A DETERMINISTIC MODEL FOR LATE PREGNANCY AMONG GHANAIAN WOMEN

[1]MICHAEL OFORI FOSU, [2]STEPHEN B. TWUM, [3]SBORNE Y.A JACKSON

[1]Kumasi Technical University, Department of Mathematics and Statistics, Kumasi Ghana
[2]Department of Mathematics, University for Development Studies Navrongo Ghana
[3]Methodist University College, Accra Ghana
E-mail: mikeoffos@yahoo.com

Abstract - Low birth weight is known to be one of the leading causes of child mortality particularly in sub-Saharan Africa. In this study, we formulate and analyze a deterministic model of aged women giving birth to low birth weight babies. The basic properties of model are studied and key parameters were also estimated. The basic reproduction number is computed and the steady states of the system also studied and found to be locally asymptotically stable. Time dependent controls are included in the model and Pontryagin's Maximum Principle is employed to characterize the necessary conditions for optimal control of low birth weight. The numerical simulation results indicate that the best strategy to minimize the low birth weight incidence is to concentrate on the three controls at the same time.

Keywords - Low birth weight, optimal control, basic reproduction number, stability, invariant region.

I. INTRODUCTION

As the saying goes; Age is nothing but a number. But when it comes to getting pregnant and having a healthy pregnancy, it can matter. Even though most healthy women who get pregnant after age 35 and even into their 40s have healthy babies. That doesn't mean that one shouldn't think about smart steps to take to maximize the woman and the baby's health during pregnancy.

There are things that need to be undertaken by aged women who want to get pregnant to enable them to reduce the risk of having unhealthy babies. This includes: preconception check-ups and counselling, early and regular prenatal care, optional prenatal tests for women over 35, taking prenatal vitamins, keeping up with other doctor appointments, maintaining a healthy and well-balanced diet,gaining the recommended amount of weight, regular exercise, avoiding smoking and drinking alcohol and askingones' doctor about medications (Traci 2017).

However, in Ghana, most women don't plan for their birth (Fosu et al 2013) and therefore preconception counselling and check-ups don't happen. Again, most women don't go for prenatal care and as a result most women deliver at home (MICS 2011). All these lead to increase in risk factors associated with old-age pregnancy which includes delivery of low birth weight babies. (in Ghana, about 22% of low birth weight babies are delivered by women who are 35+ (MICS 2011).

The results from all our earlier works (Fosu et al 2013, 2016) indicate that age (especially women aged 35+) is a major risk factor associated with low birth weight deliveries in Ghana and therefore there is a need to come up with a model that can determine or predict the outcome of old-age pregnancy.

It is expected that pregnancy, especially late pregnancy, must be well planned to reduce the risk of low birth weight deliveries and subsequent under-growth children who are more prone to diseases and mortality than normal birth weight children. However, as indicated earlier, discussions with health practitioners before getting pregnant don't happen and prenatal services by would be mothers also are not taken seriously and therefore diseases such as gestational diabetes and eclampsia which are risky for both the pregnant mother and the unborn child are not identified and managed. This may result in mothers giving birth to low birth weight babies who in most cases become under-growth.

We therefore seek to model the risk associated with unplanned pregnancies by aged women using a deterministic approach putting in treatments and controls to achieve an optimal control in predicting the outcome of birth. Some parameters based on our earlier results are estimated to simulate the numerical analysis.

In this study, we propose a new model with optimal control to examine the effect of prenatal services and effective medication to mothers. Thepurposeof thiscontrolistominimizethenumberof LBW babies among late pregnancies andunder-growth childrenwithoptimalcost of prenatal services and effective medication applying Pontryagin's Maximum Principle. This principle has been applied by Okosum et al (2010 and 2011) in modeling malaria disease. Again, Nishiura H. et al recently (2016) used this model in the study of the Zika virus.These are borne out of the review byHethcote, H. (2000) The mathematics of infectious diseases.

II. METHOD

2.1 Model Formulation

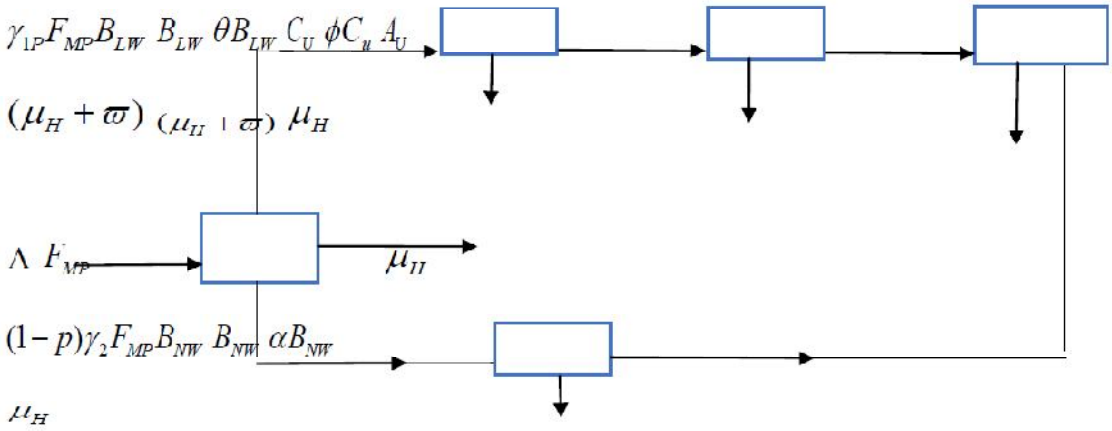
The model sub-divides the total aged women population at time t, denoted by $N(t)$, into the

following sub-populations of aged pregnant women ($F_{MP}(t)$), individual aged women are more likely to give birth to low birth weight babies ($B_{LW}(t)$), individual aged women can give birth to normal birth weight babies ($B_{NW}(t)$), individual normal babies tend to become adults ($A_U(t)$), low birth weight babies will turn to become children with under growth ($C_U(t)$), the under growth child will eventually become an adult ($A_U(t)$).

