MODELS FOR LOCATING WIRELESS SENSOR NETWORKS IN THE PRESENCE OF COOPERATIVE COVERAGE

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Abstract - In this study, we consider Wireless Sensor Network (WSN) location problems for area coverage purposes. Using a two-dimensional grid structured area setting, we develop several mathematical models which incorporate cooperative coverage of grid points under various assumptions and sensing models. The portfolio of models presented in this work covers the WSN location model for all eight combinations of continuous or discrete candidate sensor locations, binary or diffuse sensing models, and limited or unlimited resource cases with different objectives.

Index Terms - Location, Optimization, Gradual Sensing, Wireless Sensor Network

I. INTRODUCTION

A Wireless Sensor Network (WSN) comprises of spatially distributed sensors that are generally used for monitoring, sensing, communication, or surveillance purposes [1]. Under a wide range of possible different characteristics, capabilities, assumptions, objectives, and technical constraints WSNs are implemented for various purposes such as infrastructure security, area surveillance, battlefield reconnaissance, target detection, public health protection, and inventory tracking [2].

Coverage is one of the most important performance metrics used for WSNs and is categorized as: area coverage, barrier coverage, and point coverage [2]. In an area-coverage problem, the objective is generally to attain the maximum coverage performance or guarantee a specific coverage level in the area.

In a point coverage setting, on the other hand, the objective is shifted towards the coverage on or around of a finite number of discrete on the plane. Finally, in a barrier coverage problem, a planner seeks to provide a sufficient level of coverage along a border line to prevent intruders penetrating that line segment [3,4].

Location models are also classified according to the possible set of locations for sensors. In continuous location problems sensors can be placed anywhere within the region. In such WSN location problems, the set of candidate locations where sensors can be sited is not discrete but a whole continuous space [5]. In a network problem those locations are restricted to possible locations on a network. Discrete location problems only allow placing sensors to a finite set of candidate sensor locations.

The detection probability of a point (also referred as target) is usually computed by using the sensor's sensing capability and the distance between the target and sensor. Binary sensing models assume that a target is completely covered (detected) if it lies within the detection range of a sensor, otherwise not covered. Generally, the probability of detecting a target of interest decreases as the sensor-target distance increases. Such sensing models are named as diffuse sensing or probabilistic sensing model. Sometimes it is possible that a target can lie within the detection range of multiple diffuse sensing sensors, and the point can be covered by those sensors simultaneously. For such cases, the target is said to be cooperatively covered or detected and the overall detection probability can be computed by using simple probabilistic rules which implement dependent or independent detections by multiple sensors [6].

In this study, implementing the cooperative coverage concept, we provide a portfolio of mathematical models for modeling WSN location schemes under various assumptions and constraints.

Basically we vary the structure of the problem by varying assumptions related to (1) location within the space, i.e. continuous or discrete candidate sensor locations, (2) sensing model, i.e. binary or diffuse sensing, (3) resource constraint, i.e. limited or unlimited sensors. The formulations developed for each combination of the abovementioned assumptions demonstrate their impact on the structure of the mathematical models.

This paper is organized as follows. In Section II, we present preliminaries on cooperative coverage and gradual/ binary sensing models for WSNs. Next, the details of our mathematical models developed for each problem case are presented in Section III. Finally, we present the conclusion of our work as well as some possible future work directions in Section IV.

II. ASSUMPTIONS AND PRELIMINARIES

We assume that the area of interest is a two-dimensional Euclidean plane rectangular region of size a x b and is discretized into grids where the center is of each grid is indexed as $i \in I$.

