SYMMETRIC AND ANTI-SYMMETRIC SOLUTIONS FOR THE POTENTIALS OF STANDING RELATIVISTIC BRIGHT SOLITONS

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Abstract- Investigation of the standing relativistic bright solitons in two species plasmas is the main aim of this study. We derive a set of differential equations involving the scalar and vector potentials to describe the evolution of the system. Specially, standing electromagnetic solitary waves for the case of ultra relativistic two fluid plasma is obtained analytically. Also, the effect of fluid velocity at infinity on the profiles of the solitary waves is studied. It is shown that for a fixed value of the fluid velocity excited modes with more zeros in the profile of the vector potential show a higher magnitude for the scalar potential. An increase in the plasma fluid velocity also increases the magnitude of the scalar potential.

Keywords- Standing Soliton, Symmetric and Anti-symmetric Solitons, Relativistic Soliton

I. INTRODUCTION

The theoretical investigation of relativistic solitons in electron–ion plasmas is a relatively old problem in plasma physics, which has been treated by many authors in the past, and has recently gained new attention in [1–4]. The analyses have been performed mainly in the framework of the one-dimensional relativistic fluid approximation, in which solitons are described by the solutions of a set of coupled nonlinear differential equations for the electrostatic and electromagnetic potentials with suitable boundary conditions [5, 6].

Relativistic solitons have been seen in multi-dimensional particle-in-cell (PIC) as well as in fluid simulations of laser pulse interaction with the plasma [7, 8]. These solitons consist of electron density depressions and intense electromagnetic field concentrations with larger amplitude and a lower frequency than those of the laser pulse. Since a significant portion of the overall electromagnetic energy is trapped in the form of solitons, solitary waves can play an important role in the laser–plasma interaction.

Formation of solitary waves and solitons depends on a balance between the nonlinearity and dispersion of the system [9]. The former enhances the wave amplitude and narrows the wave width, while the latter has an opposite effect. Usually in nonlinear plasma there exist two types of envelope solitary waves. One is that the envelope solitary waves can be obtained by a quasi-static approximation (QSA) [10] described by an ordinary differential equation and the other is the pulsing solitary waves which can be described by a nonlinear Schrödinger (NLS) partial differential equation [11].

The corresponding solitary waves in pure electron positron plasma have been researched by many authors[12-15]. Among them some publications claimed that there is no solitary wave in neutral or quasi-neutral plasma. For example, in Ref. [7], authors have pointed out that the symmetric ponderomotive for electrons and positrons can not excite the formation of solitary waves, while the asymmetry of ponderomotive of electrons/positrons and ions allows the existence of them.

In this paper we investigate the interaction between hot electron-positron plasmas and an ultra-relativistic laser beam. A non linear coupled differential equation for the scalar and vector potential is obtained analytically. Also, It is shown that for a fixed value of the fluid velocity excited modes with more zeros in the profile of the vector potential show a higher magnitude for the scalar potential. An increase in the plasma fluid velocity also increases the magnitude of the scalar potential.

II. BASIC EQUATIONS

We describe our system by the continuity and momentum-balance equations for hot electrons and positrons

$$\frac{\partial}{\partial t}\left(n_\alpha \gamma_\alpha\right)+\nabla\cdot\left(n_\alpha \gamma_\alpha \mathbf{v}_\alpha\right)=0 \tag{1}$$

$$\frac{\partial}{\partial t}\mathbf{p}_\alpha+\left(\mathbf{v}_\alpha\cdot\nabla\right)\mathbf{p}_\alpha=-\frac{\nabla w_\alpha}{\gamma_\alpha}+q_\alpha\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_\alpha\times\mathbf{B}\right) \tag{2}$$

in which

$$\mathbf{p}_\alpha=\left(\frac{\gamma_\alpha w_\alpha}{c^2}\right)\mathbf{v}_\alpha \tag{3}$$

is the kinetic momentum, n the particle density, $\mathbf{v}$ the fluid velocity, $\gamma=\left(1-v^2/c^2\right)^{-1/2}$ the Lorentz relativistic factor, c the speed of light in vacuum, q_α the electric charge ($q_\alpha=-e$ for electrons, $q_\alpha=+e$

for positrons) and $\mathbf{E}$, $\mathbf{B}$ are the electric and magnetic fields of the wave , respectively . α represents the two species of the plasma, e for the electrons and p for the positrons; w is the enthalpy per fluid particle, $w = mc^2 + \frac{\Gamma}{\Gamma - 1}\frac{p}{n}$, where the adiabatic index has been denoted by Γ ($4/3$ for the ultra-relativistic limit and to $5/3$ for the non-relativistic case), p is the pressure and m is the rest mass of electrons and positrons.