Therefore,

$$N(t) = F_{MP}(t) + B_{LW}(t) + B_{NW}(t) + C_U(t) + A_U(t).$$

Individual aged women are recruited at a rate Λ. Individual aged women give birth to low weight babies at a rate γ_1 of proportion p. Individual aged women of a proportion (1-p) can give birth to a normal weight baby at rate γ_2. The individual normal weight babies may become adults at a rate α. Due to sicknesses which usually characterize low birth weight babies, they later become under growth at rate θ. These under growth children then become adults at a rate ϕ. The natural mortality of humans is given as μ_H



Parameter	Description	Value	Ref
Λ	The recruitment rate of the aged women (35+)	0.26	Estimated
γ_1	The transmission rate of low birth weight (proportion of women 35+ who recorded LBW babies)	0.11	Estimated
μ_H	Natural death rate in humans (CDR in Ghana)	53/365per 1000	Estimated
γ_2	The transmission rate of normal birth weight for aged woman	0.886	Estimated
θ	The rate low birth weight babies move to children under growth	0.01	*Shah et al.2016*
α	The rate normal birth weight babies move to adulthood.	0.001	Assumed
ϕ	The rate individuals move into adulthood	$100\ days^{-1}$	Assumed
ϖ	The disease induced death rate among LBW babies	0.008	Estimated

Table1. Description of variables and parameters of the aged women model

The set of differential equations corresponding to the above malnutrition transmission diagram and table 3 is:

$$\begin{cases} \frac{dF_{MP}}{dt} = \Lambda - \gamma_1 p F_{MP} B_{LW} - (1-p)\gamma_2 F_{MP} B_{NW} - \mu_H F_{MP} \\ \frac{dB_{LW}}{dt} = \gamma_1 p F_{MP} B_{LW} - (\mu_H + \theta + \varpi) B_{LW} \\ \frac{dB_{NW}}{dt} = (1-p)\gamma_2 F_{MP} B_{NW} - (\mu_H + \alpha) B_{NW} \\ \frac{dC_U}{dt} = \theta B_{LW} - (\phi + \mu_H + \varpi) C_U \\ \frac{dA_U}{dt} = \phi C_U + \alpha B_{NW} - \mu_H A_U \end{cases} \quad (1)$$

Where, $F_{MP}(0) > 0$, $B_{LW}(0) > 0$, $B_{NW}(0) > 0$, $C_U(0) > 0$ and $A_U(0) > 0$

2.2 Model Analysis

In this section, the basic properties and stability analysis of system (1) are carried out without time optimal controls incorporated.

For poor nutrition in the human life cycle to be epidemiologically meaningful, it is vital toprove that all solutions with non-negative interval data will remain non-negative for all time.

Theorem 1: If $F_{MP}(0); B_{LW}(0); B_{NW}(0)$: $C_U(0)$ and $A_U(0)$ are non-negative, then so are $F_{MP}(t); B_{LW}(t); B_{NW}(t): C_U(t)$ and $A_U(t)$ for all time t>0

Moreover, $\lim_{t\to\infty} \sup N(0) \le \frac{\Lambda}{\mu_H}$, then, $N(t) \le \frac{\Lambda}{\mu_H}$

The proof is omitted for simplicity. The feasibility region for the model is given by $D = \mu_H D \subset \mathscr{R}_+^5$

Where, $\mu_H D = \{(F_{MP}, B_{LW}, B_{NW}, C_U, A_U) \in \mathscr{R}_+^5 : F_{MP} + B_{LW} + B_{NW} + C_U + A_U\}$

D is positively in variant.

2.2.1Positivity of Solutions

We need to demonstrate by a way of proof that given any non-negative initial conditions of system (1), of $F_{MP}(0)$, $B_{LW}(0)$, $B_{NW}(0)$, $C_U(0)$, $A_U(0)$, the solutions continue to be non-negative for all $t \in [0, \infty)$. We show that the entire state variables stay non-negative. The solution of the aged women giving birth model, of system (1) having non-negative initial conditions will continue to be positive for all $t > 1$. We therefore state the following lemma.