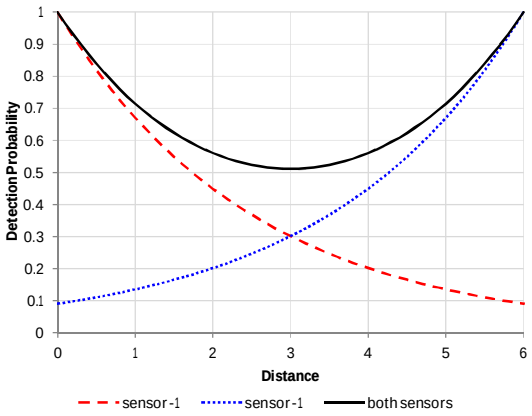


Fig. 1: Detection probability by two sensors

A. Discrete and Continuous Space Locations

In the discrete setting we assume that sensors can only be located to a finite number of pre-specified locations in the plane. In this case the general location problem reduces to the problem of selecting a number of locations among a larger set of candidate locations.

In the continuous case, on the other hand, the sensors are allowed to be located anywhere in the area of interest. The distance between each candidate location and grid in discrete problems is given or calculated with respect to the distance metric whereas it is a part of the decision problem in continuous space problems.

B. Binary and Gradual Sensing Models

In the binary sensing model, it is assumed that the sensing performance of a sensor is perfect and it always detects (covers) a target (grid) within its sensing range, r [7]. This sensing model is important in location science since it enables a basic understanding of the coverage zone, i.e. a disc shaped region of radius r around the sensor. It is also convenient for theoretical derivation and simulation experiments [8].

The diffuse sensing model is generally used for modelling imperfect sensors where the coverage probability of a target changes between 0 and 1 with respect to the sensor-target distance. In such sensing models, it is generally assumed that the gradual sensing performance is a non-increasing non-linear function of distance. Some examples include exponential, cubic, Fermi-type models.

Fig. 2 and Fig. 3 demonstrate exemplary fields containing diffuse and binary sensors, respectively.

C. Cooperative Coverage

Traditional location models assume that a point is covered by only its closest sensor or by the one which can provide the maximum coverage performance [6]. In this work, we account for the cooperative coverage possibility, in which multiple sensors can contribute to the coverage of a grid point simultaneously. This assumption is valid in many real-world applications including networks which include cell phone towers, warning sirens, radars, sonars, or other types of surveillance sensors [9,10].

For diffuse sensors, if detections of different sensors are independent of each other, the cumulative detection probability, CP, of a grid can be simply computed by the equation:

$$CP_i = 1 - \prod_{s \in S} (1 - p_{is}), \forall i \in I \qquad (1)$$

where $i \in I$ and $s \in S$ denotes the sets of grids and sensors, and p_{is} represents the coverage probability of grid i by sensor s. Fig. 1 displays an example detection probability function for two adjacent diffuse sensing sensors 6 units distance apart. It should be noted that, as expected, the cumulative detection probability (computed by eq (1)) of a grid between the two sensors is significantly enhanced by the collaborations of the two sensors.