We consider the case of a standing solitary wave. In other words, we search for a relativistic nonmoving localized structure. We obtain expressions for the momentums, the particle number densities and the relativistic factors as functions of potentials and enthalpy

$$p_{\alpha x} = \sqrt{\left(\sigma_\alpha \phi - \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_\alpha^2 - a^2} \qquad (4)$$

$$n_\alpha = \frac{\gamma_0 \beta_{x_0} w_\alpha}{\sqrt{\left(\sigma_\alpha \phi - \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_\alpha^2 - a^2}} \qquad (5)$$

$$\gamma_\alpha = \frac{\frac{\gamma_0 w_0}{mc^2} - \sigma_\alpha \phi}{w_\alpha} \qquad (6)$$

In other hand, the vector potential turns out to satisfy the following equation

$$a_{\xi\xi} + \omega^2 a = -\frac{m_e c^2}{w_0}\sum_\alpha \sigma_\alpha \gamma_\alpha n_\alpha v_{\alpha\perp} \qquad (7)$$

Also, the second equation associated with the plasma slow motion can be derived from Poisson's equation

$$\phi_{\xi\xi} = -\frac{m_e c^2}{w_0}\sum_\alpha \sigma_\alpha \gamma_\alpha n_\alpha \qquad (8)$$

Substituting equations (5-6) in equations (7-8) gives the following set of equations

$$a_{\xi\xi} + \omega^2 a = \gamma_0 \beta_{x_0}\left[\frac{1}{\sqrt{\left(\phi + \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_{e^-}^2 - a^2}} + \frac{1}{\sqrt{\left(\phi - \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_{e^+}^2 - a^2}}\right] a \qquad (9)$$

$$\phi_{\xi\xi} = \gamma_0 \beta_{x_0}\left[\frac{\left(\phi + \frac{\gamma_0 w_0}{mc^2}\right)}{\sqrt{\left(\phi + \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_{e^-}^2 - a^2}} - \frac{\left(\frac{\gamma_0 w_0}{mc^2} - \phi\right)}{\sqrt{\left(\phi - \frac{\gamma_0 w_0}{mc^2}\right)^2 - w_{e^+}^2 - a^2}}\right] \qquad (10)$$

We try to find the enthalpy as a function of the potentials a and ϕ. Through combining equation (5) and the relation $w_\alpha = n_\alpha^{1/3}$ for the ultra-relativistic case, we have

$$w_\alpha^6 + \left(a^2 - \varphi_\alpha^2\right) w_\alpha^4 + \left(\frac{mc^2}{w_0}\right)^6 \gamma_0^2 \beta_{x_0}^2 = 0 \qquad (11)$$

where $\varphi_\alpha = \sigma_\alpha \phi - \gamma_0$.

III. DISCUSSIONS ON THE SOLUTIONS

The roots of equation (23) can be easily found using the solutions of a cubic equation. The roots, in terms of the coefficients can be expressed as

$$w_{\alpha_1}^2 = -\frac{4}{3}\left(\varphi_\alpha^2 - a^2\right)\left[\sin\left(\frac{\pi}{6} + \frac{\theta_\alpha}{6}\right)\sin\left(\frac{\pi}{6} - \frac{\theta_\alpha}{6}\right)\right] \qquad (12)$$

$$w_{\alpha_2}^2 = \frac{4}{3}\left(\varphi_\alpha^2 - a^2\right)\left[\cos\left(\frac{\theta_\alpha}{6}\right)\cos\left(\frac{\theta_\alpha}{6} + \frac{\pi}{3}\right)\right] \qquad (13)$$

$$w_{\alpha_3}^2 = \frac{4}{3}\left(\varphi_\alpha^2 - a^2\right)\left[\cos\left(\frac{\theta_\alpha}{6}\right)\cos\left(\frac{\theta_\alpha}{6} - \frac{\pi}{3}\right)\right] \qquad (14)$$