Lemma 1: Suppose that the initial conditions of system (1) are non-negative, the solutions $F_{MP}(t), B_{LW}(t), B_{NW}(t), C_U(t)$ and $A_U(t)$ are positive for all $t > 0$

Proof:(omitted)

2.2.2Steady states

Here,we determine the steady states solution by setting the right-hand side of system (1) to zero. This direct calculation depicts that system (1) consequently has a low birth weight free equilibrium (LWFE) point $E_0 = \left(\frac{\Lambda}{\mu_H}, 0, 0, 0, 0\right)$ and endemic equilibrium point is $E_1 = \left(F_{MP}^*, B_{LW}^*, B_{NW}^*, C_U^*, A_U^*\right)$ in Ω where,

$$\begin{cases} F_{MP}^* = \frac{\theta + \varpi + \mu_H}{\gamma_1}, \\ B_{LW}^* = \frac{-\alpha\theta + \Lambda\gamma_1 - \alpha\mu_H - 2\theta\mu_H - 2\theta\mu_H^2}{\gamma_1(\theta + \varpi + \mu_H)}, \\ B_{NW}^* = \frac{\alpha + \mu_H}{(-1+p)\gamma_1}, \\ C_U^* = \frac{\theta(-\Lambda\gamma_1 + (\theta + \mu_H)(\alpha + 2\mu_H))}{\gamma_1(\theta + \varpi + \mu_H)(\phi + \varpi + \mu_H)}, \\ A_U^* = \frac{(1-p)\theta\Lambda\mu_H\gamma_1(\theta + \mu_H)(\alpha(\alpha + (-1+p)\theta)\phi + \mu_H(\alpha^2 + (\alpha + 2(-1+p)\theta)\phi + \alpha\mu_H))}{(-1+p)\gamma_1(\theta + \varpi + \mu_H)(\phi + \varpi + \mu_H)} \end{cases}$$

(2)

2.3The model reproduction number

A reproduction number, usually expressed in terms of average number of human infected by an index case, is an essential threshold parameter that measures whether a disease or condition persists and become extinct in a population. Let R_0 be the model reproduction number. Applying the next generation operator method by (Okosun and Makinde, 2011) to determine the reproduction number, F and V respectively, represent the matrices for the new cases of aged women giving birth to low weight babies and the transition terms then wehave;

$$F = \begin{pmatrix} \frac{p\gamma_1\Lambda}{\mu_H} & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} \theta + \mu_H + \varpi & 0 \\ -\theta & \phi + \mu_H + \varpi \end{pmatrix}$$

The basic reproduction number R_0 is defined as the spectral radius of the matrix FV^{-1} and thus,

$$R_0 = \rho\left(FV^{-1}\right) = \frac{p\gamma_1\Lambda}{\mu_H(\theta + \mu_H + \varpi)} \tag{3}$$

2.4Stability of the low birth weight freeequilibrium

Theorem 2: The low birth weight free equilibrium E_0 whenever it exists, is locally asymptotically stable if $R_0 < 1$ and unstable otherwise.

Proof: The Jacobian matrix of system (1) at the equilibrium point E_0 is obtained as;

$$J(E_0) = \begin{pmatrix} -\mu_H & -\frac{\gamma_1 p\Lambda}{\mu_H} & -\frac{\gamma_2(1-p)\Lambda}{\mu_H} & 0 & 0 \\ 0 & \frac{\gamma_1 p\Lambda}{\mu_H} - (\mu_H + \theta + \varpi) & 0 & 0 & 0 \\ 0 & 0 & \frac{\gamma_2(1-p)\Lambda}{\mu_H} - (\mu_H + \alpha) & 0 & 0 \\ 0 & \theta & 0 & -(\mu_H + \phi + \varpi) & 0 \\ 0 & 0 & \alpha & \phi & -\mu_H \end{pmatrix}$$

It can be observed from the Jacobian matrix evaluated at low birth weight free that the eigenvalues of J_{E_0} whichare $-\mu_H, -\mu_H, -(\phi + \mu_H)$ are negative and the rest are obtained by the solution of the characteristic polynomial (4):

$$p(\lambda) = [\lambda^2 + a_1\lambda + a_2] = 0 \tag{4}$$

where

$$a_1 = \alpha + \theta - \frac{p\Lambda\gamma_1}{\mu_H} - \frac{\Lambda\gamma_2}{\mu_H} + \frac{p\Lambda\gamma_2}{\mu_H} + 2\mu_H$$

$$a_2 = \frac{p\Lambda^2\gamma_1\gamma_2 + p\Lambda(\gamma_2\mu_H(\theta + \varpi + \mu_H) + \gamma_1\Lambda\gamma_2 - \mu_H)}{\mu_H\alpha} + \mu_H\alpha(1 - R_0^2)$$

The solution of $p(\lambda) = 0$ have negative real parts only if $R_0 < 1$. Thus, we conclude that the low birth weight free equilibrium is locally asymptotically stable whenever $R_0 < 1$.

2.5Stability of the endemic equilibrium

Theorem 3: If $R_0 > 1$, the endemic low birth weight equilibrium point of system (1), is locally asymptotically stable.

Proof: The Jacobian matrix of system (1) at endemic equilibrium is determined so that,

$$J_{E_1}=\begin{pmatrix} \mu_H-\gamma_1 pB_{LW}-\gamma_2(1-p)B_{NW} & -\gamma_1 pF_{MP} & -\gamma_2(1-p)F_{MP} & 0 & 0 \\ \gamma_1 pB_{LW} & \gamma_1 pF_{MP}-(\mu_H+\theta+\varpi) & 0 & 0 & 0 \\ \gamma_2(1-p)B_{NW} & 0 & \gamma_2(1-p)F_{MP}-(\mu_H+\alpha) & 0 & 0 \\ 0 & \theta & 0 & -(\mu_H+\phi+\varpi) & 0 \\ 0 & 0 & \alpha & \phi & -\mu_H \end{pmatrix}$$

The two of the eigenvalues of the endemic equilibrium of the Jacobian matrix J_{E_1} are negative $-\varpi-\phi-\mu_H, -\mu_H$ and the rest of the eigenvalues can be obtained from the characteristic equation (5).