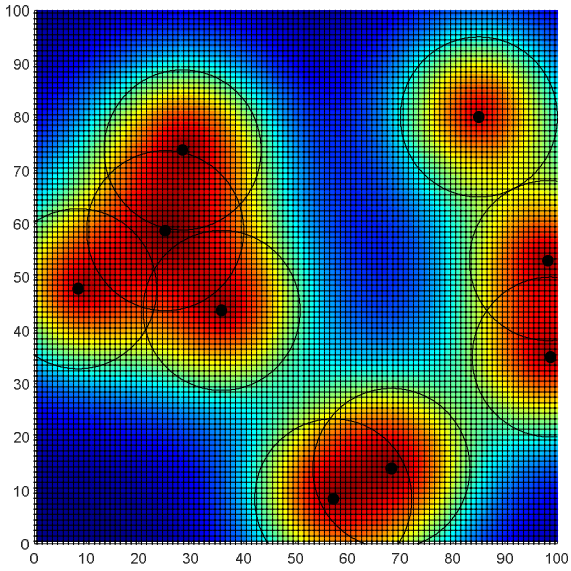


Fig. 2: Exemplary field containing diffuse sensors

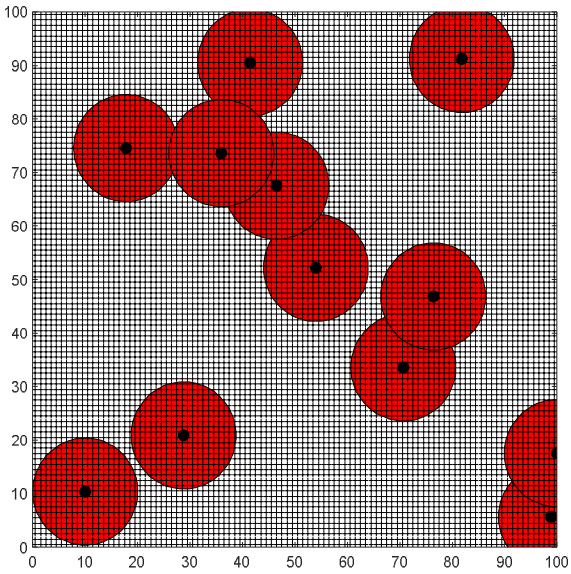


Fig. 3: Exemplary field containing binary sensors

For binary sensors, similar to the traditional approach, a grid is assumed fully covered if it lies within the range of its closest sensor. Many research papers, on the other hand, implement the backup coverage or the

k-coverage concepts which account for multiple coverage of a grid by sensors. These concepts are mostly used in dense sensor fields where a grid can be covered by multiple sensors simultaneously [11].

III. MATHEMATICAL MODELS

In this section we develop formulations for eight different WSN location problems with the assumptions & constraints summarized in Table 1. Considering the three features related to the space, sensing model, and resources, we implement a full factorial design to generate WSN location cases.

Table 1. Assumptions of location problem cases.

Model Name	Continuous/ Discrete Space?	Diffuse/ Binary Sensing?	Limited/ Unlimited Resource?
1-CDL	Continuous	Diffuse	Limited
2-CDU	Continuous	Diffuse	Unlimited
3-CBL	Continuous	Binary	Limited
4-CBU	Continuous	Binary	Unlimited
5-DDL	Discrete	Diffuse	Limited
6-DDU	Discrete	Diffuse	Unlimited
7-DBL	Discrete	Binary	Limited
8-DBU	Discrete	Binary	Unlimited

The optimization models are formulated by using the sets, indices, parameters, decision variables, objective functions and constraint sets explained below.

Sets and Indices:

$i \in I$: set of grid locations
$j \in J$: set of candidate locations
$s \in S$: set of sensors

Parameters:

m_{fix} = number of sensors available. $\left(m_{\text{fix}} = |S|\right)$
n = number of candidate locations for sensors. $\left(n = |J|\right)$
a, b = length and width of the area of interest
r = detection range of a sensor
x_i, y_i = coordinates of grid i
d_{ij} = distance between grid i and location j
$\alpha_{\min}$ = minimum level of cumulative coverage probability required at a grid $(0 \le \alpha_{\min} \le 1)$
$\pi_{\min}$ = minimum fraction of grids required to be covered $(0 \le \pi_{\min} \le 1)$

Decision Variables:

m_{var} = number of sensors to be located
x_s, y_s = coordinates of sensor s
d_{is} = distance between grid i and sensor s
p_{is} = coverage probability of grid i by sensor s
CP_i = cumulative coverage probability of grid i
c_i = 1, if grid i is covered; 0 otherwise
c_{is} = 1, if grid i is covered by sensor s; 0 otherwise
λ_s = 1, if sensor s is located; 0 otherwise

A. Continuous Space Models

In this section it is assumed that sensors can be located anywhere in the region of sizes a and b.

MODEL-1-CDL:

In this model we assume that the number of sensors available is restricted to $m_{\text{fix}} = |S|$ and the decision-maker seeks to maximize the total coverage by appropriately locating the sensors.

