Fanger et tal/1/ and Schmidt/2/ and table 4 shows the calculation results of this study for figure 5. The position of person is located in the same place with each other.

Table 3 Comparison of total room angle factor for the other study

Surface	Angle factor of Fanger (South)	Angle factor of Fanger (Without orientation)	Angle factor of Schmidt (H=0.6 m)	Angle factor of Schmidt (H=0.7 m)
Floor	0.343	0.346	0.363	0.342
Window	0.085	0.086	0.080	0.085
West wall	0.070	0.072	0.068	0.068
North wall	0.167	0.177	0.166	0.171
East wall	0.033	0.033	0.031	0.031
South wall	0.152	0.135	0.126	0.128
Heating surface	0.112	0.108	0.128	0.137
Ceiling	0.038	0.038	0.038	0.038
Sum	1.000	0.995	1.000	1.000

Table 4 Comparison of total room angle factor for this study (1•1•1 mm^3 cubic box)

Height Surface	0.6 m	0.65 m	0.7 m	0.75
Floor	0.37247	0.36188	0.35118	0.34136
Window	0.07990	0.08143	0.08300	0.08449
West wall	0.06734	0.06805	0.06852	0.06895
North wall	0.16553	0.16782	0.16995	0.17198
East wall	0.02693	0.02699	0.02702	0.02707
South wall	0.12309	0.12440	0.12558	0.12665
Heating surface	0.12542	0.12994	0.13445	0.13954
Ceiling	0.04016	0.03975	0.03063	0.03988
Sum	1.001	1.000	1.000	1.000

There is no significant difference between these study results and experimental results to evaluate practically.

4.2 Thermal comfort and view factor calculation

Thermal comfort comparison between two different human body exposures is fulfilled. The dimension of the room for simulation is 4m(X)•5m(Y)•3m(Z). The window is placed at 1m height from floor. The dimension of window is 3m•1.7m. The ceiling panel surface is 3.6m•4.6m, corresponding 80% of total ceiling surface, in the center of room. A human body can be simulated by Dubois'equation (13) for a body's surface area calculation.

$Adu = 0.203 \bullet W^{0.425} \bullet H^{coef}$ (13)

Herein, Coefficient coef is 0.696 for a sitted person. Weight W of a person is 76Kg, Height H is 110 cm for a sitted person.

As it is mentioned in the capital 4.1, a cubic box is used as a human body model like as figure 6 for simulating a sitting posture.

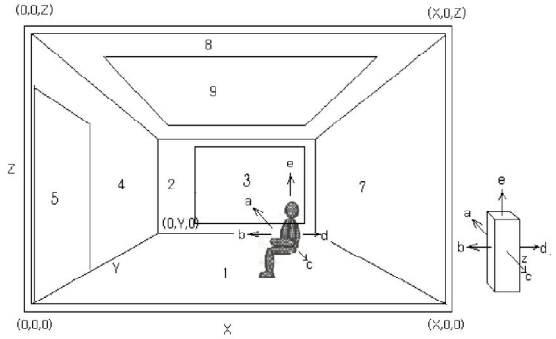


Figure 6. Cubic box for simulating a sitted person

CONCLUSIONS

Angle factor just as a geometric rational number could give us a tremendous influence on physical and social environment. An angle factor between two different inclined surfaces is expressed by a proportion of cos(β) and sin(β) to perpendicular and parallel surfaces. The simulation model "COMFORT" can describe effectively influences of surrounding built environments on human body in arectangular or non-rectangular geometry.

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NOMENCLATURE

E(I,j), $\Phi_{i,j}$, $\varphi_{i,j}$: Angle factor of a surface i to a surface j
f : over surface, u: under surface

INDICES

Pe: perpendicular, Pa: parallel, In :inclined
n : number of divided area

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