$$b(\lambda) = [\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3] = 0 \qquad (5)$$

where

$$b_1 = -\alpha-\theta-pB_{LW}\gamma_1+pF_{MP}\gamma_1-B_{NW}\gamma_2+pB_{NW}\gamma_2+F_{MP}\gamma_2-pF_{MP}\gamma_2-3\mu_H$$

$$b_2 = -\alpha\theta+(-1+p)(\alpha+\theta)B_{NW}\gamma_2+(-1+p)pF_{MP}^2\gamma_1\gamma_2-2(\alpha+\theta-(-1+p)B_{NW}\gamma_2)\mu_H -3\mu_H^2-pB_{LW}\gamma_1(\alpha+\theta+(-1+p)F_{MP}\gamma_2+2\mu_H)+F_{MP}(-(-1+p)\gamma_2(\theta+\varpi+2\mu_H)+p\gamma_1(\alpha-(-1+p)B_{NW}\gamma_2+2\mu_H))$$

$$b_2 = -pB_{LW}\gamma_1(\theta+\varpi+\mu_H)(\alpha+(-1+p)F_{MP}\gamma_2+\mu_H) -(\theta-pF_{MP}\gamma_1+\mu_H)(-(-1+p)B_{NW}\gamma_2(\alpha+\mu_H)+\mu_H(\alpha+(-1+p)F_{MP}\gamma_2+\mu_H))$$

The principles of Routh-Hurtwiz criteria is satisfied correctly as $b_1>0,\ b_2>0,\ b_3>0$ and $b_1b_2-b_3>0$ if $R_0>1$. The eigenvalues of the characteristics equations (5) have negative real parts whenever $R_0>1$. Hence, every eigenvalue of the system (1) possesses a negative real part. Thus, endemic aged women giving birth of the model system (1) at E_0 is locally asymptotically stable.

2.6Analysis of Optimal Control

In this section, Pontryagin's Maximum Principle was used to determine the necessary conditions for the optimal control of the aged women capable of giving birth. We desire to minimize the number of low birth weight babies and children under growth and the cost associated with providing the necessary public education, giving medications to enhance healthy normal baby weight and treatment of children with low birth weight. We thus, examine an optimal control problem having the objective function which is given by,

$$J(u_1,u_2,u_3)=\int_0^{t_f}\left(b_1B_{LW}+b_2C_u+\frac{A\mu_1^2}{2}+\frac{B\mu_2^2}{2}+\frac{C\mu_3^2}{2}\right)dt \qquad (6)$$

Where A, B and C denote the weighting constants for prevention (giving intensive public education, prenatal services, nutrition, eating/sleeping habits), treatment (providing effective medication for giving birth to healthy normal babies, food supplements e.g. vitamins, vaccinations) and treatment (giving treatment and care to children with low birth weight; incubation/kangaroo mother care and exclusive breast care, vaccination), respectively. The costs of the prevention and treatment tend to be nonlinear and assume quadratic function character. We seek an optimal control u_1^*, u_2^* and u_3^* such that, $J\left(u_1^*,u_2^*,u_3^*\right)=\min J\left(u_1,u_2,u_3\right)$, $\Gamma=\left\{\left(u_1,u_2,u_3\right)\middle|0\le u_i\le 1,\ i=1,2\right\}$.

$$\begin{cases} \dfrac{dF_{MP}}{dt}=\Lambda-(1-\mu_1)\gamma_1 pF_{MP}B_{LW}-\mu_2(1-p)\gamma_2 pF_{MP}B_{NW}-\mu_H F_{MP} \\ \dfrac{dB_{LW}}{dt}=(1-\mu_1)\gamma_1 pF_{MP}B_{LW}-(\mu_H+\varpi+\mu_3\theta)B_{LW} \\ \dfrac{dB_{NW}}{dt}=(1-p)\gamma_2 pF_{MP}B_{NW}-(\mu_H+\alpha)B_{NW} \\ \dfrac{dC_U}{dt}=\mu_3\theta B_{LW}-(\phi+\varpi+\mu_H)C_U \\ \dfrac{dA_U}{dt}=\phi C_U+\alpha B_{NW}-\mu_H A_U \end{cases} \qquad (7)$$

The necessary conditions that an optimal solution exist come from (Pontryagin et al, 1962) Maximum Principle. This principle converts (6) and (7) into a problem of minimizing pointwise a Hamiltonian H, with respect to u_1 and u_2.

$$\begin{aligned} H &= b_1 B_{LW} + b_2 C_u + A\mu_1^2 + B\mu_2^2 + C\mu_3^2 \\ &+ \lambda_{F_{MP}} \{\Lambda - (1-\mu_1)\gamma_1 p F_{MP} B_{LW} - \mu_2 (1-p)\gamma_2 p F_{MP} B_{NW} - \mu_H F_{MP}\} \\ &+ \lambda_{B_{LW}} \{(1-\mu_1)\gamma_1 p F_{MP} B_{LW} - (\mu_H + \varpi + \mu_3\theta) B_{LW}\} \\ &+ \lambda_{B_{NW}} \{(1-p)\gamma_2 p F_{MP} B_{NW} - (\mu_H + \alpha) B_{NW}\} \\ &+ \lambda_{C_U} \{\mu_3\theta B_{LW} - (\phi + \varpi + \mu_H) C_U\} \\ &+ \lambda_{A_U} \{\phi C_U + \alpha B_{NW} - \mu_H A_U\} \end{aligned} \tag{8}$$

Where $\lambda_{F_{MP}}$, $\lambda_{B_{LW}}, \lambda_{B_{NW}}$, λ_{C_U} and λ_{A_U} are the adjoint variables or co-state variables. The system of equations is found by taking the appropriate partial derivatives of the Hamiltonian (7) with respect to the associated state variable.