Objective function:

$$\max z = \sum_{i \in I} CP_i \quad (2)$$

s.t.

$$d_{is} = f\left(x_i, x_s, y_i, y_s\right), \forall i \in I, s \in S \quad (3)$$

$$p_{is} = f(d_{is}), \forall i \in I, s \in S \quad (4)$$

$$CP_i = 1 - \prod_{s \in S}(1 - p_{is}), \forall i \in I \quad (5)$$

$$0 < x_s < a, \forall s \in S \quad (6)$$

$$0 \le y_s \le b, \forall s \in S \quad (7)$$

$$d_{is}, p_{is} \ge 0, \forall i \in I, s \in S \quad (8)$$

$$CP_i \ge 0, \forall i \subset I \quad (9)$$

The objective function (2) maximizes the total cumulative coverage probability of grids. Constraint sets (3), (4), and (5) define the distance, coverage probability, and cumulative coverage probability values, respectively. Constraint sets (6) and (7) ensure that the sensors are located within the region. Finally, (8) and (9) declare variable domains.

MODEL-2-CDU:

We now assume that the decision-maker wishes to cover each grid in the area of interest with a probability larger than a certain threshold value, $\alpha_{\min}$, and aims to achieve this by utilizing minimum number of sensors. In this case we set $|S|$ to large number.

Objective function:

$$\min z = m_{\text{var}} \quad (2)$$

s.t.

$$d_{is} = f\left(x_i, x_s, y_i, y_s\right), \forall i \in I, s \in S \quad (3)$$

$$p_{is} = f(d_{is}), \forall i \in I, s \in S \quad (4)$$

$$CP_i = 1 - \prod_{s \in S}\left(1 - \lambda_s p_{is}\right), \forall i \in I \quad (5)$$

$$CP_i \ge \alpha_{\min}, \forall i \in I \quad (6)$$

$$m_{\text{var}} \ge \sum_{s \in S} \lambda_s \quad (7)$$

$$0 \le x_s \le a, \forall s \in S \quad (8)$$

$$0 \le y_s \le b, \forall s \in S \quad (9)$$

$$d_{is}, p_{is} \geq 0, \forall i \in I, s \in S \quad (10)$$

$$CP_i \geq 0, \forall i \in I \quad (11)$$

$$\lambda_s \in \{0,1\}, \forall s \in S \quad (12)$$

$$m_{\text{var}} \in \{0,1,2,...\} \quad (13)$$

The objective function (10) seeks to minimize the total number of sensors used while ensuring a certain level of coverage in all grids by constraint set (14). Constraint sets (11), (12), (16) and (17) are equivalent to (3), (4), (6) and (7) respectively. Note that constraint (13) is a slightly modified version of constraint (5) such that it only accounts for the sensor actually located when computing the overall coverage probability of a grid. Constraints (18)-(21) declare variable domains.

MODEL-3-CBL:

In models 1-CDL and 2-CDU we considered diffuse sensing sensors and modeled the detection probability grids by a function $f(.)$ which is generally a non-increasing function of the sensor-grid, d_{is}. Now, we modify this assumption and reformulate the location models by adopting binary sensors.

Objective function:

$$\max z = \sum_{i \in I} c_i \quad (22)$$

s.t.

$$d_{is} = f(x_i, x_s, y_i, y_s), \forall i \in I, s \in S \quad (23)$$

$$0 \leq x_s \leq a, \forall s \in S \quad (24)$$

$$0 \leq y_s \leq b, \forall s \in S \quad (25)$$

$$\sum_{s \in S} \lambda_s \leq m_{\text{fix}} \quad (26)$$

$$c_{is} d_{is} \leq \lambda_s r, \ \forall i \in I, s \in S \quad (27)$$

$$c_i \leq \sum_{s \in S} c_{is}, \ \forall i \in I \quad (28)$$

$$d_{is}, p_{is} \geq 0, \forall i \in I, s \in S \quad (29)$$

$$c_i \in \{0,1\}, \ \forall i \in I \quad (30)$$

$$c_{is} \in \{0,1\}, \ \forall i \in I, s \in S \quad (31)$$

$$\lambda_s \in \{0,1\}, \forall s \in S \quad (32)$$

The objective function (22) maximizes the total number of grids covered by at least one sensor. Constraint sets (23), (24) and (25) are equivalent to (3), (6) and (7) respectively. Constraint (26) ensures that total number of sensors used is less than available resources. Working together with (28) and (22), constraint set (27) fulfills the detection criteria by allowing take a value of 1 only if the distance between sensor s and grid i is shorter than r and sensor s is located to Note that, in constraint (28) can take a value of 1 only if for at least one of the sensors s. Constraint sets (29)-(32) declare variable domains.