where

$$\theta_\alpha = \tan^{-1}\left(\frac{\sqrt{\Delta}}{2\left(a^2 - \varphi_\alpha^2\right)^3 + 27\gamma_0^2\beta_{x_0}^2\left(\frac{mc^2}{w_0}\right)^6}\right) \qquad (15)$$

and

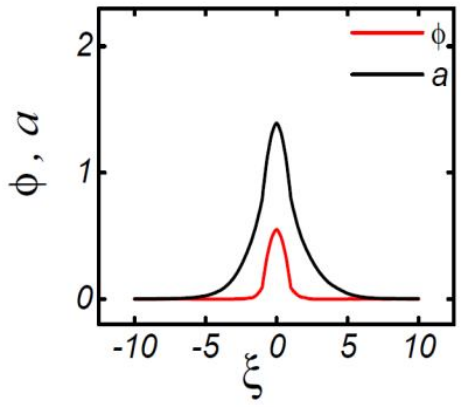
$$\Delta = 27\gamma_0^2\beta_{x_0}^2\left(\frac{mc^2}{w_0}\right)^6\left[4\left(\varphi_\alpha^2 - a^2\right)^3 - 27\gamma_0^2\beta_{x_0}^2\left(\frac{mc^2}{w_0}\right)^6\right] \quad (16)$$

IV. NUMERICAL ANALYSIS

It is clear that the set of the equations for the scalar and the vector potential cannot be solved analytically. Therefore, we attempt to solve them numerically to find the immobile localized patterns. Equations 9 and 10 are reversible under the transformation $(\xi \to -\xi,\ \phi \to \phi)$ and $(\xi \to -\xi,\ a \to \pm a)$. We therefore look for symmetric solutions for the electrostatic potential while looking for symmetric and anti-symmetric solutions for electromagnetic potential. In particular, we search for the single hump solutions for ϕ and single and multi-hump solutions for a. We denote the number of zeros in the vector potential with p. To this end, we integrate the set of equations with initial conditions so that at $\xi = 0$, we have $\phi' = 0$ and $a = 0$ for an odd number of zeros of the a profile and $\phi' = 0$ and $a' = 0$ for an even number of zeros of the a profile.

Figure 1 shows the variations of the scalar and vector potentials for the fixed value of $\beta_{x_0} = 0.43$, $\omega = 1$ and for different values of plasma temperatures.

Solutions with zero ($p = 0$), one ($p = 1$) and two ($p = 2$) nodes in the profile of the vector potential are shown. The $p = 0$ case occurs at higher temperature value. Cases for more nodes appear at lower temperature values. The first anti-symmetric mode ($p = 1$) takes place at a lower temperature than what is needed for the first symmetric mode ($p = 2$). We also note that as p increases, the magnitude of the scalar potential rises. Note that $p = 0$ mode is excited at high temperature, but other modes are excited with an increase in temperate as p increases.



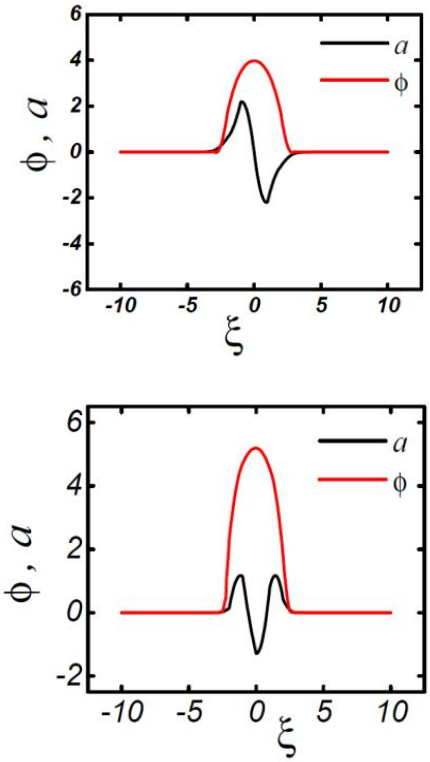
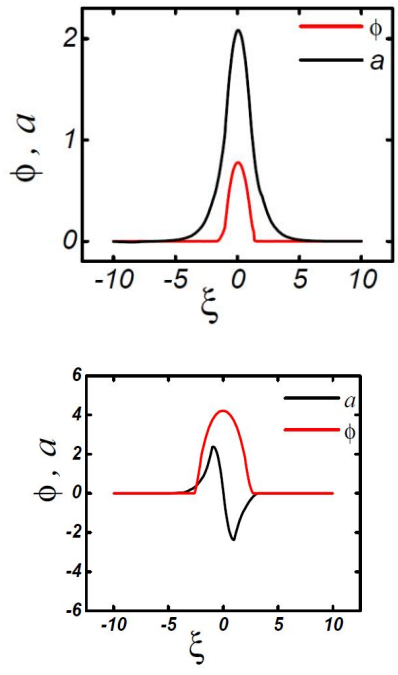