Theorem 4: Given optimal controls u_1^*, u_2^*, u_3^* and solutions F_{MP}, B_{LW}, B_{NW}, C_U, A_U of the corresponding state system (6.1)-(6.2) that minimize $J(u_1, u_2, u_3)$ over U. Then there exists adjoint variables $\lambda_{F_{MP}}, \lambda_{B_{LW}}, \lambda_{B_{NW}}, \lambda_{C_U}, \lambda_{A_U}$ satisfying.

$$\frac{-d\lambda_i}{dt} = \frac{\partial H}{\partial_i} \tag{6.0}$$

where $F_{MP}, B_{LW}, B_{NW}, C_U, A_U$ and with transversality conditions

$$\lambda_{F_{MP}}(t_f) = \lambda_{B_{LW}}(t_f) = \lambda_{B_{NW}}(t_f) = \lambda_{C_U}(t_f) = \lambda_{A_U}(t_f) = 0. \tag{6.1}$$

and

$$\mu_1^* = \min\left\{1, \max\left(0, \frac{\gamma_1 p F_{MP} B_{LW} (\lambda B_{LW} - \lambda F_{MP})}{2A}\right)\right\}, \tag{6.2}$$

$$\mu_2^* = \min\left\{1, \max\left(0, \frac{(1-p)\gamma_2 p F_{MP} B_{NW} (\lambda B_{NW} - \lambda F_{MP})}{2B}\right)\right\}, \tag{6.3}$$

$$\mu_3^* = \min\left\{1, \max\left(0, \frac{\theta B_{LW} (\lambda C_U - \lambda B_{LW})}{2C}\right)\right\}. \tag{6.4}$$

Proof:(Fleming and Rishel1975) gives the existence of an optimal control due to the convexity of the integrand of J with respect to, u_1, u_2 and u_3 a priori boundedness of the state solutions, and the Lipschitz property of the state system with respect to the state variables. The differential equations governing the adjoint variables are obtained by differentiation of the Hamiltonian function, evaluated at the optimal control.Then the adjoint equations can be written as in (Fleming et al, 1975).

$$\begin{aligned} -\frac{d\lambda F_{MP}}{dt} &= \mu_H \lambda F_{MP} + (1-\mu_1)\gamma_1 p B_{LW} (\lambda F_{MP} - \lambda B_{LW}) + \mu_2 (1-p)\gamma_2 B_{NW} (\lambda F_{MP} - \lambda B_{NW}) \\ -\frac{d\lambda B_{LW}}{dt} &= -b_1 + (1-\mu_1) p F_{MP} (\lambda F_{MP} - \lambda B_{LW}) - (\mu_H + \varpi + \theta)\lambda B_{LW} - \theta\lambda C_U \\ -\frac{d\lambda B_{NW}}{dt} &= \mu_2 (1-p)\gamma_2 F_{MP} (\lambda F_{MP} - \lambda B_{NW}) + (\mu_H + \alpha)\lambda B_{NW} - \alpha\lambda A_U \\ -\frac{d\lambda C_U}{dt} &= -b_2 + (\phi + \varpi + \mu_H) C_U - \phi\lambda A_U \\ -\frac{d\lambda A_U}{dt} &= \mu_H \lambda A_U \end{aligned} \tag{6.5}$$

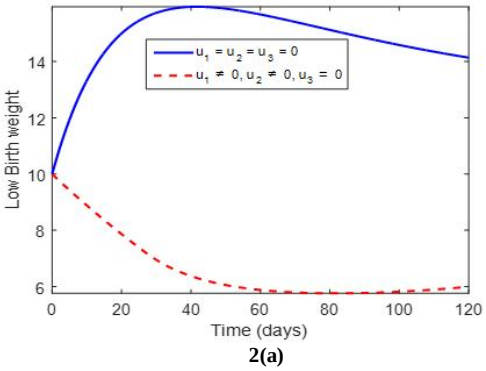
III. RESULTS

3.1: Numerical Simulations for deterministic model

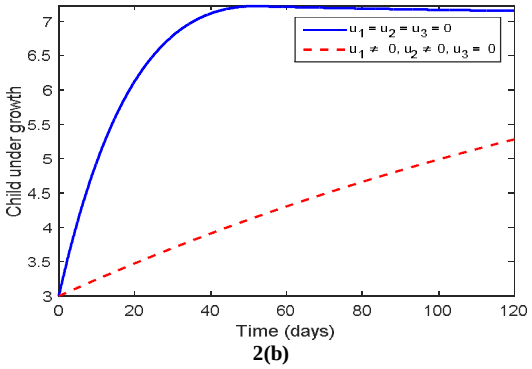
To illustrate the results of the deterministic model, numerical simulations of the model were carried out, using parameter values given in Table (1).