MODEL-4-CBU:

We now consider relax the fixed sensor number assumption, and similar to Model-2-CDU, aim to minimize the number of sensors needed to achieve a certain coverage probability. Since the binary sensing model only allows for 1 or 0 coverage per grid, in this model we consider an average coverage performance requirement instead of individual grid performances. In particular our model ensures that at least a certain fraction of grid points, $\pi_{\min}$, are covered.

Objective function:

$$\min z = m_{\text{var}} \quad (33)$$

s.t.

$$d_{is} = f(x_i, x_s, y_i, y_s), \forall i \in I, s \in S \quad (34)$$

$$0 \leq x_s \leq a, \forall s \in S \quad (35)$$

$$0 \leq y_s \leq b, \forall s \in S \quad (36)$$

$$m_{\text{var}} \geq \sum_{s \in S} \lambda_s \quad (37)$$

$$c_{is} d_{is} \leq \lambda_s r, \ \forall i \in I, s \in S \quad (38)$$

$$c_i \leq \sum_{s \in S} c_{is}, \ \forall i \in I \quad (39)$$

$$\sum_{i \in I} c_i \geq \pi_{min} \quad (40)$$

$$d_{is} \geq 0, \forall i \in I, s \in S \quad (41)$$

$$c_i \in \{0,1\}, \ \forall i \in I \quad (42)$$

$$c_{is} \in \{0,1\}, \ \forall i \in I, s \in S \quad (43)$$

$$\lambda_s \in \{0,1\} \quad (44)$$

The objective function (33) seeks to minimize the total number of sensors used. Constraint sets (34), (35), (36), (37), (38), (39) are equivalent to (3), (6), (7), (15), (27) and (28), respectively. Constraint (40) ensures that at least fraction of the area is covered. Constraint sets (41)-(44) declare variable domains.

B. Discrete Space Models

In this section we assume that sensors can only be located to a finite set of candidate locations.

MODEL-5-DDL:

In this first case of discrete space models, the location problem reduces to the selection of the m_{fix} sensor locations among n predefined candidate locations in the region. Different from the continuous model formulations, in this series of formulations we use the set $j \in J$ which represent the set of candidate locations.

Objective function:

$$\max z - \sum_{i \in I} CP_i \quad (45)$$

s.t.

$$p_{ij} = f(d_{ij}), \forall i \in I, j \in J \quad (46)$$

$$CP_i - 1 - \prod_{j \in J} (1 - \lambda_j p_{ij}), \forall i \in I \quad (47)$$

$$\sum_{j \in J} \lambda_j \leq m_{\text{fix}} \quad (48)$$

$$p_{ij} \geq 0, \forall i \in I, j \in J \quad (49)$$

$$CP_i \geq 0, \forall i \in I \quad (50)$$

$$\lambda_j \in \{0,1\}, \forall j \in J \quad (51)$$

The objective function (45) maximizes the overall cumulative detection probability of grids. Constraint sets (46) and (47) defines the detection, and cumulative detection probabilities, respectively. Similar to constraint (26), constraint (48) ensures that total number of sensors located is less than resource amount. Constraint sets
(29)-(32) declare variable domains. Note that in discrete space models the distances between candidate sensors locations and grids are no longer decision variables but parameters.

MODEL-6-DDU:

Now we relax the constraint on the number of sensors and solve the discrete location problems with unlimited number of binary sensors.