Fig.1: The variations of the electrostatic and electromagnetic potentials with $\beta_{x_0} = 0.43$ and $\omega = 1$ for different values of the plasma temperatures.

For comparison we have also shown the solutions for the fixed value of $\beta_{x_0} = 0.62$ in Fig.2. As the fluid velocity of the plasma increases, the magnitude of the electrostatic potential also slightly increases.

Furthermore, for higher values of the fluid velocity the solutions emerge at lower temperature values.



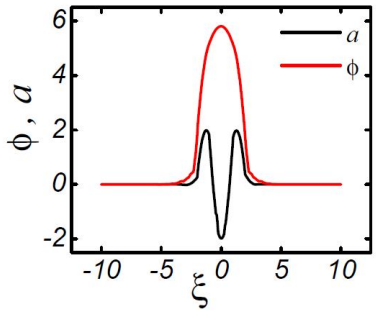


Fig.2: The variations of the electrostatic and electromagnetic potentials with $\beta_{x_0} = 0.62$ and $\omega = 1$ for different Values of the plasma temperatures.

CONCLUSION

We have studied the propagation of an intense laser in plasma consisting of hot electrons and positrons. We have presented a second- order coupled system of differential equations describing the evolution of nonlinear electromagnetic waves for two-fluid plasma. No restriction on the magnitudes of the flow velocity and the temperature of each species has been assumed. We have searched for the standing relativistic solitons. To this end, we have assumed a relativistic system in which the plasma is not at rest at infinity. We have discussed the acceptable solutions of the standing electromagnetic solitons in the hot ultra-relativistic electron positron plasmas. Acceptable solutions are presented and discussed in detail. Single hump symmetric solutions for the scalar potential and single and multi-node symmetric and anti-symmetric solutions for the vector potential are presented. The dependence of the solutions on the fluid velocity and temperature was also demonstrated.

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REFERENCES

[1] S. I. Popel, S. V. Vladimirov, and P. K. Shukla, Phys. Plasma 2, 716 (1995).

[2] P. K. Shukla, A. A. Mamun, and L. Stenflo, Phys. Scr. 68, 295 (2003).

[3] Q. Haque and H. Saleem, Phys. Scr. 69, 406 (2004).

[4] C. C. Hua, B. S. Xie, and K. F. He, Chaos, Solitons Fractals 25, 1161 (2005).

[5] B. S. Xie and S. C. Du, Phys. Plasma 13, 074504 (2006).

[6] B. S. Xie and S. C. Du, Phys. Scr. 74, 638 (2006).

[7] V. I. Berezhiani, V. Skarka, and S. Mahajan, Phys. Rev. E 48, R3252 (1993).

[8] V. I. Berezhiani, M. Y. El-Ashry, and U. A. Mofiz, Phys. Rev. E 50, 448 (1994).

[9] H. Kakati and K. S. Goswami, Phys. Plasma 7, 808 (2000).

[10] D. Farina and S. V. Bulanov, Phys. Rev. Lett. 86, 5289 (2001).

[11] D. Farina and S. V. Bulanov, Phys. Rev. E 64, 066401 (2001).

[12] G. Lehmann and K. H. Spatschek, Phys. Rev. E 83, 036401 (2011).

[13] A. Rasheed, N. L. Tsintsadze, and G. Murtaza, Phys. Plasma 18, 112701 (2011).

[14] N. L. Tsintsadze, R. Chaudhary, and A. Rasheed, J. Plasma Phys. 79, 587 (2013).

[15] D. Lu, Z. L. Li, and B. S. Xie, Phys. Rev. E 88, 033109 (2013).

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