A. Prevention (u_1) and treatment (u_2) controls only

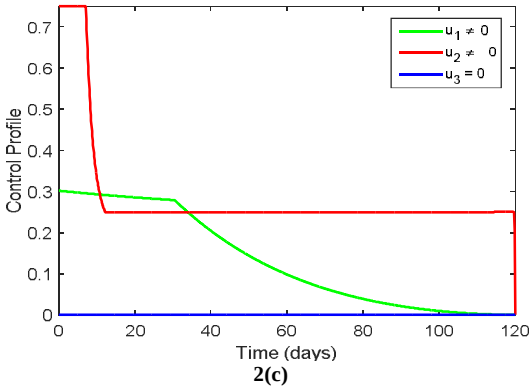
The Low birth weight prevention control u_1 and the treatment medications that enhance the chance of giving birth to healthy babies control u_2 are used to optimize the objective function J while we set the treatment of Low birth weight control (u_3) to zero. We observed in Figure 2(a) a significant difference in the number of Low birth weight babies B_{LW} under control and those without application of control. The positive effect as observed in Figure 2(a) depicts the effectiveness of educational campaigning which suggests that women of old ages giving birth actually take prenatal care with all the seriousness it deserves. In Figure 2(b) the number of children under growth C_U is not controlled as it begins to rise after some time. This indicates that this strategy is not effective in controlling the number of children under growth C_U. This is expected because there is no treatment strategy on the number of children under growth C_U as shown in Figure 2(b). The Figure 2(c) shows the control profile for the strategy used as control (u_1) requires an initial effort of 30% for about 40 days then gradually reduce during the rest of the intervention. In this strategy total effort of 100% is required for 10 days of the (u_2) which then sharply reduces to 25% which is kept for the rest of the intervention while control (u_3) is set to zero as shown in Figure 2(c).



2(a)



2(b)



2(c)

Figure 2: Simulations of the model showing the effect of aged women giving birth using prevention and treatment only on transmission.

B. Prevention (u_1) and treatment (u_3) control only

The prevention control u_1 and the treatment control u_3 are used to optimize the objective function J at the same time we set the treatment control (u_2) to zero. We can infer from Figure 3(a) that there is a significant difference in the number of Low birth weight B_{LW} in the presence of controlled cases and without control cases. In Figure 3(a) there is a substantial difference between control cases and without control cases. In fact, the positive effect of this strategy is not enough to totally control the Low birth weight as there is a rise after the intervention. This suggests that more effort is being put in the treatment cases of Low birth weight rather than prevention of the situation B_{LW}. Figure 3(b) shows that the control strategy is very efficient and robust in controlling the number of children under growth C_U. This implies that the medical facilities such incubators and drugs are of high quality and capable of improving the Low birth weight situation as shown in Figure 3(b). In Figure 3(c), the control profile for this strategy is shown as control (u_2) is set to zero.

This strategy requires a maximum effort of 100% for 37 days of control (u_1) which then gradually reduce during the rest of the intervention. Also, the strategy requires a maximum effort of control (u_3) 100% for 100 days before reducing till the end of the rest of the intervention. This effort of 100% for 100 days has manifested in the success of controlling the number of children under growth as shown in Figure 3(b).

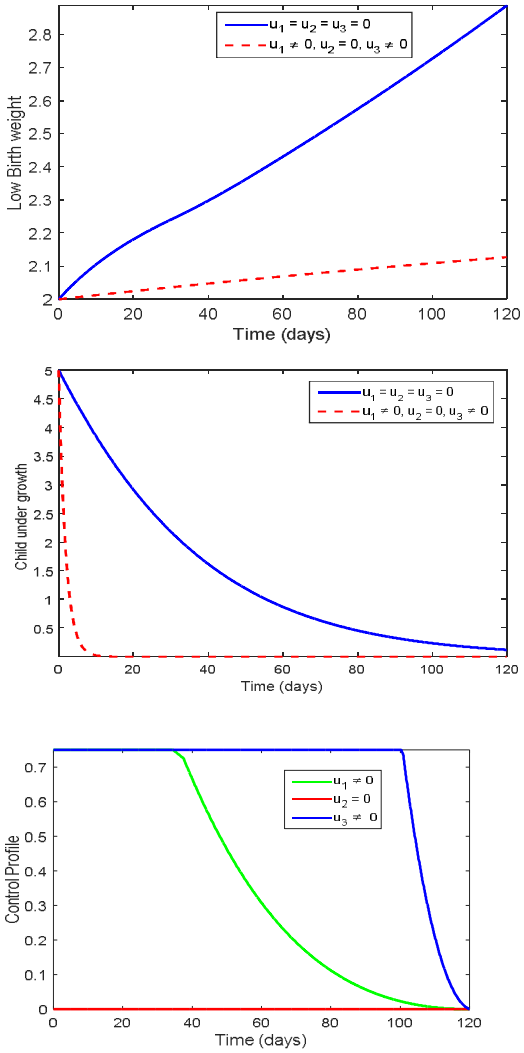


Figure 3: Simulations of the model showing the effect of prevention and treatment only on transmission

C. Treatment $(u_2),(u_3)$ controlonly

The treatment control u_2 and the treatment control u_3 are used to optimize the objective function J at the same time we set the prevention control (u_1) to zero. Thus, the two treatment controlsmechanisms are optimized without prevention. It can be deduced from Figure 4(a) that there is substantial significant difference between Low birth weight B_{LW} with control case and without application of control. This negative impact suggests that no effort is made on preventive mechanism. The necessary attentions have been focused on the treatment mechanism. In Figure 4(b) there is a significant difference between the presence of control and without control case of the number children under growth C_U. This positive effect is due to the effort made in treating the number of children under growth C_U . Therefore, this strategy is very effective in controlling the number of children under growth C_U. Figure 4(c) depicts the control profile for this strategy as control (u_1) is set to zero. It requires 100% of control (u_2) for 20 days and sharply reduce to 25% which is kept constant throughout the rest of the intervention. This effect ensures that pregnant women will take medication that ensure normal birth weight. The control (u_3) is initially set at 100% for 18 days which reduce gradually till the end of the intervention.