Objective function:

$$\min z = m_{\text{var}} \quad (52)$$

s.t.

$$p_{ij} = f(d_{ij}), \forall i \in I, j \in J \quad (53)$$

$$CP_i = 1 - \prod_{j \in J} (1 - \lambda_j p_{ij}), \forall i \in I \quad (54)$$

$$m_{\text{var}} \geq \sum_{j \in J} \lambda_j \quad (55)$$

$$CP_i \geq \alpha_{\min}, \forall i \in I \quad (56)$$

$$p_{ij} \geq 0, \forall i \in I, j \in J \quad (57)$$

$$CP_i \geq 0, \forall i \in I \quad (58)$$

$$\lambda_j \in \{0,1\}, \forall j \in J \quad (59)$$

The objective function (52), together with constraint (55) minimizes the total number of sensors used. Constraint sets (53), (54) and (56) are equivalent to (46), (47) and (14), respectively. Constraint sets (57)-(59) declare variable domains.

MODEL-7-DBL:

Now we model a discrete case sensor location problem with limited number of binary sensors. With its current form the WSN location problem is equivalent to the Maximal Coverage Location Problem (MCLP) first defined by Church and ReVelle in 1974 [12]. Similarly, MCLP seeks to locate a given number of facilities with the objective of maximizing the demand covered within a specific service distance. To achieve this, in addition to the notation defined above, we define the additional set as $N_i = \{ j \in J \,|\, d_{ij} \leq r \}$.

Objective function:

$$\max z = \sum_{i \in I} c_i \quad (14)$$

s.t.

$$\sum_{j | j \in N_i} \lambda_j \geq c_i, \ \forall i \in I \quad (15)$$

$$\sum_{j \in J} \lambda_j \leq m_{\text{fix}} \quad (16)$$

$$\lambda_j \in \{0,1\}, \forall j \in J \quad (17)$$

$$c_i \in \{0,1\}, \forall i \in I \quad (18)$$

The objective function (14) maximizes the total number of grids covered within the range of sensors. Constraint set (15) allow λ to take the value 1 only when one or more sensors are located in the set N. The number of sensors used is cannot be more than m_{fix} in constraint (16). Constraint sets (17) and (18) declare variable domains.

MODEL-8-DBU:

We now modify MODEL-7-DBL such that there are unlimited number of sensors and the decision-maker seeks to minimize the number of sensors needed to achieve a certain level of coverage.

Objective function:

$$\min z = m_{\text{var}} \quad (65)$$

s.t.

$$m_{\text{var}} \geq \sum_{j \in J} \lambda_j \quad (66)$$

$$\sum_{j | j \in N_i} \lambda_j \geq c_i, \ \forall i \in I \quad (67)$$

$$\sum_{i \in I} c_i \geq \pi_{min} \quad (68)$$

$$\lambda_j \in \{0,1\}, \forall j \in J \quad (69)$$

$$c_i \in \{0,1\}, \forall i \in I \quad (70)$$

The objective function (65) minimizes the total number of sensors utilized. Constraint set (66), (67) and (68) are equivalent to (55), (61) and (40), respectively. allow λ to take the value 1 only when one or more sensors are located in the set N. The number of sensors used is cannot be more than in constraint (62). Constraint sets (63) and (64) declare variable domains.

CONCLUSION

This work considered continuous and discrete WSN location problems for an area coverage application and developed mathematical model formulations for their variants. Assuming a two-dimensional area of interest, we discretized the region into a grid structure and for each grid allowed cooperative coverage by multiple sensors. The variants of each discrete and continuous cases are formed by combinations of binary or diffuse sensors and limited or unlimited resources. When the resources are limited, the objective is to maximize the coverage, whereas it is to minimize the number of sensors utilized while ensuring a specific coverage level when the resources are unlimited. Although most of the models mentioned here are studied individually

by several researchers, this is the first work which presents a portfolio of WSN area coverage location models and their basic differences in terms of mathematical modelling perspectives.

Disclaimer - The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of any affiliated organization or government.

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