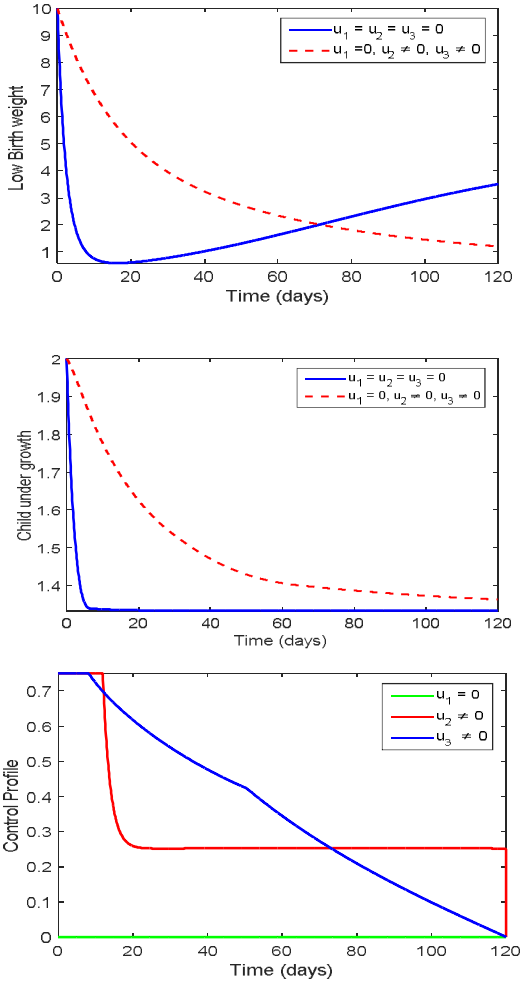
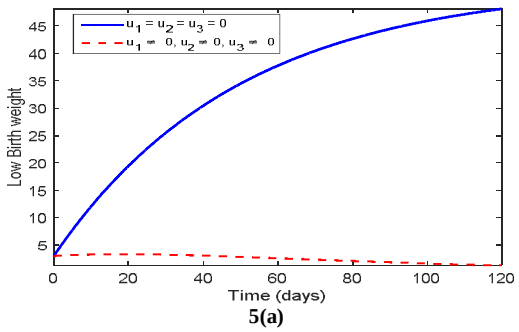


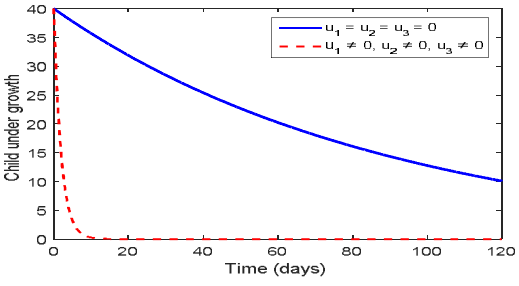
Figure 4: Simulations of the model showing the effect of treatment and insecticide only on transmission

D. Prevention and treatment (u_1, u_2, u_3)

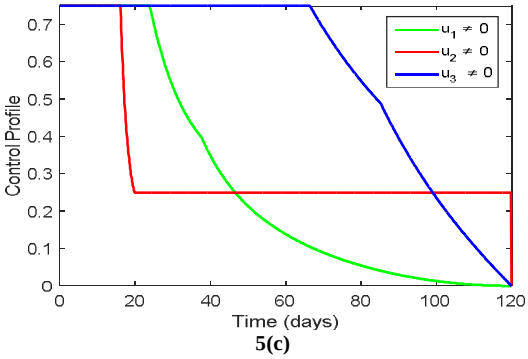
In this strategy, all the control mechanisms (u_1, u_2, u_3) are used to optimize the objective function J. That is, prevention and treatment are optimized simultaneously. There is a vast significant difference between the case controlled and without control case as shown in Figure 5(a). This positive impact observed in Figure 5(a) indicates that the strategy is very effective in controlling the number of Low birth weight incidence B_{LW} by aged women in the communities. This implies that most of the women are taking prenatal care more seriously and implementing them in their various communities. In Figure 5(b) there is also substantial difference between the case controlled and without the control application. The control strategy is more effective in controlling the number of children under growth C_U. This implies that medication given to patients have high efficacy in treating children under growth as shown in Figure 5(b). Figure 5(c) shows the control profile for this strategy as all the controls are set in motion at the same time. In this strategy, the control (u_1) is initially set to 100% for 27 days then minimize gradually during the rest of the intervention. In this mechanism, the control (u_2) is set at 100% for 20 days and reduces to 22% which is kept constant throughout the rest of the intervention. Again, the control (u_3) is maintained 100% for 70 days then gradually reduces during the rest of the intervention as shown in Figure 5(c).



5(a)



5(b)



5(c)

Figure 5: Simulations of the model showing the effect of all controls on transmission

DISCUSSION

4.1 Model interpretation

In this section, we examined and analyzed aged women giving birth and its associated risk using a model which is deterministic in nature. For the model without optimal control, we worked out the basic properties of the model. The basic reproduction number of the model was obtained, and steady states explored.

Optimal control strategy of prevention and treatment were included in the model. The Pontryagin's Maximum Principle was used to characterize the conditions for existence of optimal control to determine the best strategy in reducing the number of Low birth weight as well as children under growth. It was established in the numerical simulation result that putting all effort in treatment would help in the controlling the Low birth weight. However, the best strategy in controlling low birth weight as well as children with under growth is hinged on concentrating all effort on the three controls at the same time.

Even though Low Birth Weight is influenced by a multiplicity of factors, the incidence of LBW could be reversed if maternal risk factors are detected earlier and appropriate prevention strategies are delivered to the high-risk group.From a public health perspective, it is an advantage that most of these factors can be modified.Multi-faceted approaches could deliver better services to the pregnant mothers in Ghana. Such approaches would include health education, maternal nutrition, improvement in socio-economic indices, and more and better quality antenatal care services.

The deterministic model also shows that there is the need to focus on both prevention and treatment controls in dealing with aged women who wish to give birth in order to reduce the risk of giving birth to low birth weight and children under growth. The deterministic optimal control problem existence of optimal control was proved and solved using

Pontryagins Maximum Principle as recently employed by (Gani et al 2017) in comparing deterministic and stochastic optimal control analysis of an SIR epidemic model.

CONCLUSION

Our motivation in this study was further rekindled by the results that old age among women is a major significant factor associated with LBW and undergrowth among children. We explored further using optimal control of prevention and treatment in the model which was deterministic in nature. We employed the Pontryagin's Maximum Principle to characterize the conditions for existence of optimal control to determine the best strategy in reducing the number of Low birth weight as well as children under growth among aged women. It was established in the numerical simulation result that putting all effort in treatment would help in the controlling the Low birth weight as shown in numerical analysis. However, the best strategy in controlling low birth weight as well as children with under growth is hinged on concentrating all efforts on the three controls at the same time.

The results by all the models show that less advantaged women stand a higher risk of giving birth to low birth weight children than more advantaged women. The results of this study suggest that for reducing low birth weight, the strategy needs to focus attention on nutrition education to facilitate better weight gain during pregnancy focusing more on the girl-child education, pre pregnancy preparation among aged women, regular antenatal care visits and discouraging teenage and old age pregnancy and bearing more children as well as formulating policies that will reduce poverty among rural women. The girl child education policy must also be given all the needed resources it requires to achieve the desired set targets.Even though Low Birth Weight is influenced by a multiplicity of factors, the incidence of LBW could be reversed if maternal risk factors are detected earlier and appropriate prevention strategies are delivered to the high-risk group. From a public health perspective, it is an advantage that most of these factors can be modified.

REFERENCES

[1] Fleming, W. H. and Rishel, R. W. (1975). Deterministic and stochastic optimal

[2] control Springer Verlag, New York.

[3] Gani SR, Halawar SV (2017) Deterministic and Stochastic optimal control analysis of an SIR epidemic model. Global journal of pure and applied mathematics vol 13;9

[4] Ghana Statistical Service, GSS (2011) Multiple Indicator Cluster Survey

[5] Hethcote, H. (2000). The mathematics of infectious diseases. SIAM Review. 42(4):

[6] 599-653

[7] Lakshmikantham, V., Leela, S. and Martynyuk, A.A. (1989). Stability analysis of

[8] nonlinear systems. Marcel Dekker, Inc., New York and Basel.

[9] Michael Ofori Fosu, Ir. Peter Romeo Nyarko, Martin Anokye (2013) The desire for last birth among Ghanaian women: the determinants.Research on Humanities and Social SciencesVol.3, No.22

[10] Michael Ofori Fosu, Louis Munyakazi, N.N.N NsowahNuamah (2013) Low Birth Weight and Associated Maternal risk in Ghana.Journal of Biology, Agriculture and HealthcareVol.3, No.7

[11] Michael Ofori Fosu, Stephen B. Twum, Osborne A. Y Jackson (2016) Application of area to point kriging to low birth weight incidence in Ghana.British Journal of Applied Science & Technology15(5): 1-9

[12] Nishiura H, Kinoshita R, Mizumoto K, Yasuda Y. (2016) Transmission potential of Zika virus infection in the South Pacific. International Journal of Infectious Diseases; 45: 9597.

[13] Nita H. Shah, Foram A. Thakkar and Bijal M. Yeolekar (2016) Mathematical Analysis of Optimal Control Theory on Underweight; AIR, 8(5): 1-11

[14] Okosun K. MathematicalEpidemiology of Malaria Disease Transmission and its Optimal Control Analyses (2010) Ph.D. Thesis, University of the Western Cape, South Africa,

[15] Okosun KO,Makinde OD. (2011) Modelingtheimpactofdrugresistance inmalaria transmissionanditsoptimalcontrolanalysis. InternationalJournalofthePhysicalSciences, 6: 6479-6487

[16] Pontryagin LS, Boltyanskii VG, Gamkrelidze RV, Mishchenko EF. (1962) The mathematical theory of optimal processes.Wiley, New York,

[17] Traci C. Johnson, MD (2017)WebMD Medical Reference Reviewed

[18] Van Den Driessche P and Watmough J (2002) Reproduction number and subthreshold endemic equilibria for compartmental models of disease transmission Math. Biosci.180 29–48.

★★★